A Comparison of PAYG and Funded Pension Systems

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Abstract – The paper develops the models of pay-as-you-go (PAYG) and funded pension systems and compares the systems under certain parametrization. In deterministic environment the comparison shows that the funded pension system for the most of the considered cases has both cost and benefit advantage over the benchmark scenario of PAYG pension system. The analysis of benefits in stochastic environment shows that the distribution of replacement rate under the funded pension system first-order stochastically dominates that under PAYG pension system. Therefore, the funded pension system is more likely to deliver higher pensions at retirement.
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1. Introduction

The pay-as-you-go (PAYG) and funded pension systems are designed to provide pensions to the retirees. Under the PAYG pension system the government collects taxes from working population and distributes it among the retirees of that period (Heijdra, 2009; Blake, 2006a). This is the mechanism of intergenerational redistribution. The tax paid to the government can be viewed as a purchase of a "promise" or a "social contract" to receive pension in the retirement. The pension in this system depends solely on the ratio of retirees to contributing workers and on the growth rate of wages. The amount of tax paid by a worker during working years has no impact on the amount of pension received in the retirement. This creates an incentive to avoid tax payments. In addition, decreasing rate of fertility and increasing life expectancy have negative impact on the performance of PAYG pension system (Cipriani, 2014; WB, 1994).

Under the funded pension system (Heijdra, 2009; Blake, 2006) the worker makes contributions during working years that are being managed by professional asset managers

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and earn interest during accumulation period. Then the total amount of the fund, including individual contributions and earned interest, is used to provide pension to the retiree. This defined contribution (DC) pension system is subject to different risks (interest rate risk, longevity risk and etc.), but obviously there is a clear link between contributions made and pension received (Blake et al., 2001; Blake, 2006b). The latter has an advantage over PAYG pension system at least from the perspective of incentives to contribute into the system.

A number of empirical studies also show that historically wage growth rate has been lower than the rate of return to equity capital (Maddison, 1987; Siegel, 1992) which provides further advantage to the funded system. Given this evidence one might expect better performance of funded rather than PAYG pension system in the long run.

The factors above are the key reasons that lead to the deficit and unsustainability of PAYG pension systems. As a result, many countries now consider switching from PAYG pension system to the funded pension system or effectively mixing them.

The aim of this paper is to compare the alternative pension systems and address the question about the relative performance and hence desirability of systems under certain parametrization (consistent with the patterns described above). The paper derives and analytically describes how these two pension systems work. Then the paper computes the costs and benefits associated with the systems, performing numerical exercises, given the assumptions on the values of model parameters. The idea behind the comparison is to view it from the point of a potential participant of the systems. For instance, if participant faces the same cost during the working years under both funded and PAYG pension systems, then what matters for the preference over these two systems is the pension outcome received at retirement. Obviously, the preferred system is the one that is expected to deliver higher pensions at retirement. First, the paper compares the costs that workers face during their working years under two systems, in order to obtain identical results in the retirement and have the same replacement of final-year wage under two systems. Second,

it compares the benefits at the retirement (e.g. replacement rates) under both systems, assuming that during working years workers experience the same costs, regardless of the system they participate in. Third, the latter comparison is extended to stochastic environment. The wage rate, dependency ratio, the rate of asset return are supposed to develop randomly. Numerical exercise is performed running 10,000 Monte Carlo simulations.

The rest of the paper is organized as follows. Section 2 and 3 develop and describe the models of PAYG and funded pension systems, respectively. Section 4 compares costs and benefits of two pension systems in deterministic environment, based on numerical exercises. Section 5 compares the systems in stochastic environment and presents the outputs of simulation exercises. Section 6 concludes.

2. The Pay-as-You-Go Pension System

Under the pay-as-you-go (PAYG) financing, the amount collected through the payroll tax from contributing workers (young population, Y) is distributed among the beneficiaries-pensioners (old population, O) within the same period. That is:

$$\tau w(1+g)^n Y = pO \tag{2.1}$$

where τ is the payroll tax, $w(1+g)^n$ is the wage rate of contributing workers, n is the number of working years, p is the pension, Y and O are the numbers of contributing workers and pensioners, respectively. The wages are reset each year and they grow at a constant rate, i.e. given the monthly wage rate w for the first year, the wage rate for each month of the next year is $w(1+g)$ and thus the final-year wage rate is given by $w(1+g)^{n-1}$.

