# FORECASTING ARMENIAN KEY MACROECONOMIC INDICATORS USING FACTOR-BASED DYNAMIC MODELS

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Abstract – Two model averaging approaches are used and compared in estimating and forecasting dynamic factor models, the well-known Bayesian model averaging (BMA) and the recently developed weighted average least squares (WALS). Both methods propose to combine frequentist estimators using Bayesian weights. We apply our framework to the Armenian economy using quarterly data from 2000-2010, and we estimate and forecast real GDP growth and inflation.
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## 1. INTRODUCTION

In the recent macroeconomic literature, factor-based dynamic models have become popular. The idea underlying these models is that, while there are potentially a very large number of explanatory variables, most of the movement in the dependent variable can be explained by only a few variables or linear combinations thereof. One of the reasons why this happens it that the explanatory variables are often highly correlated.

We mention three recent examples where this approach has been successfully applied. Stock and Watson (2002) performed forecasting experiments for key USA macroeconomic variables using 215 explanatory variables. From this large number of variables they extracted a few factors to forecast key macroeconomic indicators. Forni et al. (2000, 2003) provided a time-series forecasting method based on spectral analysis, and applied this method to forecast Euro-area industrial production and inflation using 447 explanatory variables. Finally, Bernanke et al. (2005) took a VAR model and augmented it with factors

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based on 120 macroeconomic variables. All three papers find that the mean squared errors of estimates and forecasts based on factor models are lower than those obtained from vector autoregressive models.

After extracting factors, these models are typically estimated in the traditional econometric way, that is, separating model selection and estimation. Recent advances in econometric theory allow us to combine model selection and estimation into one procedure, thus avoiding the undesirable problem of pretesting. This procedure is called 'Bayesian model averaging'. The purpose of the current paper is to extend the basic model averaging framework to include dynamics and factor extraction, and to apply this extended framework to explain and forecast Armenian real GDP and inflation dynamics.

In addition, we wish to compare the standard Bayesian model averaging (BMA) approach to the 'weighted average least squares' (WALS) approach, recently developed in Magnus et al. (2010). The WALS approach has both theoretical and computational advantages over BMA. Theoretical, because it generates bounded risk and contains an explicit treatment of ignorance; computational, because its computing time increases linearly rather than exponentially with the dimension of the model selection space. Magnus et al. (2010) applied WALS to growth empirics, but without dynamics or lagged dependent variables.

Estimation and forecasting in factor-based dynamic models using the BMA algorithm was first applied by Koop and Potter (2004) to US data. The current paper follows their general approach, but also reports on experiments where the two model averaging methods (WALS and BMA) are compared.

The paper is organized as follows. The factor-based dynamic model is introduced in Section 2. In Section 3 we present the WALS and BMA model averaging methods. Section 4 describes macroeconomic time series dynamics. We report results from two experiments: an estimation simulation in Section 5, and a forecast experiment in Section 6. Section 7 concludes.

## 2. The dynamic factor model

We consider the dynamic regression model

$$y_t = \alpha(L)y_{t-1} + \beta(L)x_{t-1} + \xi_t \qquad (t = 1, \ldots, T), \tag{2.1}$$

where $y_t$ is a scalar dependent variable, $x_t$ is a $k \times 1$ vector of nonrandom explanatory variables, $\alpha(L)$ and $\beta(L)$ are polynomials in the lag operator of dimensions $p_1$ and $p_2$, respectively, and $\xi_t$ is a random vector of unobservable disturbances, independently and identically distributed as $N(0, \sigma^2)$.

We have $p_1 + kp_2$ explanatory variables, which may be a large number. Moreover, many of the parameters may be close to zero. These two factors make it difficult to apply standard estimation methods (Koop and Potter, 2004). It is then common in the macroeconometric literature to replace the $k$ explanatory variables with a much smaller number of variables. This can be achieved by using principal component or factor analysis algorithms. After extracting the principal components, Model (2.1) can be rewritten as

$$y_t = \alpha(L)y_{t-1} + \gamma(L)f_{t-1} + \epsilon_t \qquad (t = 1, \ldots, T), \tag{2.2}$$

where $f_t$ ($m \times 1$) is the vector of extracted principal components and $\gamma(L)$ is a polynomial in the lag operator (Stock and Watson, 2002). We assume that $m < k$ and $m < T$. As noted by Koop and Potter (2004, p. 553), there is a cost in this type of transformation, namely that the interpretation of the variables is more difficult.

Koop and Potter (2004) were the first to show how Bayesian model averaging can be applied to estimation and forecasting using dynamic factor models. Their study applies BMA to the problem of forecasting GDP and inflation using quarterly US data on 162 time series. Our paper follows their approach, but also compares two competing estimation procedures: BMA and WALS. This will not only tell us something about the power of the two algorithms, but will also provide information about the robustness of our results.

## 3. Estimation methodology

The idea behind combining estimators (or forecasts) is to use information from all models within a given family in a continuous fashion. In contrast to standard econometrics - where one first selects a model and then estimates the parameters within the chosen model, a discrete procedure - we combine the estimators from all models considered, where some models get a higher weight than others, based on priors and diagnostics. One advantage of this procedure is that we avoid the well-known pretest problem: our procedure is a joint procedure, where model selection and estimation are combined.

