ON THE APPLICABILITY OF DYNAMIC FACTOR MODELS FOR MACROECONOMIC FORECASTING IN ARMENIA

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Abstract: In this paper, we are trying to find out whether advanced short-term forecasting techniques that have been proved successfully applied in advanced countries can be employed to a developing country to forecast key macroeconomic variables. We compare the forecasting performance of factor-augmented models such as FAAR, FAVAR and Bayesian FAVAR with its small-scale benchmark counterpart models (AR, VAR and Bayesian VAR). Based on the out-of-sample recursive forecast evaluations and using RMSFE's, we conclude that when we apply advanced forecasting techniques to forecast real GDP growth then in the prevailing part of experiments the factor-augmented models outperform small-scale benchmark models, but when we apply to forecast inflation, then we conclude that factor-augmented models outperform small-scale benchmark models only in a few cases. Therefore, factor-augmented models can be included in the forecasting practice at the Central Bank of Armenia mainly to improve forecasting accuracy for the real GDP growth. From the other side the application of these models can be extended to any other developing country without large modifications.

Keywords: small-scale models, factor-augmented models, static and dynamic factors, recursive and rolling regression scheme, out-of-sample forecast evaluation, Armeni

JEL classification codes: E17, E37, C11, C15, C32, C53, C55

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1. Introduction

Forecasting the key macroeconomic variables such as real GDP growth and inflation, has always been one of the key research interests in macroeconomics. Empirical studies for advanced economies have a long history and the forecasting methodology is being improved constantly in the recent years. For example, since the early 2000's, factor models are became a popular tools for Central Banks for producing short-term forecasts. There are many applications of factor models to forecasting macroeconomic and financial variables. For example Forni et al., (2000), Stock and Watson (2002), Artis et al., (2002) Schneider and Spitzer (2004), Matheson (2006), Gupta and Kabundi (2009), Cristadoro et al. (2005). The main finding of these applications is that the forecasts generated from the factor-augmented models are superior to traditional small-scale benchmark models, like AR and VAR. Of course, these results are informative, but it does not mean that we can automatically apply these results to other economies, since they are based on the country specific data. In the current paper, we want to consider the applicability of the factor-augmented models to developing economies and particularly to Armenian macroeconomic variables. For that, we consider well-known small-scale models, namely Univariate autoregression, vector autoregression and Bayesian vector autoregression (hereafter AR, VAR and BVAR) and their factor-augmented counterpart, particularly, Factor-augmented autoregression, Factor-augmented vector autoregression and Bayesian Factor-augmented vector autoregression (hereafter FAAR, FAVAR and BFAVAR).

As a rule, the factor-augmented models can be constructed in two steps: factor extraction, followed by model estimation and forecasting. Following Barhoumi, Darne & Ferrara, 2014 there are three main algorithms for extracting factors, namely static principal component as in Stock and Watson, (2002), dynamic principal components estimated in the time domain, as in Doz, Gianonne and Reichlin (2011, 2012) and dynamic principal components in the frequency domain, as in Forni, Hallin, Lippi & Reichlin (2000, 2004). All mentioned methods for factor extraction have the same purpose, namely, given a large number of initial variables, to extract only a small number of factors, which summarize the most part of information contained in the whole dataset. In this paper, we use all mentioned methods to extract the dynamics of unobserved factors. After extracting the unobservable factors in a usual manner, they included into standard small scale forecasting models such as AR, VAR and Bayesian VAR and then using them we can produce the forecasts for the key macroeconomic variables.

To extract the dynamics of the factors we using Armenian actual quarterly macroeconomic time series from 1998Q1 to 2018Q4. In the additional dataset included 40 macroeconomic variables, comprising information on national accounts and consumer price indices, labor force variables, monetary and financial variables and international macroeconomic variables. The main sources for our dataset is the Central Bank of Armenia (https://www.cba.am/) and the National Statistical Agency (https://www.armstat.am/) internal databases as well as external source databases, like OECD (https://data.oecd.org/) and IndexMundi (https://www.indexmundi.com/). Using these additional macroeconomic time series, we calculate the dynamics of unobservable factors with help of static and two dynamic algorithms (time and frequency domain). After extracting the dynamics of factors, we estimate the unknown parameters of the AR, VAR and Bayesian VAR models. Then we design out-of-sample forecast evaluation experiments based on the recursive regression scheme. As a result of out-of-sample forecast evaluation we calculate root mean squared forecast error (RMSFE) indices. The indices we calculate for all competing models included in our analysis. To keep robustness of our conclusions we conduct out-of-sample forecast experiments for different lag lengths and various combinations of dynamic and static factors. Based on the out-of-sample forecast evaluations and using calculated RMSFE indices we conclude that factor-augmented models are outperform small-scale benchmark models, especially when we apply for forecasting the real growth of GDP. Therefore, these methods can be included in the system of the short-term forecasting models in the Central Bank of Armenia to improve real GDP growth forecasts. We also conclude that these methods without significant changes can be applied to other developing economies.