Taking into account that the final-year monthly wage rate of a pensioner is $w(1+g)^{n-1}$, from the equation (2.1) the following is obtained:

$$r \equiv \frac{\tau(1+g)}{d} \tag{2.2}$$

where r is the rate at which initial pension replaces final-year wage, $d \equiv \frac{O}{Y}$ denotes the system dependency ratio and shows the ratio of beneficiaries to contributing workers. Equation (2.2) shows that the replacement rate increases with an increase in wage growth rate, g, and payroll tax rate, τ, and with a decrease in system dependency ratio, d.

Current contributing workers support the PAYG pension system. This can be viewed as a purchase of a "promise" from the government (or signing a "social contract") to receive pension when they retire. Thus, there is no relationship between the amount they contribute into the system in the current period and the amount they will get when they retire. As shown above, the only things that matter for the latter are the tax rate, wage growth rate and the number of contributing workers in the retirement period.

An absence of a clear link between current contributions and future pension has an important implication. When the PAYG pension system is in place (thus the "promise" is already purchased), there is an incentive by current workers to avoid paying into the system. Regardless of their evasion, the system pays them pension after they retire since there are other workers who have contributed and supported the system.[2] The design of the system can be adjusted, for instance, to reflect the numbers of working years but the incentive of avoiding contributions or reporting lower wages remains in place.

In addition, empirical evidence on demographic development (Lutz et al., 2008) shows that with decreasing rate of fertility and increasing life expectancy the dependency ratio increases over time adding a heavy burden on the system. These shortcomings, among others, are the reasons why many countries now consider switching from PAYG to funded pension systems (or, more often, effectively combining two of them).

3. THE FUNDED PENSION SYSTEM

Under the funded financing, the pension is derived from contributions made by a worker on a personal account during working years (accumulation period), and its amount depends on the rate of return that pension fund managers earn during those years. The

[2]This is the well-known "free riding" behavior in economics (see, for instance, Mas-Colell et al. (1995)).

amount of contribution in turn is determined by the rate of contribution and the wage of a worker. Total amount of contribution depends on how long the worker has been accumulating those pension savings.

The contributions are supposed to be made monthly and contribution rate is denoted by c. As in Section 2, initial monthly wage of a pension plan member is w and total number of working years is n. The wage rate each year grows at a rate of g and final-year monthly wage is $w(1+g)^{n-1}$. The yearly rate of return on savings is i_y which corresponds to monthly return of $i = (1+i_y)^{\frac{1}{12}} - 1$.

The timing of the savings is as follows: at the end of the month worker receives the wage and makes the contribution into the personal account. At the end of the following month the saving earns monthly interest rate i. Thus, in the first year, when the monthly wage rate is w, capital accumulation will be given as:

$$wc(1+i)^{11} + wc(1+i)^{10} + ... + wc = wc\sum_{k=1}^{12}(1+i)^{k-1} = wc\frac{(1+i)^{12}-1}{i} \tag{3.1}$$

Similarly, the capital accumulation during second year and the year n is given by $w(1+g)c\frac{(1+i)^{12}-1}{i}$ (this is only for the second year contributions, without taking into account those for the first year) and $w(1+g)^{n-1}c\frac{(1+i)^{12}-1}{i}$, respectively. By the year of retirement the capital accumulation (the value of the funds) is given by:

$$F_n = \frac{(1+i)^{12}-1}{i}wc\frac{(1+i_y)^n - (1+g)^n}{(1+i_y)-(1+g)} \tag{3.2}$$

Equation (3.2) shows that accumulated capital F_n is increasing in the parameters w, c, i, i_y, g, n.

