RUBINSTEIN BARGAINING AT THE COURT

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Abstract – This paper extends Rubinstein's well known sequential bargaining process to end exogenously after a finite number of stages such as in the case of court deliberations between two opposing parties. The court may allow these two players to bargain. But because the two parties may have different preferences they may not be able to reach an agreement. The presence of a judge, the mediator, may bring about a final decision if the two parties fail to arrive to an agreement.
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1. INTRODUCTION

An interesting topic of analysis for game theorists is how individual agents reach mutually beneficial agreements and what can hinder these agreements. There exist various factors that make mutually beneficial improvements unattainable. This can be due to incomplete or asymmetric information. To simplify matters, economists usually narrow their analysis to rational agents with infinite computational capacity and complete information. Both arguments are unrealistic, but are often justified. Though in the short run (i.e. with time and capacity constraints) people's behavior can follow irrational patterns, in the long run they will have enough time and information to behave optimally.

This work will discuss bargaining situations where there is both complete and incomplete information.[1] Under both situations the actions of individuals are based on optimizing decisions. The specific setting that I introduce can occur in a courtroom. There are two parties, or players, and a judge. Both players want a share of some object. Players

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[1]A bargaining situation with complete information means that everything related to the bargaining is known to the parties bargaining. Consequently, a bargaining situation with incomplete information means that there are some information that are not known to the parties bargaining.

have preferences, which are not necessarily the same.[2] Each of them wants to gain maximal utility given their preferences and their beliefs[3] of the other player's preferences. I will analyze two settings, one with perfect information and the other with imperfect information, with the latter related to the court's decision. The court allows the players to bargain.[4] The bargaining can end internally if the players reach an agreement. However, there is also an externally set upper bound on the number of bargaining periods. After some period, if the parties will not have arrived at an agreement, the court will make a decision. It is interesting to analyze how the presence of the judge will influence the decision of players in both settings, where there is complete and incomplete information.

In section 2, I review the literature by introducing relevant papers for my thesis. Section 3 will describe the basic model for my thesis: the Rubinstein bargaining model. Finally, section 4 will analyze the model in a courtroom, with perfect and imperfect information, which represents a modified version of Rubinstein bargaining model.

2. Literature Review

Bargaining theory is two-pillared, embracing both Nash's axiomatic bargaining solution[5], and Rubinstein's solution to the infinite-horizon bargaining with alternating offers.[6] Rubinstein (1982) has shown that an infinite-horizon bargaining game with alternating offers has a unique subgame-perfect equilibrium. Nash's and Rubinstein's solutions become close in the case when the discount rate approaches 1 (Binmore, Rubinstein, and Wolinsky, 1986). There have been various modifications of Rubinstein's original model. For instance, Rubinstein himself (1985) considered a game of incomplete information. He showed that there is a connection between the equilibrium and what expectations (beliefs) Player 1 has about Player 2. Muthoo (1989) relaxed the assumption about commitment,

[2]In my analysis I consider that preferences of players are described by the same utility function for simplicity.

[3]This is in the case when there is imperfect information during bargaining.

[4]The order of bargaining will be described later.

[5]This work is closely related to Rubinstein bargaining model. Therefore, I will not discuss Nash bargaining problem.

[6]In the next section, I will describe the Rubinstein bargaining model in detail and will also introduce the results.

i.e. now the proposer is allowed to change his offer after proposing without incurring any costs. The main result of this paper is that if the discount factor of the players is greater than 0.7, then any division of the pie can be the subgame-perfect equilibrium of this game. Moreover, in equilibrium, a proposer does not change his mind. In the paper by Ponsati and Sakovics (1998) players are allowed to leave the game after not coming to an agreement, however both receive zero payoffs. They showed that there are infinitely many subgame-perfect equilibrium outcomes.

The situation described in the courtroom has similarities[7] with the extensions mentioned above. However, the main assumption, through which this model differs from the others, is the presence of the social planner (which is the judge in the courtroom).

3. Rubinstein Bargaining Model

There are two players N = $\{1,2\}$ who will divide a pie of size one. The players know about each others preferences (i.e. there is complete information in the game). The game is played in rounds, and in each round, players make offers to each other sequentially concerning how to divide the pie between them. The game ends whenever one player accepts the offer of the other (i.e. the game is infinite horizon). An outcome of the game is a division $x \in [0,1]$, where x is the share of Player 1 and $(1-x)$ is the share of Player 2. The utility of Player 1, if an agreement is achieved in round t, is given by $U_1(x,t) = x + \delta^t$, where $t \in N$ is the round of the game, when the players agreed, and x is the share of Player 1. Accordingly, the utility of Player 2 is $U_2(x,t) = (1-x)\,\delta^t$.