The remaining paper is organized as follows. In section 2 we briefly present the main idea of the forecasting models, namely AR, VAR and Bayesian VAR as well as we explain the steps of extracting unobservable factors by above mentioned three methods. In section 3 we present the dynamics of actual macroeconomic variables and give some explanations for their fluctuations during last years. In this section, we also consider the preliminary treatment of the additional explanatory variables, which we uses for extraction of unobservable factors. In this section, we also analyze some descriptive statistics. In section 4, we explain in details the experimental design that we use for out-of-sample forecast evaluation. Here we explain the steps of recursive regressions scheme and explain how to calculate the RMSFE indices. In section 5 we present some additional experiments that we have conducted to keep robustness of our conclusions. In this section, we also present the results of out-of-sample recursive forecast evaluations both for real GDP growth and for inflation. Last section concludes.

2. Models

In this section, we present the basic forecasting models, particularly AR, VAR and Bayesian VAR and their factor-augmented counterpart models, in particular FAAR, FAVAR and Bayesian FAVAR. The three small-scale models we use in order to evaluate the out-of-sample forecast performances of the three factor-augmented models. Below we briefly present the main characteristics of the mentioned models in turn.

It is well known that the univariate AR model can be estimated by using the following regression model: $y_t = c + \sum_{j=1}^{p} \rho_j y_{t-j} + \varepsilon_t$. The unknown parameters of the model can be consistently estimated by using traditional OLS algorithm.

We estimate an unrestricted vector autoregressive model $y_t = A_0 + A(L) y_t + \varepsilon_t$, where y_t is a $(n \times 1)$ vector of variables to be forecasted, A_0 is a $(n \times 1)$ vector of constant terms, $A(L)$ is a $(n \times n)$ polynomial matrix in the backshift operator L with lag length p, ε_t is a $(n \times 1)$ vector of error terms. In our case we assume that $\varepsilon_t \sim N\left(0, \sigma^2 I_n\right)$, where I_n is a $(n \times n)$ identity matrix. The unknown parameters of the VAR model can be consistently estimated by using traditional OLS algorithm. However, from the other side in the VAR model very often we need to estimate many parameters. This over parametrization could cause inefficient estimates and hence a large out-of-sample forecast error. Thus, to overcome this over parametrization we also implement the BVAR algorithm. In order to use BVAR first we need to identify the priors. In this paper, we uses the "Minnesota" type priors according of which the prior mean and standard deviation of the BVAR model can be set as follows:

1. The parameters of the first lag of the dependent variables follow an AR(1) process while parameters for other lags are equal to zero.
2. The variances of the priors can be specified as follows:

$$\left(\frac{\lambda_1}{l^{\lambda_3}}\right)^2 \text{ if } i = j, \quad \left(\frac{\sigma_i \lambda_1 \lambda_2}{\sigma_j l^{\lambda_3}}\right) \text{ if } i \neq j, \quad \left(\sigma_1 \lambda_4\right)^2 \text{ for the constant term}$$

Where, i refers to the dependent variable in the j-th equation and j to the independent variables in that equation, σ_i and σ_j are standard errors from AR regressions estimated via OLS. The ratio of σ_i and σ_j controls for the possibility that variable i and j may have different scale (l is the lag length). The λ's set by the researcher, that control the tightness of the prior. Thus, having "Minnesota" type priors it is possible to calculate the posterior parameters using Bayesian approach to estimation (Hamilton, 1994, pp. 351-371).

As it was mentioned above in this paper, we mainly concentrate on the forecasting performance of the large-scale forecasting models, such as FAAR, FAVAR and BFAVAR. Unlike small-scale benchmark models, the large-scale factor-augmented models are includes static or dynamic factors. As a rule the factor-augmented models are estimated in two steps, at the first step, we estimate the dynamics of unobservable factors using static and two dynamic approaches and then at the second step we estimate factor-augmented model and producing forecasts. The question is now how to determine the unobservable factors?

There are three main algorithms for extracting factors, namely 1. The static principal components as in Stock and Watson (2002) 2. The dynamic principal component (frequency domain) approach as in Forni et al. (2005) and 3. The dynamic principal component approach (time domain) as in Doz et al. (2011, 2012). There are a number of papers where in very details are presented the computational steps of the mentioned factor models (Barhoumi, Darne & Ferrara, 2014)[3]. The Stock and Watson approach consist of deriving the static principal components using variance-covariance matrix of the additional vector of time series. The Doz, Gianonne & Reichlin uses state-space model and Kalman filter to extract the dynamic principal components. The Forni, Hallin, Lippi & Reichlin approach estimate the dynamic principal components using spectral density matrix of the data. In our paper to extract the dynamics of principal components, we uses all three mentioned approaches.

3. Data and descriptive statistics

For estimating small-scale benchmark models, namely AR, unrestricted VAR and BVAR we use the following four macroeconomic time series, particularly real GDP growth, CPI inflation, short-term nominal interest rate and unemployment rate. To select the macroeconomic time series to be included in the small-scale benchmark models we closely follow the paper by Pirshel and Wolters (2014). Thus, our dataset includes four key macroeconomic variables, which we mainly use in the small-scale benchmark models (AR, VAR and BVAR). Besides, of the key macroeconomic variables our dataset also includes 40 additional set of macroeconomic variables, which we mainly use to extract the dynamics of unobservable factors. We select 40 additional macroeconomic variables because studies have shown that smaller datasets with includes about 40 series outperform larger datasets with disaggregated data, with more than 100 series (Bai and Ng, 2002, Watson, 2003, Boivin and Ng, 2006, Barhoumi, Darne & Ferrara, 2014). Our dataset is balanced and it start with 1996Q1 to 2018Q4, quarterly time series. Each variable in our dataset includes 92 observations.