As our framework, we choose the linear regression model

$$y = X_1\beta_1 + X_2\beta_2 + \epsilon = X\beta + \epsilon, \qquad \epsilon \sim N(0, \sigma^2 I_n),$$

where $y$ $(n \times 1)$ is the vector of observations, $X_1$ $(n \times k_1)$ and $X_2$ $(n \times k_2)$ are matrices of nonrandom regressors, $\epsilon$ is a random vector of unobservable disturbances, and $\beta_1$ and $\beta_2$ are unknown parameters which we need to estimate. We assume that $k_1 \geq 1$, $k_2 \geq 0$, $k = k_1 + k_2 \leq n - 1$, that $X = (X_1 : X_2)$ has full column-rank, and that the disturbances are independent and identically distributed.

The reason for distinguishing between $X_1$ and $X_2$ is that $X_1$ contains variables that we want to be in the model (whatever $t$-values or other diagnostics we find), while $X_2$ contains variables that may or may not be in the model. The columns of $X_1$ are called 'focus' regressors, the columns of $X_2$ 'auxiliary' regressors. The uncertainty about each auxiliary regressor, that is whether we should or should not include the regressor in our model, is a very common situation, and the application of model averaging is then a natural procedure. Rather than choosing one model by some preliminary diagnostic tests, we assume that each model tells us something of interest about our focus parameters. We do not, however, trust each model to the same degree, and the resulting weights are determined by priors and data. In this paper we concentrate on two model averaging algorithms, the well-known BMA algorithm and the recently introduced WALS algorithm. We briefly summarize each in turn. Full details and background references are provided in Magnus et al. (2010).

***Bayesian model averaging (BMA)***

With the exception of Magnus et al. (2010), the literature on Bayesian model averaging considers the case $k_1 = 1$. We summarize the approach of Magnus et al. (2010). Since there are $k_2$ auxiliary regressors, we have $2^{k_2}$ different models to consider, because each auxiliary regressor can either be included or not. Model $\mathcal{M}_i$ is defined as the model where a specific subset of the $k_2$ auxiliary regressors is selected. If we let $p(\mathcal{M}_i)$ denote the prior probability that $\mathcal{M}_i$ is the true model, then the posterior probability for model $\mathcal{M}_i$ is given by

$$\lambda_i = p(\mathcal{M}_i|y) = \frac{p(\mathcal{M}_i)p(y|\mathcal{M}_i)}{\sum_j p(\mathcal{M}_j)p(y|\mathcal{M}_j)} \qquad (i = 1, \dots, 2^{k_2}),$$

and if we take $p(\mathcal{M}_i) = 2^{-k_2}$, which is the common assumption, then $p(\mathcal{M}_i)$ does not depend on $i$, and we have simply $\lambda_i \propto p(y|\mathcal{M}_i)$, the marginal likelihood of $y$ in model $\mathcal{M}_i$. If we adopt Zellner's $g$-prior, then

$$\lambda_i \propto \left(\frac{g_i}{1+g_i}\right)^{k_{2i}/2} (y' M_1 A_i M_1 y)^{-(n-k_1)/2},$$

where

$$A_i = \frac{g_i}{1+g_i} M_1 + \frac{1}{1+g_i}(M_1 - M_1 X_{2i}(X_{2i}' M_1 X_{2i})^{-1} X_{2i}' M_1),$$
$$M_1 = I_n - X_1(X_1' X_1)^{-1} X_1',$$

and $X_{2i}$ is the $n \times k_{2i}$ matrix containing the observations of the $k_{2i}$ auxiliary regressors included in model $\mathcal{M}_i$. We specify $g_i$ as

$$g_i = \frac{1}{\max(n, k_2^2)}.$$

The $\lambda_i$'s are the required weights to obtain the BMA estimates and precisions. For example, the BMA estimator of $\beta_1$ is given by

$$\mathrm{E}(\beta_1|y) = \sum_{i=1}^{k_2} \lambda_i \mathrm{E}(\beta_1|y, \mathcal{M}_i).$$

There are several problems with BMA. First, all $2^{k_2}$ models have to be evaluated implying a huge computational effort; second, the priors are based on the normal distribution, leading to unbounded risk; and third, the treatment of 'ignorance' is ad hoc and unsatisfactory. These problems are avoided in an alternative model averaging procedure, called WALS.

***Weighted average least squares (WALS)***

In the WALS algorithm, developed in Magnus et al. (2010), we first 'orthogonalize' the columns of $X_2$ such that $P'X_2'M_1X_2P = \Lambda$, where $P$ is orthogonal and $\Lambda$ is diagonal. Then we define $X_2^* = X_2P\Lambda^{-1/2}$ and $\beta_2^* = \Lambda^{1/2}P'\beta_2$, so that $X_2^*\beta_2^* = X_2\beta_2$. Our prior will be on $\beta_1$ and $\beta_2^*$ (rather than on $\beta_2$), and this gives us enormous computational advantage, because all models which include $x_{2j}^*$ as a regressor will have the same estimator of $\beta_{2j}^*$, irrespective of which other $\beta_2^*$'s are estimated.

The second ingredient is the 'equivalence theorem' (Magnus and Durbin, 1999; Danilov and Magnus, 2004), which tells us that the WALS estimator $b_1$ of $\beta_1$ will be 'good' (in the mean squared error sense) if and only if $W\hat{\beta}_2^*$ is a good estimator of $\beta_2^*$, where $\hat{\beta}_2^*$ denotes the least squares estimator of $\beta_2^*$ in the unrestricted model, and $W$ is a random diagonal matrix of order $k_2 \times k_2$. The diagonal elements $w_j$ of $W$ will depend on the weights $\lambda_i$, but while there are $2^{k_2}$ $\lambda$'s, there are only $k_2$ $w$'s. This is where the computational advantage comes from.