Rubinstein (1982) considered two types of models:

1. players incur fixed bargaining costs for each period ($c_1$and c_2)

2. players have fixed discounting factors (δ_1 and δ_2).

Rubinstein's results for the first model are as follows: when players incurred bargaining costs for each period (c_1 and c_2), then

→if $c_1 < c_2$, then Player 1 gets the whole pie,

→if $c_1 > c_2$, then Player 1 receives only c_2,

[7] These are incomplete information, outside option.

→if $c_1 = c_2$, then any partition from which Player 1 receives at least c_1 is a perfect equilibrium partition (P.E.P.).

The results for the second model, when players have fixed discounting factors (δ_1 and δ_2), are that there is one P.E.P. Player 1 obtaining $\frac{1-\delta_2}{1-\delta_1\delta_2}$.

Notations

T - duration of the game set by the court

x_T - share decided by court to give to Player 1

$(1 - x_T)$ - share decided by court to give to Player 2

x_1 - share that Player 1 will get

x_2 - share that Player 2 will get

δ_1 - discount factor of Player 1

δ_2 - discount factor of Player 2

4. Adding a Judge to Rubinstein's Model

The game I am offering is similar to Rubinstein's. I made the game finite horizon by introducing a social planner (judge). A Round is defined as "*Player 1 makes an offer to Player 2 and Player 2 either accepts or rejects*". Therefore, the action of Player 1 is to make an offer in every round, and the action of Player 2 is to accept or reject. Formally,

$A_1 = \{(x_t, 1 - x_t)\, s.t.\, x \in [0, 1]\,,\, t \in [0, T]\}$

$A_2 = \{y_t \in \{Y, N\}\,,\, t \in [0, T]\}$

An agreement has to be found in a Round $t < T$. If an agreement has not been found until Round T, then the judge will make a decision in Round T: give x_T to Player 1 and $(1 - x_T)$ to Player 2.

I will analyze this modified version of Rubinstein's model by considering several cases about x_T:

√ x_T is given exogenously

√ x_T is stochastic.

Let us start observing the model in the case when x_T is given exogenously. I will also assume that there is complete information in the game. Players know each others preferences, know x_T , and the court knows the preferences of the players.

Theorem 1: *Let $\delta_1 = \delta_2 = 1$.*[8] *Then $\exists$ equilibrium s.t. Player 1 proposes not less than x_T , and Player 2 accepts not less than $(1 - x_T)$.*

Proof: Any offer of Player 1 to Player 2, which is less than $(1 - x_T)$, will be rejected.

The proof is quite trivial. Player 2 has a strategy to ensure $(1 - x_T)$: "reject all offers of Player 1, which give him less than $(1 - x_T)$, by getting $(1 - x_T)$ in the final period, when the court decides". Therefore, Player 1 will propose a partition, where he will get x_T , leaving $(1 - x_T)$ share for Player 2.

So, we proved that $(x_T, 1 - x_T)$ partition is an equilibrium.

Theorem 2: *Let $\delta_1 \neq \delta_2 > 0$. Then in every SPE, the agreement will be achieved in Round 1, when the first offer of Player 1 is accepted by Player 2.*

Proof: I will use the backward induction solution method. This is a common solution method when there is a perfect information in the game, which is exactly the case that we have. There is no uncertainty about the preferences of the players and the court's decision. Following the steps of backward induction, we start solving the game from the end. The solution at the final T round is $(x_T, 1 - x_T)$. At $T - 1$ round Player 1 will offer a partition, which will give him $x_1 \geq \delta_1 x_T$ (i.e. at least as much as he will get in the end). So, Player 2 will get from the offer $x_2 = 1 - x_1 \geq 1 - \delta_1 x_T$. However, Player 2 has a strategy: "reject all offers that will give him less than $\delta_2 (1 - x_T)$". Therefore, Player 2 will accept all offers such that $x_2 \geq \delta_2 (1 - x_T)$.