Under the funded pension system, the amount of pension is based on the capital accumulated during working years. We assume a constant level of pension, p, that can be withdrawn at the end of each month. The duration of retirement life is assumed to be N_1 (in months). The interest rate that the remaining funds earn each month is equal to i_1. We assume that the funds are fully used during the retirement life, that is:

$$F_n(1+i_1)^{N_1} - p(1+i_1)^{N_1-1} - ... - p = F_n(1+i_1)^{N_1} - p\sum_{k=1}^{N_1}(1+i_1)^{k-1} = 0$$

hence, p is equal to:

$$p = \frac{F_n(1+i_1)^{N_1}}{\frac{(1+i_1)^{N_1}-1}{i_1}} \tag{3.3}$$

In the special case when the remaining funds are assumed to earn no interest, $i_1 = 0$, the pension is equal to: $p = \frac{F_n}{N_1}$. Equation (3.3) shows that the pension increases with an increase in capital accumulated during working years, and an increase in the interest rate that the funds earn during retirement period. On the other hand, the pension decreases if the retirement lifetime is longer. Thus, replacement rate under the funded pension system is equal to:

$$r = c\frac{(1+i)^{12}-1}{i}\frac{1}{a}\frac{\frac{(1+i_y)^n}{(1+g)^n}-1}{\frac{(1+i_y)}{(1+g)}-1} \tag{3.4}$$

where $a \equiv \frac{\frac{(1+i_1)^{N_1}-1}{i_1}}{(1+i_1)^{N_1}}$. Replacement rate is higher if interest rates during accumulation and payout phases, i and i_1, contribution rate c, and duration of accumulation phase n are higher. The replacement rate decreases with an increase in wage growth rate g and the duration of retirement lifetime N_1. The impact of these parameters seems to be natural, except perhaps for the impact of g. The intuition behind it is the following: accumulated assets are not increasing as fast as wages and since the other parameters (the contribution rate, among others) are fixed the replacement rate decreases.

4. Comparison of Pension Systems in Deterministic Environment

This section compares the pension systems in deterministic environment. Since the systems are different (in terms of both parameters and variables), we base our comparison on the fact that an individual prefers the system that requires lower costs during working years assuming the same pension outcome or provides higher pension at retirement assuming the same costs. The first and second cases are studied in subsections 4.1. and 4.2, respectively. All computations are performed in MatLab.

4.1. **Costs of Pension Systems.** We examine the costs of two pension systems that generate the same replacement rate which is assumed to be equal to 40% throughout this section. This means that the systems are identical in the replacement rates, but are different in terms of costs they require. More specifically, numerical examples below compare the cost for a contributing worker during working years under different pension systems in order to provide the same replacement of the final-year wage rate.

Table 1 reports the tax rates under PAYG pension system that ensure replacement rate $r = 0.4$ for different values of dependency ratio and wage growth rate. In general, the results in Table 1 suggest that (i) increasing the dependency ratio increases the tax rate; and (ii) increasing wage growth rate decreases the tax rate. The lowest value of required tax rate (11%) is obtained when dependency ratio takes on the lowest value ($d = 0.3$, 1 retiree for 3 contributing workers) and wage growth rate is at the maximum ($g = 0.05$). In the worst scenario, when the dependency ratio is 1.5 (1 retiree for ≈ 0.67 contributing workers) and there is no wage growth, the required payroll tax rate is 60%. The moderate scenario is the one where the dependency ratio is 0.8 (1 retiree for 1.25 contributing workers). In this case the required rates of payroll tax are in the range of 30-32%, depending on the wage growth rate. The latter scenario is used as a benchmark in comparison of costs associated with different pension system.[3]

TABLE 1. Required Rates of Payroll tax (τ) to provide replacement rate $r = 0.4$ under PAYG pension system.

	$g = 0$	$g = 0.01$	$g = 0.02$	$g = 0.03$	$g = 0.04$	$g = 0.05$
$d = 0.3$	0.12	0.12	0.12	0.12	0.12	0.11
$d = 0.5$	0.20	0.20	0.20	0.19	0.19	0.19
$d = 0.8$	0.32	0.32	0.31	0.31	0.31	0.30
$d = 1$	0.40	0.40	0.39	0.39	0.38	0.38
$d = 1.5$	0.60	0.59	0.59	0.58	0.58	0.57

The results from the same exercise performed for the funded pension system (assuming 12-year retirement period) are reported in Table 2. Required contribution rates are computed for different values of interest rates during accumulation and payout phases, i

[3]This is chosen as a benchmark based on prior knowledge of actual dependency ratio.