The third ingredient is the treatment of ignorance. Based on the fact that a $t$-value of one indicates that including an auxiliary regressor gives us the same mean squared error of the estimated focus parameter as excluding the auxiliary regressor, we define ignorance on an auxiliary parameter $\eta$ by the properties

$$\Pr(\eta > 0) = \Pr(\eta < 0), \qquad \Pr(|\eta| > 1) = \Pr(|\eta| < 1),$$

and we propose the Laplace distribution

$$\pi(\eta) = (c/2)\exp(-c|\eta|)$$

with $c = \log 2$. The WALS estimator is a Bayesian combination of frequentist estimators, and possesses major advantages over standard Bayesian model averaging (BMA) estimators: the WALS estimator has bounded risk, allows a coherent treatment of ignorance, and its computational effort is negligible. The sampling properties of the WALS estimator as compared to BMA estimators have been examined in Magnus et al. (2011), where Monte Carlo evidence shows that the WALS estimator performs significantly better than standard BMA and pretest alternatives. Because of the light computational cost, extensions are possible in many directions. For example, Magnus et al. (2011) extend the WALS theory to allow for nonspherical disturbances.

In this paper we consider a broader class of linear models than before, by allowing the regressors to include lagged dependent variables. The $y_t$ will then be correlated with the current and all previous disturbances, but uncorrelated with all future disturbances. Hence, the regressor $y_{t-1}$ will be uncorrelated with the current disturbance and all future disturbances, although it will be correlated with all previous disturbances. Therefore, the standard OLS assumptions do not hold, and the finite-sample properties of the least squares estimators are not valid. However, as shown by Mann and Wald (1943), these properties will hold asymptotically.

We shall assume that the lagged dependent variables are always focus regressors. But the extracted principal components can be either focus or auxiliary. Thus we write

$$y = X_1\beta_1 + X_2\beta_2 + \epsilon, \tag{3.1}$$

where $X_1$ contains the lagged dependent variables and a subset (possible empty) of the principal components, and $X_2$ contains the remainder of the principal components. In this form we can apply BMA and WALS to this system.

## 4. Data description and preliminary analysis

Our data consist of quarterly time series of 42 macroeconomic variables from 2000 (second quarter) to 2010 (fourth quarter), in total 43 observations for each variable. This set includes national accounts data (9 variables) and consumer prices and exchange rate data (13 variables), listed in Table 1; and financial and monetary policy indicators (13 variables) and international macroeconomic indicators (7 variables), listed in Table 2.

Table 1. National accounts, consumer prices, and exchange rates

| National accounts | Price indices | Price indices and exchange rates |
|---|---|---|
| GDP | Consumer price index | Wheat price index |
| Consumption | Food price index | Fuel price index |
| Investment | Nonfood price index | Imported food price index |
| Exports | Services price index | Imported nonfood price index |
| Imports | Home food price index | Administrative price index |
| Industrial output | | AMD/USD exchange rate |
| Agricultural output | | AMD/EURO exchange rate |
| Construction | | AMD/RR exchange rate |
| Services | | |

All variables in Table 1 are in logarithmic form, in first differences. The variables in column 1 are all real. The variables in columns 1 and 2 are seasonally adjusted.

Table 2. Financial, monetary, and international indicators

| Financial policy indicators | Interest rates | International indicators |
|---|---|---|
| Cash money | AMD deposits | USA real GDP |
| Money aggregate, M0 | USD deposits | EU real GDP |
| Money aggregate, M1 | AMD loans | USA consumer price index |
| Money aggregate, M2X | USD loans | EU consumer price index |
| Total deposits | Central Bank interbank | Gasoline price index |
| Loans to economy | | Petroleum price index |
| Loans to enterprizes | | Wheat price index |
| Loans to households | | |

The variables in Table 2 are also in logarithmic form, in first differences, and the variables in columns 1 and 3 are seasonally adjusted. The international indicators in column 3 are taken from the International Financial Statistics (IFS) published by the IMF and are already seasonally adjusted.

TABLE 3. Characteristics of the extracted principal components

| Principal components | Rotated eigenvalue | % of total variance | Cumulative % | Correlation with real GDP | Correlation with inflation |
|---|---|---|---|---|---|
| *Int_rate* | 5.11 | 12.78 | 12.78 | 0.04 | −0.21 |
| *Ex_rate* | 5.00 | 12.51 | 25.29 | −0.03 | 0.28 |
| *Invest* | 3.90 | 9.74 | 35.03 | 0.65 | 0.07 |
| *Mon_agg* | 3.60 | 9.00 | 44.03 | 0.43 | 0.01 |
| *Credit* | 3.19 | 7.98 | 52.01 | 0.02 | 0.10 |
| *Pr_index* | 2.62 | 6.54 | 58.55 | 0.23 | 0.62 |
| *ImpExp* | 2.58 | 6.46 | 65.01 | 0.22 | −0.03 |
| *Nat_acc* | 2.37 | 5.93 | 70.93 | 0.31 | −0.27 |
| *GDP_star* | 2.01 | 5.01 | 75.95 | 0.29 | −0.12 |
| *Hfood_pr* | 1.69 | 4.23 | 80.18 | 0.09 | 0.47 |

We estimate and forecast factor-based dynamic models using principal components. These principal components are based on the underlying data set of 40 variables (without including dependent variables, that is real GDP and inflation). The extracted principal components have been given names, based on the correlation coefficients between the extracted principal components and the underlying time series. Some important characteristics of the extracted principal components are presented in the Table 3. The first principal component is *Int_rate* and its contribution to the total variance of the underlying variables is 12.78%. The second principal component is *Ex_rate* with a contribution of 12.51%, and the third is *Invest* with a contribution of 9.74%. The ten most important principal components (those with a rotated eigenvalue larger than 1) explain more than 80% of the variance of the underlying variables, which we consider to be sufficient.