Now let's first present the dynamics of the four key macroeconomic variables and then to present the dynamics of additional 40 macroeconomic variables, that we using for factor extraction. The dynamics of the four key macroeconomic variables are presented in below Figures from 1 to 4.

[3] The best way to understand the computational steps of the factor models is to use MATLAB codes. The corresponding MATLAB codes for factor model proposed by Doz, Gianonne & Reichlin (2001, 2012) can be found here https://www.newyorkfed.org/research/economists/giannone/pub, MATLAB codes for factor model proposed by Forni, Hallin, Lippi & Reichlin (2005) can be found here http://www.barigozzi.eu/Codes.html

Figure 1. Real GDP growth, %-th to the previous quarter

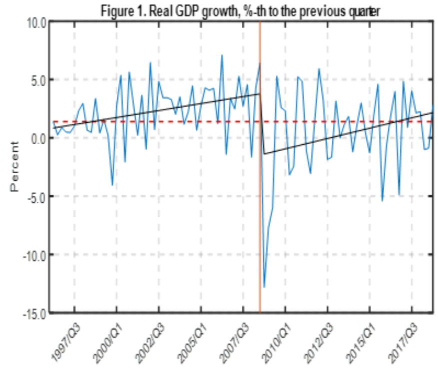


Figure 2. Inflation, %-th to the previous quarter

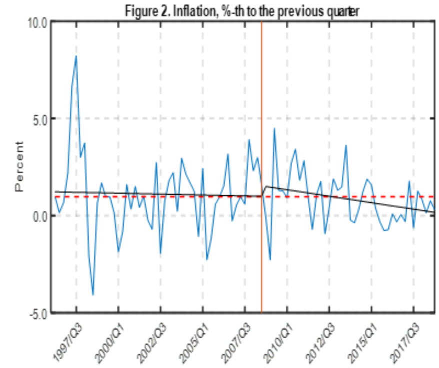


Figure 3. Short-term nominal interest rates, %-th

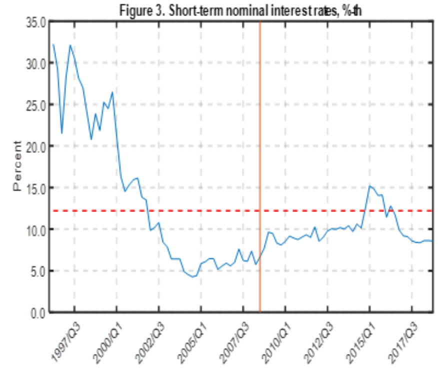
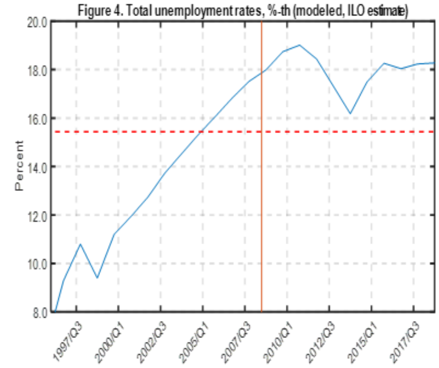


Figure 4. Total unemployment rates, %-th (modeled, ILO estimate)



In the Figures 1 to 4 presented the dynamics of the Armenian key macroeconomic variables. The first important macroeconomic variable is the real GDP growth. To obtain real GDP growth rates with respect to the previous quarter the following preliminary calculations have been done. First absolute values of real GDP calculated at average prices of 2005 were logged and then seasonally adjusted (using X12ARIMA seasonal adjustment algorithm). Then, using seasonally adjusted logged values we have calculated the first differences, that is real GDP growth rates. In 2009 the real GDP growth rate decreased to about 14%, which was caused mainly by the world financial crisis. Dividing the whole sample of real GDP growth on the two sub-periods, before and after financial crisis, we can calculate the coefficient of variations for two sub-periods, before and after the financial crisis. Our calculations show that the coefficient of variation before 2008 was 2.26%, while after 2008 it already was 3.92%, therefore the volatility of real GDP growth is increased.

In Figure 2 we present the dynamics of CPI inflation. The CPI has been calculated since 1993 on a monthly basis. The CPI in the Republic of Armenia is the only indicator characterizing inflation in the Republic of Armenia. The central bank of Armenia before 2006 targeting the monetary aggregates, but after 2006 has switched to inflation targeting through interest rates, as managing the monetary aggregates has been proved ineffective due to the large inflow of remittances from abroad. The inflation targets was initially 3.0 % for 2006, and changed only once in 2007; from 2007 onward it is maintained at 4.0 % with a confidence band of $\pm 1.5\%$. In the Figure 2 presented the dynamics of seasonally adjusted inflation in %-th to the previous quarter. The preliminary treatment of the inflation dynamics includes the following procedures. First, we recalculate the CPI chain indices to the base quarter. Then we take the logged values and apply seasonal adjustment procedure (X12ARIMA seasonal adjustment algorithm), after that we calculate the first differences.