TABLE 2. Required contribution rate (c) to provide replacement rate $r = 0.4$ under the funded pension system ($i_1 = i$).

	$g = 0$		$g = 0.01$		$g = 0.02$		$g = 0.03$		$g = 0.04$		$g = 0.05$	
	$n = 20$	$n = 40$	$n = 20$	$n = 40$	$n = 20$	$n = 40$	$n = 20$	$n = 40$	$n = 20$	$n = 40$	$n = 20$	$n = 40$
$i = 0$	0.24	0.12	0.26	0.14	0.29	0.17	0.31	0.20	0.34	0.23	0.37	0.27
$i = 0.01$	0.20	0.09	0.23	0.11	0.25	0.14	0.27	0.16	0.29	0.19	0.32	0.22
$i = 0.02$	0.17	0.07	0.19	0.09	0.21	0.11	0.23	0.13	0.25	0.15	0.27	0.18
$i = 0.03$	0.15	0.05	0.16	0.07	0.18	0.08	0.20	0.10	0.22	0.12	0.24	0.14
$i = 0.04$	0.13	0.04	0.14	0.05	0.16	0.06	0.17	0.08	0.19	0.09	0.21	0.11
$i = 0.05$	0.11	0.03	0.12	0.04	0.13	0.05	0.15	0.06	0.16	0.07	0.18	0.09

and i_1, wage growth rate g, and for a contributing worker with 20 and 40 working years. Interest rates in the accumulation and payout phases are assumed to be equal. The results in Table 2 suggest that (i) increasing the duration of accumulation period from 20 to 40 years substantially reduces the contribution rate; (ii) increasing interest rates both in accumulation and decumulation phases decreases contribution rate; and (iii) increasing wage growth rate increases contribution rate.

The costs associated with the funded pension system are the biggest (37%) when the funds earn no interest during accumulation and decumulation phases ($i = i_1 = 0$), wage growth rate is at the maximum possible level (5%) and the duration of accumulation phase is 20 years. The highest contribution rate, when saving period is extended to 40 years, is 27%, whereas the maximum payroll tax rate in the benchmark scenario of PAYG pension system is 32%. Thus, when the accumulation period is 40 years, the funded pension system outperforms the benchmark in all the considered cases.

In the most optimistic scenario, when the fund performance is the best (5% per annum both in the accumulation and payout phases) and there is no wage growth rate, monthly contributions over 20-year period are 11%. In this sense, the funded pension system outperforms the PAYG benchmark scenario (where in the optimistic scenario payroll tax rate is 30%), but as shown in Table 1, the minimum tax rate over all reported cases is also 11%. Furthermore, Table 2 reports that monthly contributions over 40-year period that yield 40% replacement rate at the retirement are only 3%.

TABLE 3. Replacement rate under PAYG pension system, given payroll tax $\tau = 0.1$.

	$g = 0$	$g = 0.01$	$g = 0.02$	$g = 0.03$	$g = 0.04$	$g = 0.05$
$d = 0.3$	0.33	0.34	0.34	0.34	0.35	0.35
$d = 0.5$	0.20	0.20	0.20	0.21	0.21	0.21
$d = 0.8$	0.13	0.13	0.13	0.13	0.13	0.13
$d = 1$	0.10	0.10	0.10	0.10	0.10	0.11
$d = 1.5$	0.07	0.07	0.07	0.07	0.07	0.07

In conclusion, when the accumulation period is 40 years, the funded pension system has a cost advantage over PAYG for all the considered cases (not only the benchmark). When accumulation period is 20 years, the funded pension system outperforms the benchmark scenario for all cases, except for three out of 36 (first, $i = 0$, $g = 0.04$ and $n = 20$; second, $i = 0$, $g = 0.05$ and $n = 20$; third, $i = 0.01$, $g = 0.05$ and $n = 20$). Moreover, in the considered optimistic scenarios the funded pension system performs much better than the benchmark. The common factor that affects both systems is wage growth rate that reduces the costs of PAYG, but increases the costs of the funded system.