Each of the extracted principal components could be used for estimation in our factor-based dynamic models. However, we use our knowledge of economic theory and Armenian setting to include only those principal components which contain important information about real GDP and inflation dynamics. Thus, based on this criterion we choose

TABLE 4. Focus and auxiliary variables ($j = 1, \ldots, 4$)

| Real GDP | | | Inflation | | |
|---|---|---|---|---|---|
| Regressor | Model 1.1 | Model 1.2 | Regressor | Model 2.1 | Model 2.2 |
| *Intercept* | focus | focus | *Intercept* | focus | focus |
| $GDP_{t-j}$ | focus | focus | $INF_{t-j}$ | focus | focus |
| $Invest_{t-j}$ | auxiliary | focus | $Int\_rate_{t-j}$ | auxiliary | auxiliary |
| $Mon\_agg_{t-j}$ | auxiliary | auxiliary | $Ex\_rate_{t-j}$ | auxiliary | focus |
| $Pr\_index_{t-j}$ | auxiliary | auxiliary | $Credit_{t-j}$ | auxiliary | auxiliary |
| $ImpExp_{t-j}$ | auxiliary | focus | $Pr\_index_{t-j}$ | auxiliary | focus |
| $Nat\_acc_{t-j}$ | auxiliary | focus | $Nat\_acc_{t-j}$ | auxiliary | auxiliary |
| $GDP\_star_{t-j}$ | auxiliary | auxiliary | $GDP\_star_{t-j}$ | auxiliary | auxiliary |
| — | — | — | $Hfood\_pr_{t-j}$ | auxiliary | focus |

the following principal components. Regarding real GDP, the highest correlations are obtained by *Invest*, *Mon_agg*, *Pr_index*, *ImpExp*, *Nat_acc* and *GDP_star*. Regarding inflation, the highest correlations are obtained by *Int_rate*, *Ex_rate*, *Credit*, *Pr_index*, *Nat_acc*, *GDP_star* and *Hfood_pr*.

These choices then lead to the four models in Table 4. Model 1 refers to real GDP and Model 2 to inflation. Each model has two variants. In variant 1 (Models 1.1 and 2.1) we take only the lagged values of the dependent variable (and the intercept) as our focus variables, while all other variables are auxiliary, that is, we are uncertain whether they should be in the model or not. The same type of specification is found in Koop and Potter (2004). In variant 2 (Models 1.2 and 2.2) we have more focus variables. Here we argue that some of the extracted principal components must always be in the model, so that they should be treated as focus variables. For Model 1.2 we found that *Invest*, *ImpExp* and *Nat_acc* can be considered as focus variables. This is because the level of the real GDP directly depends on the current level of the mentioned principal components. For Model 2.2 we found that *Ex_rate*, *Pr_index* and *Hfood_pr* can be considered as additional focus variables. The reason is that these principal components directly impact the current level of inflation. Having specified the four factor-based dynamic models, we now turn to their estimation and forecasting simulation experiments.

## 5. An estimation simulation experiment

We conduct a comparison between BMA and WALS estimation results using Monte-Carlo simulations. We assume that we know the true data-generating process (DGP) and can therefore compare the estimates to the truth. We conduct the simulation experiments for one, two, and three lags, so that we gain insight on the performance of the WALS and BMA algorithms for various lag lengths. Tables 5 and 6 present the parameter values in the data-generating processes for the growth and inflation models, respectively.

TABLE 5. True data-generation process, Model 1 (Growth), Version 2

| Version 1 | One lag | Two lags | Three lags |
|---|---|---|---|
| *Intercept* | 0.50 | 1.20 | 3.00 |
| $GDP_{t-1}$ | 0.75 | 0.95 | 0.80 |
| $Invest_{t-1}$ | −0.30 | −0.70 | −0.40 |
| $ImpExp_{t-1}$ | −0.15 | 0.00 | 0.10 |
| $Nat\_acc_{t-1}$ | 0.60 | 0.15 | 0.55 |
| $Mon\_agg_{t-1}$ | 0.30 | 0.40 | 0.40 |
| $Pr\_index_{t-1}$ | 0.20 | 0.35 | 0.30 |
| $GDP\_star_{t-1}$ | −0.35 | 0.10 | −0.30 |
| $GDP_{t-2}$ | — | −0.60 | −1.30 |
| $Invest_{t-2}$ | — | 0.30 | 1.30 |
| $ImpExp_{t-2}$ | — | 0.30 | 0.40 |
| $Nat\_acc_{t-2}$ | — | 0.95 | 1.15 |
| $Mon\_agg_{t-2}$ | — | 0.40 | 0.70 |
| $Pr\_index_{t-2}$ | — | −0.25 | 0.90 |
| $GDP\_star_{t-2}$ | — | 0.05 | 0.90 |
| $GDP_{t-3}$ | — | — | −0.10 |
| $Invest_{t-3}$ | — | — | 0.75 |
| $ImpExp_{t-3}$ | — | — | 0.45 |
| $Nat\_acc_{t-3}$ | — | — | 0.60 |
| $Mon\_agg_{t-3}$ | — | — | 0.90 |
| $Pr\_index_{t-3}$ | — | — | −0.30 |
| $GDP\_star_{t-3}$ | — | — | −0.30 |
| $\sigma^2$ | 2.25 | 2.25 | 2.25 |