So, we have two inequalities for Player 2:

(1) $$x_2 \leq 1 - \delta_1 x_T$$

[8] *There can happen such a situation, when the court procedure is short (1 or 2 periods).*

(2) $$x_2 \geq \delta_2 (1 - x_T)$$

If this system of inequalities does not have a solution, it will mean the game will not end with the players agreeing. Consequently, the necessary condition for having an equilibrium is:

$\delta_2 (1 - x_T) < 1 - \delta_1 x_T$

$\Rightarrow (\delta_1 - \delta_2) x_T < 1 - \delta_2$

$1 - \delta_2 > 0 \Rightarrow (\delta_1 - \delta_2) x_T > 0$

$\Rightarrow (\delta_1 - \delta_2) > 0 \Rightarrow \delta_1 > \delta_2$

So, $\delta_1 > \delta_2$ is the necessary condition for equilibrium.

Solving the system of inequalities (1) and (2) for Player 2, we will get that any offer of Player 1, such that $x_2 \in [\delta_2 (1 - x_T), 1 - \delta_1 x_T]$, is acceptable by Player 2. Now, let us think what offer Player 1 will make from the interval mentioned above. Clearly, Player 1 has an advantage because in each round he always makes an offer. Therefore, he will offer the minimal share $\delta_2 (1 - x_T)$ for Player 2 and get a maximal share $1 - \delta_2 (1 - x_T)$. Continuing the solution by going back until first round, we will get an equilibrium interval for Player 2 $x_2 \in \left[\delta_2^T (1 - x_T), 1 - \delta_1^T x_T\right]$. Again, Player 1 will make an optimal offer by giving minimal share $\delta_2^T (1 - x_T)$ for Player 2 and therefore receiving a maximal share $1 - \delta_2^T (1 - x_T)$.

So, agreement will be found in Round 1 with each player receiving respectively $x_1 = 1 - \delta_2^T (1 - x_T)$and $x_{2=}\delta_2^T (1 - x_T)$, provided that $\delta_1 > \delta_2$. Otherwise the game will end at period T by the court's decision $(x_T, 1 - x_T)$.

Let us make some observations about the equilibrium: *"who will get the bigger share in equilibrium and under what conditions?"*

Let us compare x_1 and x_2 and find out when Player 1 will get the bigger share. Mathematically,

$x_1 > x_2 \Rightarrow 1 - \delta_2^T (1 - x_T) > \delta_2^T (1 - x_T)$

$\Rightarrow 1 > 2\delta_2^T (1 - x_T)$

$\Rightarrow \delta_2^T (1 - x_T) < \frac{1}{2}$

$\Rightarrow x_T > 1 - \frac{1}{2\delta_2^T}$

So, when $x_T > 1 - \frac{1}{2\delta_2^T}$, Player 1 will get bigger share of the pie in equilibrium.

Similarly, let us find when Player 2 will get the bigger share. Mathematically,

$x_1 < x_2 \Rightarrow 1 - \delta_2^T (1 - x_T) < \delta_2^T (1 - x_T)$

$\Rightarrow 1 < 2\delta_2^T (1 - x_T)$

$\Rightarrow \delta_2^T (1 - x_T) > \frac{1}{2}$

$\Rightarrow x_T < 1 - \frac{1}{2\delta_2^T}$

So, when $x_T < 1 - \frac{1}{2\delta_2^T}$, Player 2 will get more than Player 1 in equilibrium.

Let us look at the case when $x_T = 1 - \frac{1}{2\delta_2^T}$. What will each player get?

$x_1 = 1 - \delta_2^T (1 - x_T) = 1 - \delta_2^T \left(1 - \left(1 - \frac{1}{2\delta_2^T}\right)\right)$

$= 1 - \delta_2^T \frac{1}{2\delta_2^T} = 1 - \frac{1}{2} = \frac{1}{2}$

$x_2 = \delta_2^T (1 - x_T) = \delta_2^T \left(1 - \left(1 - \frac{1}{2\delta_2^T}\right)\right) = \delta_2^T \frac{1}{2\delta_2^T} = \frac{1}{2}$

Corollary 1: *When $\delta_1 = \delta_2 = \delta > 0$, the equilibrium shares for players will be respectively $x_1 = 1 - \delta^T (1 - x_T)$and $x_2 = \delta^T (1 - x_T)$.*

Proof: Follows from Theorem 2 by taking δ_1 instead of δ_2 in the equilibrium solution.