4.2. **Benefits of Pension Systems.** We now turn to examine the benefits of two pension systems that generate the same costs. We assume that workers face the same costs during their working years under both pension systems throughout this subsection. In particular, the contribution rate in the funded pension system and payroll tax rate in PAYG pension system are assumed to be equal. Numerical exercises assume the payroll tax and contribution rate are equal to 10%. This makes two systems identical in terms of their costs, but they are different in terms of generated benefits as we show below.

Table 3 reports replacement rates of PAYG pension system. Some general patterns emerge from these results: (i) increasing dependency ratio decreases replacement rate; and (ii) increasing wage growth rate increases replacement rate. Among the considered scenarios, the highest replacement rate is 35% when dependency ratio is the lowest ($d = 0.3$, i.e. 3 contributing workers support 1 retiree) and wage growth rate is the highest (5%). This scenario is quite optimistic. On the other hand, the lowest replacement rate is 7% when dependency ratio is the highest ($d = 1.5$, i.e. ≈ 0.67 contributing workers

TABLE 4. Replacement rate under the funded pension system, given contribution rate $c = 0.1$ ($i_1 = i$).

	$g = 0$		$g = 0.01$		$g = 0.02$		$g = 0.03$		$g = 0.04$		$g = 0.05$	
	$n = 20$	$n = 40$	$n = 20$	$n = 40$	$n = 20$	$n = 40$	$n = 20$	$n = 40$	$n = 20$	$n = 40$	$n = 20$	$n = 40$
$i = 0$	0.17	0.33	0.15	0.28	0.14	0.23	0.13	0.20	0.12	0.17	0.11	0.15
$i = 0.01$	0.20	0.43	0.18	0.36	0.16	0.30	0.15	0.25	0.14	0.21	0.13	0.18
$i = 0.02$	0.23	0.57	0.21	0.46	0.19	0.38	0.17	0.31	0.16	0.27	0.15	0.23
$i = 0.03$	0.27	0.76	0.24	0.60	0.22	0.49	0.20	0.40	0.18	0.33	0.17	0.28
$i = 0.04$	0.32	1.01	0.29	0.80	0.26	0.64	0.23	0.52	0.21	0.43	0.19	0.36
$i = 0.05$	0.37	1.36	0.33	1.06	0.30	0.84	0.27	0.67	0.25	0.55	0.23	0.45

support 1 retiree), regardless of the level of wage growth rate. This scenario is another extreme case with the highest ratio of system dependency. The moderate scenario can be considered the one that assumes dependency ratio $d = 0.8$ (1.25 contributing workers support 1 retiree) which yields replacement rate of 13%. We use this scenario as a benchmark to compare with the results for the funded pension system.

Table 4 reports replacement rates for the funded pension system assuming the duration of retirement life is equal to 12 years. The results indicate that (i) increasing the duration of accumulation phase increases the replacement rate; (ii) increasing interest rates in accumulation and distribution phases increases the replacement rate; and (iii) increasing the wage growth rate decreases the replacement rate.

In the worst scenario, when interest rates in both accumulation and payout phases are equal to zero, wage growth rate is the highest, and accumulation period is 20 years, the funded pension system provides 11% replacement rate. This is slightly below the benchmark under PAYG pension system. Moreover, when the saving period is extended to 40 years, under the same assumptions the funded pension system outperforms PAYG pension system, providing 15% replacement of final-year salary.

In the most optimistic scenario, when interest rates in both accumulation and payout phases are the highest (5%) and there is no wage growth rate, the funded pension system provides 37% and 136% replacement rates to the worker who contributes over 20 and 40 years, respectively.

The two systems yield exactly the same benefit rate equal to 13% when (i) in both accumulation and payout phases funds earn no interest, wage growth rate is 3%, and saving period is 20 years; and (ii) in both accumulation and payout phases interest rates are equal to 1%, wages grow at 5% per annum and accumulation period is 20 years. Thus, the funded pension system has a benefit advantage over the benchmark of the PAYG pension system in all considered scenarios except for two (first, $i = i_1 = 0$, $g = 0.04$ $n = 20$; second, $i = i_1 = 0$, $g = 0.05$, $n = 20$). The common factor under both systems is wage growth rate which has a positive impact on the benefits under PAYG system and a negative impact on the benefits under the funded system.