We randomly draw the errors $\{u_t\}$ from a standard normal distribution. Then, given the data-generating process and the values of the regressors, we generate the time series for real GDP growth or inflation, the dependent variables. Now that we have all the data, we estimate the parameters using the models and the BMA and WALS estimation

TABLE 6. True data-generation process for Model 2 (Inflation), Version 2

| Version 2 | One lag | Two lags | Three lags |
|---|---|---|---|
| *Intercept* | 1.00 | 0.10 | −2.00 |
| $INF_{t-1}$ | 0.10 | 0.80 | 1.40 |
| $Ex\_rate_{t-1}$ | 0.20 | 0.60 | −0.10 |
| $Pr\_index_{t-1}$ | 0.55 | −0.15 | −0.15 |
| $Hfood\_pr_{t-1}$ | 0.10 | −0.15 | −0.85 |
| $Int\_rate_{t-1}$ | −0.20 | 0.00 | 0.50 |
| $Credit_{t-1}$ | 0.10 | −0.55 | −0.50 |
| $Nat\_acc_{t-1}$ | −0.10 | −0.70 | 0.70 |
| $GDP\_star_{t-1}$ | −0.30 | −0.40 | 0.40 |
| $INF_{t-2}$ | — | 0.50 | 1.00 |
| $Ex\_rate_{t-2}$ | — | −0.50 | −0.15 |
| $Pr\_index_{t-2}$ | — | −0.50 | −0.75 |
| $Hfood\_pr_{t-2}$ | — | 0.40 | −0.50 |
| $Int\_rate_{t-2}$ | — | 0.50 | 0.60 |
| $Credit_{t-2}$ | — | 0.40 | 0.60 |
| $Nat\_acc_{t-2}$ | — | 0.40 | 0.80 |
| $GDP\_star_{t-2}$ | — | 0.40 | 0.40 |
| $INF_{t-3}$ | — | — | 0.30 |
| $Ex\_rate_{t-3}$ | — | — | 0.25 |
| $Pr\_index_{t-3}$ | — | — | −0.65 |
| $Hfood\_pr_{t-3}$ | — | — | −0.20 |
| $Int\_rate_{t-3}$ | — | — | −0.50 |
| $Credit_{t-3}$ | — | — | 0.60 |
| $Nat\_acc_{t-3}$ | — | — | −0.50 |
| $GDP\_star_{t-3}$ | — | — | 0.50 |

algorithms. This gives us parameter estimates. Next we draw new errors $\{u_t\}$, obtain new values for the dependent variable, and hence new parameter estimates. We repeat this 1000 times, and compute the simulation root mean squared errors (RMSE):

$$\mathrm{RMSE}_k^{wals} = \sqrt{\frac{1}{1000}\sum_{l=1}^{1000}(\beta_k^{wals_l} - \beta_k^{true})^2},$$

$$\mathrm{RMSE}_k^{bma} = \sqrt{\frac{1}{1000}\sum_{l=1}^{1000}(\beta_k^{bma_l} - \beta_k^{true})^2},$$

where $\beta_k^{true}$ denotes the true value of $\beta_k$, and $\beta_k^{wals_l}$ and $\beta_k^{bma_l}$ are the WALS and BMA estimates, respectively, for the $l$-th iteration.

TABLE 7. RMSE for the estimation simulations, Model 1 (Growth), Version 2

| Version 1 | WALS | BMA | WALS | BMA | WALS | BMA |
|---|---|---|---|---|---|---|
| *Intercept* | 0.0373 | 0.0375 | 0.0293 | 0.0270 | 0.0330 | 0.0399 |
| $GDP_{t-1}$ | 0.0192 | 0.0193 | 0.0292 | 0.0283 | 0.0247 | 0.0222 |
| $Invest_{t-1}$ | 0.0340 | 0.0341 | 0.0366 | 0.0362 | 0.0212 | 0.0216 |
| $ImpExp_{t-1}$ | 0.0129 | 0.0130 | 0.0116 | 0.0118 | 0.0101 | 0.0099 |
| $Nat\_acc_{t-1}$ | 0.0172 | 0.0173 | 0.0190 | 0.0199 | 0.0132 | 0.0122 |
| $Mon\_agg_{t-1}$ | 0.0164 | 0.0206 | 0.0142 | 0.0151 | 0.0134 | 0.0180 |
| $Pr\_index_{t-1}$ | 0.0099 | 0.0110 | 0.0100 | 0.0124 | 0.0104 | 0.0119 |
| $GDP\_star_{t-1}$ | 0.0154 | 0.0126 | 0.0132 | 0.0064 | 0.0202 | 0.0148 |
| $GDP_{t-2}$ | — | — | 0.0163 | 0.0170 | 0.0380 | 0.0395 |
| $Invest_{t-2}$ | — | — | 0.0146 | 0.0141 | 0.0480 | 0.0516 |
| $ImpExp_{t-2}$ | — | — | 0.0092 | 0.0099 | 0.0113 | 0.0119 |
| $Nat\_acc_{t-2}$ | — | — | 0.0105 | 0.0109 | 0.0161 | 0.0171 |
| $Mon\_agg_{t-2}$ | — | — | 0.0082 | 0.0098 | 0.0150 | 0.0194 |
| $Pr\_index_{t-2}$ | — | — | 0.0097 | 0.0088 | 0.0259 | 0.0259 |
| $GDP\_star_{t-2}$ | — | — | 0.0124 | 0.0065 | 0.0254 | 0.0288 |
| $GDP_{t-3}$ | — | — | — | — | 0.0056 | 0.0053 |
| $Invest_{t-3}$ | — | — | — | — | 0.0263 | 0.0244 |
| $ImpExp_{t-3}$ | — | — | — | — | 0.0124 | 0.0134 |
| $Nat\_acc_{t-3}$ | — | — | — | — | 0.0138 | 0.0140 |
| $Mon\_agg_{t-3}$ | — | — | — | — | 0.0161 | 0.0202 |
| $Pr\_index_{t-3}$ | — | — | — | — | 0.0104 | 0.0087 |
| $GDP\_star_{t-3}$ | — | — | — | — | 0.0109 | 0.0088 |