The third important macroeconomic variable is the short-term nominal interest rate for time deposits in national currency. The preliminary treatment for this variable include only first differences (in percentage points). The short-term nominal interest rate shows an overall downward trend. For example as we see from Figure 3 before 2005 the nominal interest rate is characterizing with relatively large fluctuations, but since 2006 the fluctuations of interest rates becoming smaller. Such behavior can be explained by fact that before 2006 the Central bank conducting monetary aggregates targeting policy, while after 2006 the inflation-targeting regime.

The next important macroeconomic variable is total unemployment rate. The official values for unemployment (in persons) has been taken in yearly terms from the World Bank development indicators. This indicator is the International Labor Organization (ILO) estimate. Then using temporal decomposition method, particularly Boot Faibes and Lisman mechanical projection algorithm the yearly unemployment data were decomposed to quarterly data. After that, the unemployment data were logged and calculated the first differences.

Besides of above mentioned four key macroeconomic variables we also have 40 additional variables. These variables we uses for extracting the unobservable factors and then including extracted factors into standard small scale forecasting models such as AR, VAR or BVAR models we producing short-term forecast for the key macroeconomic variables. In the results of including extracted factors into small-scale forecasting models, we getting FAAR, FAVAR and BFAVAR models. The factor-augmented models we want to compare with traditional small-scale AR, VAR and BVAR models to see whether factor-augmented models, could improve the forecast accuracy.

The name and some another important characteristics of the additional dataset are presented in Appendix A. As we can see from Appendix A the additional dataset comprising information on national accounts (production and expenditure components) consumer and producer price indices, employment variables, monetary and interest rates as well as international indicators on growth rates and prices. The additional set of variables were selected from different sources, particularly from the https://stats.oecd.org/ and https://www.indexmundi.com/. As we can see from the appendix for some of the additional variables, the seasonal adjustment procedures have been applied. All no stationary time series are made stationary through the first differencing.

All calculations and forecasts experiments have been done using the MATLAB (r2018b) codes. Some of the MATLAB codes are taken from the internet sources, for example codes for extracting dynamic factors in frequency and time domain have been taken from internet[4]. Some part of MATLAB codes are written by the authors. In addition, we have C# codes for time domain factor model, as well as recursive and rolling regressions, which can be conducted directly from MS Excel spreadsheet. To run the models from MS Excel we have created a specific interface using VBA (Visual basic for Application), which can be downloaded from https://github.com/KarenPoghos/ForecastXL.

4. Experimental design

To conduct out-of-sample forecast experiments we use recursive regressions scheme. For out-of-sample forecast evaluations, we divide the whole sample on the two part, particularly in-sample and out-of-sample periods. In our experiments in-sample periods includes 70 % of observations, while out-of-sample periods 30 % of observations. This means that if the whole sample includes period from 1996Q1 to 2018Q4 (92 observations), then in sample period includes 1996Q1 to 2012Q1 (65 observations), while out-of-sample period includes observations from 2012Q2 to 2018Q4 (27 observations).

The recursive simulation scheme proceeds as follows: First, we estimate the models using subsample 1996Q1 – 2012Q1 (65 observations) and generate 1 to 4 steps-ahead forecasts. Then we increase the sample size by one (66 observations, 1996Q1 – 2012Q2) and generate again 1 to 4 steps-ahead forecasts. We continue increasing the sample size by one and generating 1 to 4 steps-ahead forecast until the sample size 84 (1996Q1 – 2017Q4). Then we increase the sample size by one but only generate 1 to 3 steps-ahead forecasts (since we only have 88 observations in total). We continue increasing the sample size until we have 87 observations in the sample, in which case we can only compute the one step-ahead forecast. In such way, we obtain 27 one-step-ahead forecasts, 26 forecasts for 2-steps-ahead, 25 for 3-steps – ahead and finally 24 forecast for 4-steps-ahead.

Next, we use the out-of-sample forecasts from recursive regression to compute the corresponding root mean squared forecast error (RMSFE) indices for each of the fourth forecasting horizons. More formally let us denote the out-of-sample period by T^* (in our case $T^*=27$) and forecast horizons $h=1,2,3,4$. Then the RMSFE index is calculated by the following formula:

[4] MATLAB codes for time domain factor model can be found https://www.newyorkfed.org/research/economists/giannone/pub, while MATLAB codes for frequency domain factor model can be found http://www.barigozzi.eu/Codes.html

$$RMSFE_{ih} = \sqrt{\frac{1}{T^{*}-(h-1)} \sum_{t=1}^{T^{*}-(h-1)} \left(\hat{y}_{it} - y_{it}\right)^{2}},$$

Where y_{it} denotes the actual value of the $i-th$ dependent variable (in our case we have four core variables and therefore $i=1,2,3,4$), $\hat{y}_{it}$ is the forecasted value of the $i-th$ dependent variable, and $RMSFE_{ih}$ is the root mean squared error calculated for the $i-th$ dependent variable and the $h-th$ forecast horizon.