5. Comparison of Pension Systems in Stochastic Environment: Simulation Output

This section compares the benefits under PAYG and funded pension systems in stochastic environment, performing simulation exercises. We assume that (1) the payment into the system is equal to 10% of the wage; (2) the wage profile is determined by the process outlined in Blake et al. (2013) and is assumed to be the same for the considered pension systems; (3) the asset's rate of return follows lognormal distribution; (4) the dependency ratio follows lognormal distribution; (5) in the payout phase under the funded pension system the individual is supposed to purchase an annuity, which guarantees a fixed pension for the infinite lifetime; (6) the dependency ratio under PAYG pension system is simulated only once in the year of retirement and is assumed to be the same for the entire retirement lifetime. The latter assumption is realistic, given the evidence on decreasing dependency ratio.

The simulation exercise generates pension outputs for each of the considered pension systems. Each of these outputs consists of wage and pension distributions (illustrated by constructing fan charts), replacement rates distribution (illustrated by drawing empirical distribution function and histograms), as well as the relevant statistics. The outputs

are generated by running 10,000 Monte Carlo simulations and using specifically written scripts in MatLab.

We assume that the individual is 23 years old and is expected to retire at age 63. Thus, accumulation period is 40 years. Under the funded and PAYG pension systems, lognormal distributions are assumed for asset's rate of return, $i \sim \log N(0.03, 0.07)$, and for dependency ratio, $d \sim \log N(0.8, 0.07)$, respectively. The rate of return on average is assumed to be 3% (which is a quite moderate assumption) and dependency ratio on average is 0.8. The latter is true under PAYG system, based on the actual data, especially in aging countries.

Figure 5.1 plots the wage and pension profile that the two systems are expected to deliver based on simulation. Even though the wage profile is the same, the expected pension under the funded and PAYG pension systems are substantially different with the former providing a considerably better outcome. By visual inspection of Figure 5.1, one can conclude that the worst scenario under the funded system seems to outperform almost all the scenarios under PAYG system.

FIGURE 5.1. Wage and Pension under the Funded and PAYG pension systems

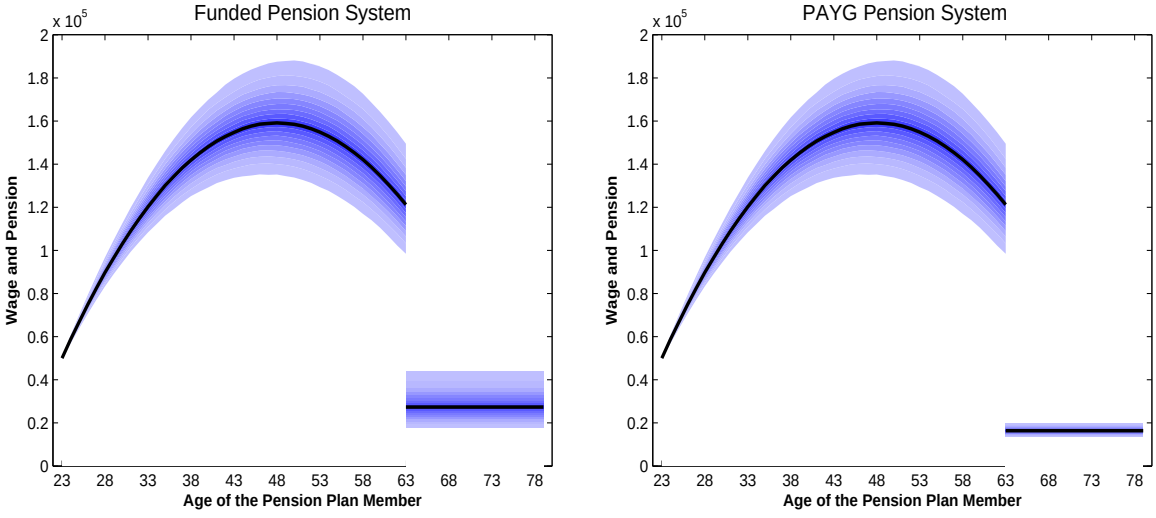

Figure 5.2 shows the empirical distribution of pension ratios. The simulated replacement rate under the funded system varies from about 0.1 at one extreme and to 0.6 at the