The results of the Monte-Carlo simulation are presented in Tables 7 (Growth) and 8 (Inflation). The main purpose of the simulations is to compare BMA and WALS estimates. WALS has certain theoretical and computational advantages, but does it in fact perform better than BMA? The simulations suggest that this might be the case, altough the difference is small. In the growth simulations, WALS achieve a lower RMSE than BMA for 88% (one lag), 53% (two lags), and 61% (three lags) of the parameters, thus outperforming BMA. In the inflation simulations, the percentages are somewhat lower: 39% (one lag), 59% (two lags), and 48% (three lags). Hence, WALS has a slight advantage over BMA.

The above estimation simulations were based on the assumption that the data-generating process and the model coincide. For example, if the DGP has one lag, then we use a model with one lag. This, of course, is not realistic situation, since in practice we don't know the true DGP and therefore the chance that our chosen model happens to be the DGP is small.

TABLE 8. RMSE for the estimation simulations, Model 2 (Inflation), Version 2

| Version 2 | WALS | BMA | WALS | BMA | WALS | BMA |
|---|---|---|---|---|---|---|
| *Intercept* | 0.0095 | 0.0089 | 0.0529 | 0.0538 | 0.0882 | 0.0910 |
| $INF_{t-1}$ | 0.0067 | 0.0062 | 0.0261 | 0.0257 | 0.0409 | 0.0416 |
| $Ex\_rate_{t-1}$ | 0.0061 | 0.0061 | 0.0126 | 0.0124 | 0.0174 | 0.0180 |
| $Pr\_index_{t-1}$ | 0.0065 | 0.0065 | 0.0161 | 0.0163 | 0.0269 | 0.0252 |
| $Hfood\_pr_{t-1}$ | 0.0067 | 0.0067 | 0.0200 | 0.0216 | 0.0341 | 0.0354 |
| $Int\_rate_{t-1}$ | 0.0052 | 0.0057 | 0.0197 | 0.0184 | 0.0455 | 0.0253 |
| $Credit_{t-1}$ | 0.0047 | 0.0038 | 0.0138 | 0.0163 | 0.0210 | 0.0202 |
| $Nat\_acc_{t-1}$ | 0.0046 | 0.0038 | 0.0117 | 0.0138 | 0.0198 | 0.0209 |
| $GDP\_star_{t-1}$ | 0.0052 | 0.0070 | 0.0101 | 0.0127 | 0.0171 | 0.0123 |
| $INF_{t-2}$ | — | — | 0.0195 | 0.0204 | 0.0341 | 0.0351 |
| $Ex\_rate_{t-2}$ | — | — | 0.0093 | 0.0073 | 0.0127 | 0.0107 |
| $Pr\_index_{t-2}$ | — | — | 0.0189 | 0.0187 | 0.0297 | 0.0318 |
| $Hfood\_pr_{t-2}$ | — | — | 0.0127 | 0.0120 | 0.0210 | 0.0215 |
| $Int\_rate_{t-2}$ | — | — | 0.0169 | 0.0135 | 0.0318 | 0.0252 |
| $Credit_{t-2}$ | — | — | 0.0115 | 0.0127 | 0.0164 | 0.0132 |
| $Nat\_acc_{t-2}$ | — | — | 0.0089 | 0.0092 | 0.0185 | 0.0224 |
| $GDP\_star_{t-2}$ | — | — | 0.0064 | 0.0092 | 0.0120 | 0.0108 |
| $INF_{t-3}$ | — | — | — | — | 0.0111 | 0.0105 |
| $Ex\_rate_{t-3}$ | — | — | — | — | 0.0117 | 0.0084 |
| $Pr\_index_{t-3}$ | — | — | — | — | 0.0134 | 0.0108 |
| $Hfood\_pr_{t-3}$ | — | — | — | — | 0.0115 | 0.0115 |
| $Int\_rate_{t-3}$ | — | — | — | — | 0.0289 | 0.0176 |
| $Credit_{t-3}$ | — | — | — | — | 0.0161 | 0.0161 |
| $Nat\_acc_{t-3}$ | — | — | — | — | 0.0084 | 0.0124 |
| $GDP\_star_{t-3}$ | — | — | — | — | 0.0097 | 0.0106 |