5. Forecast results

In this section, we present the out-of-sample forecast evaluation results for 15 competing short-term forecasting models. To keep robustness of our results we have estimated models with different lags length and different combinations of static and dynamic factors. Then we choose the one model that yields the best forecasting performance in the sense of minimization the RMSFE indices. Following the paper by Pirshel and Wolters (2014), we vary the number of lags in the models from one up to four lags. In addition, we vary the number of static and dynamic principal components in the factor-augmented models. Thus varying both number of lags and number of static and dynamic factors we comparing estimated models to each other and select only that model which provides a minimum value of the RMSFE index. Now let us explain how we determine the number of static and dynamic factors.

First, we have to determine the number of static principal components. For that, we use a simple approach: we retain the principal components with eigenvalues more than 1. To select the appropriate number of dynamic factors first we should take into account the fact that the number of dynamic factors cannot exceeds the number of static factors. For example, to extract principal components we have additional 40 variables (Appendix A). Using these variables, we can extract maximum 40 principal components. Using the rule of eigenvalues more than 1, we retain in our analysis only first 14 principal components. Thus, we already determined the maximum number of static factors. Then we need to find the optimal number of static principal components. For that we estimate models with 1,2,…,14 principal components using different lag lengths 1,2,…,4 (56 models in total). Then we select the one that yields the minimum RMSFE. Thus in such way we can estimate the optimal number of static factors. Then using the rule that the number of dynamic factors cannot exceeds the number of static factors, we can construct different combinations of the dynamic and static factors. To be more clear let's note the optimal number of static principal components r, then using above rule we can estimate models with 1 dynamic and r static components, 2 dynamic and r static components. Continuing in a such way the final model that we can estimate is the model with q dynamic factors and r static factors (in this case q = r). Thus having all possible combinations of principal components we estimate models for different lags length and select the one that give the minimum value of RMSFE index. In the below Tables 1 and 2 we present the results of out-of-sample forecast evaluation for 15 competing models for recursive regression scheme. Let us first present the results for real GDP growth and then for inflation.

1. Real GDP growth: As we can see from Tables 1 the factor-augmented models are outperform small-scale models for all forecast horizons. For one-step ahead forecast horizon FAAR_TS model, outperform all small-scale benchmark models producing the minimum value of RMSFE's. In the case of two, three and four steps ahead forecast horizons the FAVAR_QML, FAVAR_SW and BFAVAR_TS are outperform all small-scale benchmark models. Thus as we can conclude that factor-augmented AR, VAR and BVAR models are better suited for short-term forecasting of real GDP growth.
2. Inflation: From Table 2 we can see that again factor-augmented models are outperform small-scale benchmark models for all forecast horizons with exception only two steps ahead forecast horizon. For one-step ahead forecast horizon the FAVAR_TS is outperform all small-scale models producing the lowest RMSFE's value. For two steps ahead forecast horizon the small-scale BVAR model is outperform all factor-augmented models. For three and four steps, ahead forecast horizons the FAVAR_TS and BFAVAR_SW are outperform all small-scale models producing lowest RMSFE's values. Thus, for inflation dynamics as we see it is more appropriate to use both factor-augmented models and small-scale BVAR model.

Thus, our general conclusion is that the factor-augmented models are able to produce more accurate short-term forecast for the key macroeconomic variables than small-scale benchmark models and therefore these models can be included in the short-term forecasting practice at the Central bank of Armenia.

In order to check whether the obtained results for RMSFE's are significantly different among of different models we also perform across models tests. For that, we comparing factor-augmented models with its counterpart small-scale models. The across model test is based on a statistic proposed by Diebold and Mariano (1995). In this paper the Diebold-Mariano statistic, we calculate by regressing of the loss differential on an intercept, using heteroscedasticity autocorrelation robust (HAC) standard errors (Diebold, 2013)). The time-t loss differential between forecast 1 and 2 can be calculated as $l_t = \left(\varepsilon_t^{AR}\right)^2 - \left(\varepsilon_t^{i}\right)^2$, where, ε_t^{AR} is the forecast error from AR model at time t, ε_t^i denote the forecast error from the alternative factor augmented counterpart model ($i = FAAR\_FAAR\_FHLR, FAAR\_TS, FAAR\_QML$). In the same way, we can calculate the loss differentials for VAR and BVAR models errors. Thus, we regressing of the loss differential on an intercept using HAC standard errors.