other. Under the PAYG pension system, the pension ratio ranges approximately from 0.1 to 0.2. Thus, the minimum value of pension ratio under the funded pension system is greater than under PAYG pension system. Moreover, the funded pension system allows for a possibility of 60% replacement of the final-year wage, whereas the best scenario under PAYG pension system promises only up to 20% replacement.

FIGURE 5.2. Empirical Distribution of Pension Ratio under the Funded and PAYG pension systems

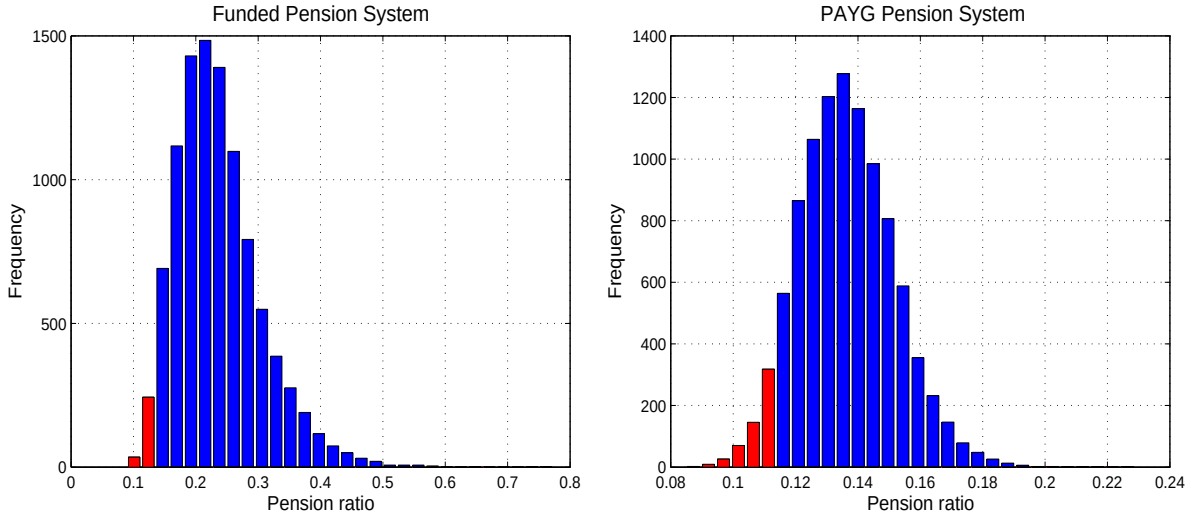

FIGURE 5.3. Empirical cumulative distribution functions of pension ratio under the Funded (dotted line) and PAYG (solid line) pension systems