TABLE 9. RMSE for coefficient simulation in the case of misspecification, Models 1 and 2, Version 2

| Version 2 | Real GDP | | Inflation | | |
|---|---|---|---|---|---|
| Regressor | WALS | BMA | Regressor | WALS | BMA |
| *Intercept* | 0.0297 | 0.0302 | *Intercept* | 0.0520 | 0.0513 |
| $GDP_{t-1}$ | 0.0201 | 0.0202 | $INF_{t-1}$ | 0.0442 | 0.0437 |
| $Invest_{t-1}$ | 0.0356 | 0.0357 | $Ex\_rate_{t-1}$ | 0.0154 | 0.0153 |
| $ImpExp_{t-1}$ | 0.0131 | 0.0132 | $Pr\_index_{t-1}$ | 0.0297 | 0.0297 |
| $Nat\_acc_{t-1}$ | 0.0181 | 0.0182 | $Hfood\_pr_{t-1}$ | 0.0263 | 0.0264 |
| $Mon\_agg_{t-1}$ | 0.0175 | 0.0223 | $Int\_rate_{t-1}$ | 0.0107 | 0.0127 |
| $Pr\_index_{t-1}$ | 0.0103 | 0.0121 | $Credit_{t-1}$ | 0.0101 | 0.0123 |
| $GDP\_star_{t-1}$ | 0.0162 | 0.0121 | $Nat\_acc_{t-1}$ | 0.0167 | 0.0188 |
| — | — | — | $GDP\_star_{t-1}$ | 0.0107 | 0.0115 |

We now consider one case where the model is underspecified. More specificially, the DGP has three lags, but the model has only one lag. We estimate the parameters in the one-lag model and compare with the corresponding (true) parameters in the three lag DGP model. The results are presented in Table 9. Here also WALS appears to be outperforming BMA. For 88% (growth) and 61% (inflation) of the parameters, WALS achieves a lower RMSE than BMA.

## 6. A FORECAST EXPERIMENT

We conduct a second experiment, this time in forecasting rather than estimation. Suppose we use $T_1 < T = 42$ quarters on which we base our estimates. This leaves us $T_2 = T - T_1 > 0$ quarters for forecast experiments. The $h$-period forecast is given by

$$\hat{y}_{T_1+h} = \hat{\alpha}(L) y_{T_1+h-1} + \hat{\gamma}(L) f_{T_1+h-1} \qquad (h = 1, \ldots, T_2),$$

where $y$ denotes either GDP or inflation. In practice we would not know $f_{T_1+h-1}$ and $y_{T_1+h-1}$, when $h \geq 2$. So we would have to forecast these as well. In the experiment we use the observed values of $f_{T_1+h-1}$ and $y_{T_1+h-1}$, and not the forecasted value $\hat{y}_{T_1+h-1}$ when $h \geq 2$. Then we compute

$$\mathrm{RMSE}_{T_1} = \sqrt{\frac{1}{T - T_1} \sum_{h=1}^{T-T_1} (\hat{y}_{T_1+h} - y_{T_1+h})^2},$$

which depends on the estimation period $T_1$, the model, and the method (BMA or WALS). The results are presented in Tables 10 and 11.

In this case we have calculated the RMSE not only for BMA and WALS, but also for two additional algorithms of estimation: general-to-specific (GtS) model selection followed by estimation of the selected model, and ordinary least squares (OLS) of the unrestricted model. Including these standard forecasting algorithms allows us to compare model averaging algorithms with more traditional methods.

In general, the smaller is the estimation period $T_1$, the less accurate are the estimates and the forecasts, that is, the RMSE increases as $T_1$ decreases. This is to be expected, but it does not always happen. In particular the behavior for $T_1 = 35$ deviates. The explanation

TABLE 10. RMSE for ex-post forecast accuracy (Model 1 (GDP))

| | | | $T_1$ | | | | |
|---|---|---|---|---|---|---|---|
| Number of lags | Version | Method | 38 | 37 | 36 | 35 | 34 |
| 1 lag | 1 | WALS | 0.8557 | 0.9967 | 2.5503 | 6.1158 | 3.6533 |
| | | BMA | 0.8338 | 0.9726 | 2.4993 | 6.3675 | 3.5549 |
| | | GtS | 0.9606 | 1.1124 | 2.6739 | 6.0492 | 3.4711 |
| | | OLS | 0.9847 | 1.0726 | 3.2020 | 5.6782 | 3.6570 |
| | 2 | WALS | 0.8597 | 1.0181 | 2.8576 | 5.8070 | 3.6330 |
| | | BMA | 0.9416 | 1.1265 | 2.5818 | 5.9392 | 3.6009 |
| | | GtS | 1.1311 | 1.2979 | 2.3720 | 6.0667 | 3.5998 |
| | | OLS | 0.9847 | 1.0726 | 3.2020 | 5.6782 | 3.6570 |
| 2 lags | 1 | WALS | 2.2203 | 2.8415 | 3.6176 | 2.7037 | 2.6341 |
| | | BMA | 1.8333 | 2.3204 | 3.2558 | 2.4816 | 1.7849 |
| | | GtS | 2.0147 | 2.9072 | 3.1916 | 2.4471 | 1.5800 |
| | | OLS | 2.6610 | 3.3104 | 3.5279 | 3.2889 | 3.2062 |
| | 2 | WALS | 2.2162 | 2.8118 | 3.5271 | 3.1048 | 2.7043 |
| | | BMA | 2.2155 | 2.6711 | 3.6429 | 3.0343 | 2.5689 |
| | | GtS | 2.9367 | 3.4904 | 3.7139 | 3.1051 | 2.8986 |
| | | OLS | 2.6610 | 3.3104 | 3.5279 | 3.2889 | 3.2062 |
| 3 lags | 1 | WALS | 2.3276 | 2.5872 | 4.3087 | 4.5578 | 2.8051 |
| | | BMA | 2.1199 | 1.9616 | 3.2988 | 3.8783 | 3.0844 |
| | | GtS | 2.0535 | 2.5983 | 4.1221 | 4.5073 | 3.5460 |
| | | OLS | 2.5757 | 3.1871 | 4.4612 | 4.9230 | 3.1832 |
| | 2 | WALS | 2.2043 | 2.7098 | 4.1208 | 4.6082 | 3.2474 |
| | | BMA | 2.1169 | 2.8038 | 4.2715 | 4.6699 | 3.6258 |
| | | GtS | 2.8060 | 1.5199 | 3.8202 | 4.4603 | 4.3567 |
| | | OLS | 2.5757 | 3.1871 | 4.4612 | 4.923 | 3.1832 |