Similarly, the conditions for players getting a bigger share in equilibrium are:

$x_T > 1 - \frac{1}{2\delta_2^T}$ for Player 1,

$x_T < 1 - \frac{1}{2\delta_2^T}$ for Player 2,

and when $x_T = 1 - \frac{1}{2\delta_2^T} \Rightarrow x_1 = x_2 = \frac{1}{2}$

Particular case: Let us denote by $(\alpha, 1 - \alpha)$ the solution, that the court wants the players to achieve privately. The question of interest is "what rule (x_T) should the court choose in order to influence the players negotiations in achieving the solution $(\alpha, 1 - \alpha)$". This problem is similar to the initial situation when players know x_T, exogenously given. Recall the solution of that problem:

$x_1 = 1 - \delta_2^T (1 - x_T)$

$x_2 = \delta_2^T (1 - x_T)$.

The proposed question becomes a simple mathematical problem, which we can write as the following system of equations:

$$x_1 = \alpha \tag{3}$$

$$x_2 = 1 - \alpha \tag{4}$$

$\Rightarrow$

$$1 - \delta_2^T (1 - x_T) = \alpha \tag{5}$$

$$\delta_2^T (1 - x_T) = 1 - \alpha \tag{6}$$

Solving this system with respect to x_T we will obtain the desired solution for the court: $x_T = 1 - \frac{1-\alpha}{\delta_2^T}$

So far, the court modified model has been analyzed by assuming perfect information in the game. Let us relax that assumption. Now I will assume that players do not know which decision rule the court will apply.

Extension 1*: Court applies stochastic rule (x_T).*

Theorem 3*: If the beliefs of players about the division rule of the court are greater than $\frac{1}{2}$, then the only equilibrium that can be achieved is the court's decision at final T round.*

Proof: Assume players have the following beliefs about court's decision: Player 1 thinks that he will get the bigger share in final T round, conversely, in his turn Player 2 also thinks that he will get the bigger share at the end of the game. Clearly, whatever o_er Player 1 makes, it will be less than Player 2 expects to get from the court in the end.

Therefore, no agreement can be found between players i.e. the game will end by the court's decision. _

Let us imagine the following situation. There are two parties. Each party has a lawyer, who can make a private call to the judge. It could happen that the judge gives different information depending on the lawyer's question or that the lawyer might misunderstand the information coming from the judge. This will be translated in my model as the judge sends a noisy signal to each player. This will become a new setting of the model in a court, where players do not know the decision rule of the judge and also the judge sends noisy signals to the players.

Let me formalize this new setting as Extension 2, for which I have no complete solution. I will make some predictions about this new setting.

Extension 2*: The rules of the game are the same. In this case I again assume that players do not know the decision rule of the court. I will further complicate the model by assuming that the judge sends noisy signals to players.*

Let us denote by s_1 the noisy signal for Player 1 and by s_2 the noisy signal for Player 2. The dynamics of the game is the following. When Player 1 receives the signal s_1, he forms an expectation about what he will get from the court at the end of the game. Let us denote Player 1's expectation by μ_1. Then, consistent to his expectation μ_1, Player 1 will make an offer, denoted by o_1.

$s_1 \rightarrow \mu_1 (s_1) \rightarrow o_1 (\mu_1)$

Player 2 also receives a signal, s_2. However, unlike Player 1, Player 2 does not form his expectation based only on the noisy signal coming from the court. He also observes the offer of Player 1, o_1. So, after having the information about s_2 and o_1, Player 2 forms his expectation denoted by μ_2. Then, Player 2 will decide whether to accept or reject o_1.

$s_2, o_1 \rightarrow \mu_2 (s_2, o_1) \rightarrow \{Y, N\}$

The question of interest is "*What offer will Player 1 make?*"

Let us suppose that the game is just one period. However, even in this case it is difficult to find what o_1 will be. Will Player 1 offer the partition $(\mu_1 1 - \mu_1)$? What if Player 2 expects to get more from the court? So, even in the one-period game, it is uncertain what o_1 will be.

Let us expand the game into$T$ periods. In this case one can make the following prediction. Round by round players will start to learn the expectations of each others and there will exist some $t \in [0, T]$, when players will guess completely each others preferences and will come to a reasonable agreement. So, the bigger is T, the more time players will have to negotiate and learn each others expectations.

However, these are only predictions. It will be interesting to look into this problem in a more detailed way and try to find o_1 and $t \in [0, T]$.

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