Table 1. RMSFE indices for the real GDP growth[5]

Forecasting models	Forecast horizons			
	1	2	3	4
AR (p = 4)[6]	2.588	2.539	2.512	2.452
VAR (p = 1)	2.544	2.566	2.518	2.481
BVAR (p = 2, w = 0.3, d = 1)[7]	2.639	2.546	2.510	2.482
FAAR_SW (p = 3, r = 2)[8]	2.051	2.556	2.422	2.489
FAAR_FHLR (p = 3, q = 1, r = 2)[9]	2.039	2.593	2.420	2.452
FAAR_TS (p = 3, q = 1, r = 2)	***2.036***	2.501	2.485	2.462
FAAR_QML (p = 3, q= 2, r = 2)	2.181	2.474	2.433	2.504
FAVAR_SW (p = 1, r = 2)	2.375	2.627	***2.298***	2.404
FAVAR_FHLR (p = 1, q = 2, r = 2)	2.327	2.562	2.333	2.420
FAVAR_TS (p = 1, q = 2, r = 2)	2.338	2.555	2.317	2.394
FAVAR_QML (p = 1, q = 2, r = 2)	2.329	***2.453***	2.399	2.423
BFAVAR_SW (p = 1, r = 2, w = 0.3, d = 1)	2.406	2.609	2.356	2.402
BFAVAR_FHLR (p = 1, q = 2, r = 2, w = 0.3, d = 1)	2.392	2.551	2.367	2.456
BFAVAR_TS (p = 1, q = 2, r = 2, w = 0.3, d = 1)	2.326	2.554	2.319	***2.390***
BFAVAR_QML (p = 1, q = 2, r = 2, w = 0.3, d = 1)	2.328	2.458	2.400	2.413

[5] FAAR_SW is a FAAR model with static principal components, FAAR_FHLR is a FAAR model with principal components estimated in the frequency domain, FAAR_TS is a FAAR model estimated in the time domain with using two steps Kalman filter approach, FAAR_QML is a FAAR model estimated in the time domain with using quasi-maximum likelihood algorithm. In the same way, we can explain the abbreviations for the FAVAR and BFAVAR models.

[6] The number in the brackets means that the minimum RMSFE indices for AR model has achieved in case of four lags. For all other models in the brackets are presented the number of lags where the particular model achieved its minimum RMSFE.
[7] W = 0.3 and d = 1, the coefficients that we use for BVAR and BFAVAR models estimation. The first coefficient (overall tightness) is implementing to the diagonal matrix of the variances, while the second coefficient (decay) is implemented to the lags. In this paper, we set the overall tightness equal to 0.1, 0.2 and 0.3 and lag decay equal to one and two. These parameters are chosen so that they are consistent with the ones used in Gupta and Kabundi (2009). Thus we vary the coefficients of the overall tightness and decay parameters and estimate the BVAR models for different lags lengths. Then we select the one model that yields the best ex post forecast performance in the sense of the minimization of RMSFE.

8 The numbers in the brackets p = 3, r = 2 means that FAAR model with number of lags equal to 3 and number of static principal components equal to 2 yields the best out of sample forecast performance, in the sense of minimum RMSFE.

[9] Comparing with the FAAR_SW numbers in the brackets we see that here we have one additional parameter, q = 1, which says that FAAR_FHLR model yields the minimum value of RMSFE when we apply three lags, one dynamic and two static principal components

Table 2. RMSFE indices for inflation

Forecasting models	Forecast horizons			
	1	2	3	4
AR (p = 1)	1.146	1.135	1.165	1.189
VAR (p = 1)	1.118	1.110	1.137	1.131
BVAR (p = 1, w = 0.2, d = 1)	1.150	***<u>1.059</u>***	1.096	1.108
FAAR_SW (p = 4, r = 1)	1.154	1.131	1.122	1.120
FAAR_FHLR (p = 4, q = 1, r = 1)	1.188	1.175	1.169	1.179
FAAR_TS (p = 4, q = 1, r= 1)	1.194	1.166	1.160	1.170
FAAR_QML (p = 4, q = 1, r = 1)	1.297	1.249	1.253	1.274
FAVAR_SW (p = 3, r = 2)	1.140	1.139	1.149	1.083
FAVAR_FHLR (p = 3, q = 2, r = 2)	1.147	1.150	1.156	1.110
FAVAR_TS (p = 3, q = 1, r = 2)	***<u>1.117</u>***	1.154	***<u>1.064</u>***	1.081
FAVAR_QML (p = 3, q = 2, r = 2)	1.124	1.167	1.191	1.207
BFAVAR_SW (p = 3, r = 2, w = 0.3, d = 2)	1.184	1.169	1.100	***<u>1.075</u>***
BFAVAR_FHLR (p = 3, q = 2, r = 2, w = 0.3, d = 1)	1.162	1.195	1.146	1.174
BFAVAR_TS (p = 3, q = 1, r = 2, w = 0.3, d = 1)	1.160	1.163	1.113	1.101
BFAVAR_QML (p = 3, q = 1, r = 2, w = 0.3, d = 1)	1.172	1.205	1.136	1.143

The results of calculations for different forecast horizons are presented in Tables 3 and 4.