lies in the global financial crisis, which affected Armenia greatly. From the third quarter of 2008 (quarter 34 in our data set) to the second quarter of 2009 (quarter 37) Armenia's real growth of GDP decreased by 18%. The largest decrease (around 9%) in real GDP took place in the fourth quarter of 2008 (quarter 35). This large decrease in real GDP causes a large deviation of real GDP from its long-term trend, and this may explain (in part) why the RMSE values calculated for $T_1 = 35$ are relatively large, and for $T_1 = 36$ somewhat smaller. Two main conclusions emerge from the Tables 10 and 11. First, we see that the model averaging algorithms (WALS and BMA) outperform the more traditional methods like GtS and OLS. But the choice between WALS and BMA is still ambiguous. While in the estimation simulations we found that WALS performs better than BMA, we find in the

TABLE 11. RMSE for ex-post forecast accuracy (Model 2 (Inflation))

| | | | $T_1$ | | | | |
|---|---|---|---|---|---|---|---|
| Number of lags | Version | Method | 38 | 37 | 36 | 35 | 34 |
| 1 lag | 1 | WALS | 0.8542 | 0.7807 | 0.9050 | 0.8545 | 0.8993 |
| | | BMA | 0.8851 | 0.8810 | 0.9949 | 0.9373 | 0.9697 |
| | | GtS | 0.9906 | 1.0484 | 1.0819 | 1.0232 | 1.0198 |
| | | OLS | 0.9060 | 0.8448 | 0.8741 | 0.8788 | 0.8842 |
| | 2 | WALS | 0.8923 | 0.8061 | 0.9000 | 0.8813 | 0.8865 |
| | | BMA | 0.9579 | 0.8718 | 0.9787 | 0.9291 | 0.9421 |
| | | GtS | 1.0024 | 0.9252 | 1.0051 | 0.9481 | 0.9559 |
| | | OLS | 0.9060 | 0.8448 | 0.8741 | 0.8788 | 0.8842 |
| 2 lags | 1 | WALS | 1.6452 | 1.6568 | 1.5987 | 1.7970 | 1.5262 |
| | | BMA | 1.0726 | 0.9829 | 0.8997 | 1.0536 | 0.9385 |
| | | GtS | 1.0536 | 0.9371 | 0.7445 | 1.0959 | 0.8920 |
| | | OLS | 2.0139 | 2.1006 | 2.0722 | 2.3070 | 1.8852 |
| | 2 | WALS | 1.6357 | 1.6463 | 1.5935 | 1.7557 | 1.5967 |
| | | BMA | 1.1079 | 1.0094 | 1.0292 | 1.1293 | 1.3293 |
| | | GtS | 1.0076 | 0.9035 | 1.0614 | 1.1510 | 1.2761 |
| | | OLS | 2.0139 | 2.1006 | 2.0722 | 2.3070 | 1.8852 |
| 3 lags | 1 | WALS | 4.5016 | 3.8276 | 4.1335 | 3.9527 | 2.7138 |
| | | BMA | 1.2269 | 1.1040 | 1.0326 | 0.9662 | 1.1329 |
| | | GtS | 6.1520 | 1.1017 | 4.7159 | 4.7402 | 4.5196 |
| | | OLS | 6.1409 | 5.2268 | 5.7689 | 5.4534 | 3.5626 |
| | 2 | WALS | 4.2447 | 3.4789 | 3.9977 | 3.8076 | 2.4278 |
| | | BMA | 1.3646 | 1.2628 | 1.3900 | 1.2499 | 1.9155 |
| | | GtS | 0.9806 | 1.0851 | 2.0882 | 1.9551 | 1.7758 |
| | | OLS | 6.1409 | 5.2268 | 5.7689 | 5.4534 | 3.5626 |

forecasting simulations that BMA performs better than WALS in 2/3 of the 30 forecasts, both for growth and for inflation.

## 7. CONCLUDING REMARKS

In this paper we have applied two alternative model averaging algorithms (WALS and BMA) to the problem of estimating factor-based dynamic models. The estimated models have also been used to forecast two key macroeconomic variables, namely real GDP and inflation in Armenia. The advantage of using model averaging is that it allows all models to play a role in the estimation and forecasting, thus avoiding the problem of pretesting. A comparison of the WALS to the BMA algorithm does not reveal large differences in performance. The WALS algorithm has a stronger theoretical appeal, but in the current context there is not sufficent evidence to prefer one over the other. The simulations experiments show, however, that both model averaging algorithms outperform the more traditional algorithms such as GtS and OLS.

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