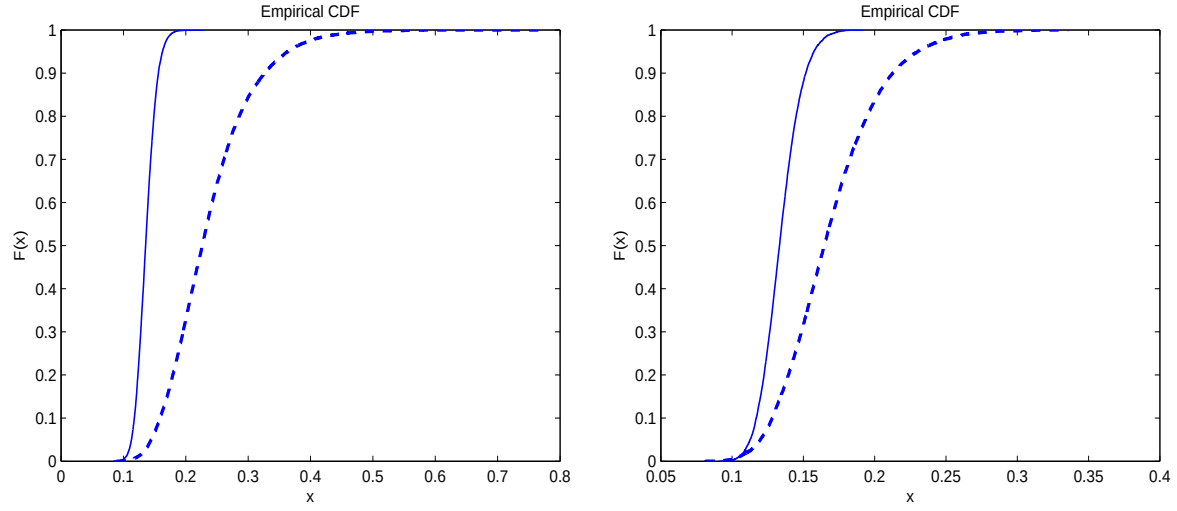

TABLE 5. Different statistics associated with simulated pension ratio

	Median	Mean	Std	VaR confidence level		
				80%	90%	95%
Funded Pension System	0.2264	0.2362	0.0682	0.1792	0.1590	0.1446
PAYG Pension System	0.1356	0.1364	0.0153	0.1233	0.1175	0.1129

The graph on the left in Figure 5.3 shows the empirical cumulative distribution function of pension ratio. The distribution of pension ratio under the funded pension system first-order stochastically dominates that under PAYG pension system, e.g. the probability that pension ratio falls below the certain level is always greater under PAYG pension system than under the funded pension system. For instance, the probability that pension ratio is lower than 20% is equal to 1 under PAYG pension system and is approximately 0.4 under the funded pension system.

Table 5 presents the relevant statistics. Under the funded pension system, the mean and median of pension ratio are approximately 24% and 23%, respectively, which are greater than the ones obtained under PAYG pension system. At the same time, examining the values of standard deviations one can conclude that the funded pension system is riskier than PAYG pension system. The other indicator of risk is VaR computed at different confidence levels. For instance, the 95% VaR for funded and PAYG systems are 0.14 and 0.11, respectively. The upper 5% quantiles of pension ratio are equal to 0.36 and 0.16 under the funded and PAYG pension systems, respectively. Therefore, the funded pension system outperforms PAYG system in the considered tail events as well.

When the age of the participant is changed to 43, so that the accumulation period is 20 years, the funded pension system is likely to deliver higher pensions (more precisely, the distribution of replacement ratio first-order stochastically dominates that under PAYG pension system) only if the provided mean return is set to greater or equal to 9.5% (Figure 5.3, the graph on the right).

In conclusion, although there is more risk associated with the funded pension system (given our assumptions on values of parameters), the distribution of pension ratio under the funded pension system first-order stochastically dominates that under PAYG system,

and therefore, the funded pension system is more likely to deliver higher pensions at retirement.

6. Concluding Remarks

The paper develops and analytically describes the models of PAYG and funded pension systems, and addresses issues concerning the relative performance of two pension systems. Since the model parameters are not the same, the comparison of two systems is somewhat difficult. The systems are compared from the standpoint of an individual participant. In particular, the comparison is based on the fact that the individual prefers the systems that requires lower costs during working years with the same pension outcome or provides higher pensions at retirement with the same costs.

The analysis of the costs under both pension systems that provide the same replacement rate reveals that the funded pension system has a cost advantage over the benchmark scenario under PAYG pension system in most of the considered cases. Moreover, an increase in the duration of accumulation phase strengthens the obtained results. The analysis of the benefits under both pension systems that require the same costs shows that the funded pension system again outperforms benchmark scenario under PAYG pension system in most of the considered cases. These computations, however, are done in deterministic environment which might not hold true for long term horizon. Thus, the paper also provides the comparison of benefits in deterministic and stochastic environments, assuming that there is a randomness in the development of key variables of pension systems. Under the basic parametrization the analysis of simulation outputs shows that distribution of replacement rate under the funded pension system first-order stochastically dominates that under PAYG pension system, and thus, the funded pension system is more likely to deliver higher pensions at retirement.

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