Table 3. Relative RMSFE's for real GDP growth models

Models	Forecast horizons			
	1	2	3	4
FAAR_SW versus AR	**0.792**	1.007	**0.964**	1.015
FAAR_FHLR versus AR	**0.788***	1.021	**0.963**	1.000
FAAR_TS versus AR	**0.787***	**0.985**	**0.989**	1.004
FAAR_QML versus AR	**0.843**	**0.974**	**0.969**	1.021
FAVAR_SW versus VAR	**0.934**	1.024	**0.913**	**0.969**
FAVAR_FHLR versus VAR	**0.915**	**0.999**	**0.926**	**0.975**
FAVAR_TS versus VAR	**0.919**	**0.996**	**0.920**	**0.965**
FAVAR_QML versus VAR	**0.915**	**0.956**	**0.953**	**0.977**
BFAVAR_SW versus BVAR	**0.912**	**1.025**	**0.938**	**0.968**
BFAVAR_FHLR versus BVAR	**0.907**	**1.002**	**0.943**	**0.990**
BFAVAR_TS versus BVAR	**0.881**	**1.003**	**0.924**	**0.963**
BFAVAR_QML versus BVAR	**0.882**	**0.965**	**0.956**	**0.972**

Diebold Mariano t-statistics that are statistically significant at the confidence levels of 90%, 95% and 99% respectively are denoted by *,**,***.

Table 4. Relative RMSFE's for Inflation models

Models	Forecast horizons 1	2	3	4
FAAR_SW versus AR	1.007	**0.997**	**0.964**	**0.942**
FAAR_FHLR versus AR	1.037	1.036	1.003	**0.991**
FAAR_TS versus AR	1.042	1.028	**0.996**	**0.984**
FAAR_QML versus AR	1.132	1.101	1.075	1.071
FAVAR_SW versus VAR	1.020	1.026	1.010	**0.957**
FAVAR_FHLR versus VAR	1.026	1.037	1.016	**0.981**
FAVAR_TS versus VAR	**0.975**	1.017	**0.913**	**0.909**
FAVAR_QML versus VAR	**0.981**	1.028	1.023	1.015
BFAVAR_SW versus BVAR	1.030	1.104	1.004	**0.970**
BFAVAR_FHLR versus BVAR	1.010	1.129	1.046	1.060
BFAVAR_TS versus BVAR	1.008	1.098	1.016	**0.994**
BFAVAR_QML versus BVAR	1.019	1.138	1.037	1.032

In the Tables 3 and 4 presented relative RMSFE's for real GDP growth and inflation. To calculate the relative RMSFE's we use the following ratio: $RRMSFE_{i,h}^{M} = \dfrac{RMSFE_{i,h}^{M}}{RMSFE_{i,h}^{M_0}}$.

Now let us to analyze the relative RMSFE indices first for real GDP growth and then for inflation. where $RRMSFE_{i,h}^{M}$ is the relative RMSFE for i-th variable (real GDP growth, inflation) calculated at h forecast horizon with model M-th, $RMSFE_{i,h}^{M}$ is the RMSFE value for i-th variable calculated at h forecast horizon with model M-th, $RMSFE_{i,h}^{M_0}$ is the RMSFE value for i-th variable calculated at h forecast horizon with using M_0 small-scale benchmark model ($M_0 = AR,\ VAR,\ BVAR$). The relative RMSFE should be below 1 to outperform the benchmark counterpart model.

As we see from Table 3 when we comparing the FAAR model with its counterpart AR model then we see that relative RMSFE values are below 1 in a 10 cases out of 16. However, when we use Diebold – Mariano test we see that only in two cases the relative RMSFE values are significantly differ. When we comparing FAVAR with its counterpart VAR model, then we see that relative RMSFE values are below 1 in 15 cases out of 16. However, when we apply Diebold-Mariano test we see that the differences between RMSFE values are not statistically significant. In the case of comparing BFAVAR with the small-scale BVAR we see that in all cases we have RMSFE values below 1, but when we apply Diebold-Mariano test we see that the differences are not significant.

In Table 4 presented relative RMSFE values calculated for inflation models. From this table we see that there is no significant differences between RMSFE values when we apply DM test. In addition, we see that the number of cases where relative RMSFE values below 1 is relatively less comparing with the real GDP growth results. For example when we comparing FAAR models with AR model we see that the relative RMSFE values are below 1 only in a six cases out of 16, while in the case of real GDP growth this combination is 10/16. The same picture we also observe in case of FAVAR model, where we have the same combinations 6/16. Finally, when we compare the BFAVAR with the BVAR without factors we see that only in two cases the RMSFE values are below 1, while in the prevailing cases we have the opposite situation.

Thus, based on the used dataset and results of calculations we can see that when we apply factor-augmented models to real GDP growth then in the prevailing part of the total cases the factor augmented models are outperform their corresponding small-scale models (41 out of 48). Even in two cases, the differences between RMSFE values are statistically significant. When we apply, factor-augmented models to inflation then we see that the number of outperforming cases are radically decreasing (14 out of 48). Therefore for the Armenian dataset the factor-augmented models are better suited for the real GDP growth forecasting and using these models we can more accurately to forecast real GDP dynamics, even some models are able to increase the statistically significance of the forecast improvements. Thus, these models can be included in the system of short-term forecasting models at the Central bank of Armenia to improve the forecast accuracy of the real GDP growth.

6. Conclusions

In this paper, we evaluate the forecasting performance of the 15 competing models for short-term forecasting the key macroeconomic variables in Armenia. We comparing the factor-augmented AR, VAR and BVAR with their counterparts AR, VAR and BVAR models. As a result of comparisons we want to see whether the advanced models developed and succefully applied in a developed countries could be still useful to forecast the key macroeconomic variables in developing countries like Armenia. For that using Armenian actual quarterly macroeconomic variable from 1996Q1 to 2018Q4 we conduct estimation for a 15 models with and without additional factors. Using recursive regression scheme we conduct out-of-sample forecast evaluation experiments. We calculate the RMSFE's values for each model and for different forecast horizons. Thus based on the used dataset and calculated RMSFE values we conclude that when we apply factor-augmented models to forecast real GDP growth then in the prevailing part of experiments the factor-augmented models outperform their counterpart small-scale models. Even in some cases factor augmented models outperform traditional small-scale models with statistically significant improvements. However, when we apply the same database to forecast inflation dynamics then we conclude that factor augmented models are able to outperform small-scale benchmark models only in a few cases. Therefore, advanced forecasting techniques can be more successfully applied for forecasting real GDP growth, even applying these methods we can significantly improve forecast accuracy. Thus we suggest to include factor augmented algorithms in the system of short-term forecasting in the Central bank of Armenia to improve real GDP growth forecasting accuracy.

Appendix A: Macroeconomic variables

Id	Series description	SA	Transf.
	National accounts		
1	Industry value added (at average prices of 2005 year), mln AMD	Yes	Ln and Δ
2	Agriculture, forestry and fishing value added (at average prices of 2005 year), mln AMD	Yes	Ln and Δ
3	Construction value added, (at average prices of 2005 year), mln AMD	Yes	Ln and Δ
4	Services value added, (at average prices of 2005 year), mln AMD	Yes	Ln and Δ
	Taxes on production (minus subsidies),(at average prices of 2005 year) , mln Armenian drams		
5	Final Consumption (at average prices of 2005 year), mln AMD	Yes	Ln and Δ
6	Private consumption (at average prices of 2005 year), mln AMD	Yes	Ln and Δ
7	Government consumption (at average prices of 2005 year), mln AMD	Yes	Ln and Δ
8	Gross accumulation (at average prices of 2005 year), mln AMD	Yes	Ln and Δ
9	Exports of goods and services(at average prices of 2005 year), mln AMD	Yes	Ln and Δ
10	Imports of goods and services (at average prices of 2005 year), mln AMD	Yes	Ln and Δ
	Consumer and Producer Prices		
11	Food price index (including alcohols and tobacco), in %-th with respect to the previous period	Yes	Ln and Δ
12	Non-food price index, in %-th with respect to the previous period	No	Ln and Δ
13	Services price index, in %-th with respect to the previous period	No	Ln and Δ
14	Industrial production price index, with respect to the previous period, %-th	No	Ln and Δ
15	Construction price index, with respect to the previous period, % - th	No	Ln and Δ
16	Tarifs for transportation, with respect to the previous period, % - th	No	Ln and Δ
	Employment		
17	Employed population, aged 15 and over, persons	No	Ln and Δ
18	Employment in agriculture, persons	No	Ln and Δ
19	Employment in industry, persons	No	Ln and Δ
20	Employment in services, persons	No	Ln and Δ
21	Self-employed, persons	No	Ln and Δ
	Money and Interest rates		
22	Cash money outside of banking system, mln AMD	Yes	Ln and Δ
23	Monetary base, mln. AMD	Yes	Ln and Δ
24	Broad money, mln. AMD	Yes	Ln and Δ
25	Total deposits in the banking system, mln. AMD	Yes	Ln and Δ
26	Loans to economy with accumulated interest rates, mln. AMD	Yes	Ln and Δ

27	Interest rate on attracted deposits (from 15 days to 1 year) of enterprises in national currency, %-th	No	Δ
28	Interest rate on attracted deposits (from 15 days to 1 year) of households in national currency, %-th	No	Δ
29	Interest rate on loans (from 15 days to 1 year) granted by enterprises in national currency	No	Δ
30	Interest rate on loans (from 15 days to 1 year) granted by households in national currency	No	Δ
	International indicators		
31	US, gdp Growth rate compared to previous quarter, seasonally adjusted	No	Δ
32	EU (28 countries), GDP Growth rate compared to previous quarter, seasonally adjusted	No	Δ
33	EU (28) Industrial production, s.a., growth previous period	No	Δ
34	US Industrial production, s.a., growth previous period	No	Δ
35	Russia Industrial production, s.a., growth previous period	No	Δ
36	Commodity Food Price Index, (2005 = 100), includes Cereal, Vegetable Oils, Meat, Seafood, Sugar, Bananas, and Oranges Price Indices	No	Ln and Δ
37	Crude oil, UK Brent 38° API.US Dollars per Barrel		
38	Russian Natural Gas Monthly Price - US Dollars per Million Metric British Thermal Unit	No	Ln and Δ
40	Crude oil, average spot price of Brent, Dubai and West Texas Intermediate, equally weighed	No	Ln and Δ

Note: AMD is the short name of the Armenian national currency. SA – means seasonal adjustment, if Yes then series is seasonally adjusted, Transf. – means preliminary transformation applied, for example if we see Ln and Δ, then it means that a particular series have been logged and then first differenced, if we see Δ then the series was only first differenced. The main source of the data is internal databases of the Central bank of Armenia and National Statistical Agency, as well as https://stats.oecd.org/ and https://www.indexmundi.com/.

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