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Optimal Portfolio Structure for Investments in the International Financial Market: The Example of the Central Bank of Armenia

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Abstract

The present research focuses on theoretical and practical issues of portfolio management, particularly constructing an optimal investment portfolio which will best suit the specific preferences of the investor. Central factors to be considered in the context of investor preferences would be the latter's willingness to be exposed to risk and his expectations in terms of return from these investments. Expected portfolio return and standard deviation are used as quantitative measurements of investment decision making factors. At the outset of research, the Markowitz's mean-variance model is used to determine the optimal investment portfolio, utilizing time series data on a number of financial instruments available to the Central Bank of Armenia (as the exemplary investor) to estimate the efficient investment frontier and evaluating the investor's degree of risk aversion on the basis of previous research. Principal shortcomings of the model and its use for the CBA are identified to serve as directions toward further improvement.

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INTRODUCTION

Sophisticated financial markets and a well-developed financial infrastructure are an integral part of all developed market economies worldwide. They provide the investment environment in which those who hold excess funds (investors, which comprise a wide spectrum of individual, corporate and governmental bodies) offer those funds to economic entities in need of such funds. This intermediary role of the financial system is highlighted by the fact that separate economic entities are not self-sufficient in terms of fulfilling their needs in investment resources, as well as by common practice of timing discrepancies between financial flows among market agents. To facilitate these flows, the processes of lending and borrowing, as well as investing in real assets, are most often intermediated by various types of financial instruments, such as equities, debt instruments, derivatives, etc.

In essence, a strong financial system serves for investors as a means to better utilize their free financial resources and be more flexible in their profit-making activities at various market conditions. One may consider such issues as yet irrelevant in terms of timing and priority in a developing economy such as that of Armenia, where the transition to market is not yet complete and where the financial markets are mal-developed. Further, this reasoning may equally be applied not only to Armenia, but other "transitional" countries which undergo similar paths of development. At its present position in the process of transition, Armenia has reported significant successes in the development of its banking sector, which represents the single most mature part of the country's financial infrastructure and serves as a most popular destination for individual investors' funds. Further, this sector provides for much less uncertainty in terms of investment decision making-factors, in that credit ratings, financial conditions and indicators of reliability of banks are transparent and comparatively stable along with prospects of return on investment. Whereas the evaluation of the risk-return trade-off is much more complicated and less deterministic in the case of financial markets, the outlook – much less predictable.

All of the above being said, we nevertheless find that knowledge and skills in optimal portfolio construction present a weighty milestone in raising confidence towards the financial markets and motivation for investment. More importantly, numerous investors are at present capable of entering the international financial market with their investments. Further, the successful and economically justified activity of capable investors in the international financial market should, at the end of the day, have an immeasurable positive impact on the development of the country's financial infrastructure itself. Studies indicate that the successful performance of the country's key agents in the international financial markets can be an important factor in securing its financial position, boosting its economic credibility and attracting investments into the country[1].

The history of financial markets in the Western countries over the last century has produced numerous theoretical and practical approaches to constructing optimal investment behavior. The risk-return trade-off and ways to determine the best combination of these have been the topic of many economists, financial analysts and consultants. They have successfully addressed empirical return patterns for groups of financial instruments, the logic and the psychology of investor behavior and the optimal portfolio solutions for specific types of investors. Most renowned works in these directions include those of H. Markowitz, D. Tobin, J. Pratt, K. Arrow, W. Sharpe, Z. Bodie, R. Pindyck, I. Friend and M. Blume, E. Fama and others.

[1] *See, for example,* K. Avanesyan, "Problems of state reserve management in RA," Ph.D. thesis, Yerevan 2002; M. Galstyan, "Management of international reserves and the foreign exchange rate," Ph.D. thesis, Yerevan 2003.

We have chosen as the exemplary investor for this research the Central Bank of Armenia for a number of reasons. First of all, the CBA is one of the few economic entities in the country who are active on the international financial markets. Secondly, the motivational circumstances and decision-making factors for the Central Bank's investments are overly interesting for consideration in terms of their uniqueness (the CB being a state non-commercial entity with a mandate for macroeconomic financial stability and its monetary functions) and applicability to standard economic reasoning (investment activity being commercial in its nature anyway) at the same time.

On one hand, the CBA is an investor in the international financial market willing to attract gains from its investment activity and looking to optimize its investment position. In this respect the CBA differs little from the classic profit-seeking investor. However, factors determining the Bank's optimal investment position differ from that of the purely commercial institution in that they are influenced by stronger security constraints, as well as the Central Bank's economic stabilization and monetary objectives.

Generally, there is widespread agreement on the "classic trilogy of objectives"[2] for investment decision making: security, liquidity and return. The objective of investment decisions, in this context, should be to maximize return *subject to* the maintenance of sufficient security of the assets and adequate liquidity. The constraints on security, liquidity and other applicable indicators vary from investor to investor. Essentially those altogether determine the extent of an investor's risk aversion, which determines how inclined is the investor to take extra risk in order to obtain higher returns on his investment portfolio.

In the case of our exemplary investor, it can be said that "it is generally accepted that central banks are fully entitled to so invest their reserves as to maximize the gain they can make on them,"[3] even though we note that an investor such as the CBA should always conduct its investment activity in such a way that it does not conflict with its mandate, destabilize markets or take advantage of privileged information. Meanwhile, the classic trilogy of objectives exercised towards a central bank suggests that the objective of return is essentially only the third most important of the three. The Armenian Law on the Central Bank states that "the primary criterion for allocation of reserves shall be the security and liquidity of the allocated resources."[4] At least at the intuitive level, it appears from this that the CBA can be characterized with a relatively risk-averse approach to its reserve management operations, and that risk minimization should most likely play a major role in its investment decision making.

Taking into account the above reasoning coupled with the Central Bank's capital allocation principles, as well as possible subjective constraints dictated by the specifics of CBA's mandate and objectives as a state body, further in the present research we try to determine the optimal portfolio structure for CBA financial market investments according to the Markowitz model. The research further aims to discuss the implications of the abovementioned model, identify its advantages and drawbacks as applied to specific circumstances of Armenia and the CBA, point to acknowledged shortcomings and directions of further improvement of the presented analysis. The next section presents a theoretical background on Markowitz's mean-variance optimal portfolio selection model. We then put the theory to work by applying it to a CBA-specific portfolio and determining the optimal choice of the investor using an intuitive indifference function. To conclude the analysis, we

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[2] J. Nugee, "Foreign Exchange Reserves Management," Bank of England CCBS Handbook No. 19, Nov. 2000; p. 12.
[3] Op. cit., p. 29.
[4] The Republic of Armenia Law on The Central Bank of the Republic of Armenia (adopted 30.06.1996, amended and revised), Article 52.2.

identify the shortcomings of the utilized model, which will in the future serve as priority directions to improve the theoretical and practical worth of the presented research.

THEORETICAL BACKGROUND

Markowitz's mean-variance efficient portfolio selection is one of the most widely used approaches in solving portfolio diversification problems. Published back in 1952[5], this model is still viewed as one of the origins of the modern portfolio theory. Harry Markowitz proposes a single holding period model, at the beginning of which the investor possesses a certain amount of money (initial wealth) to be invested in securities. The investor has an objective of maximizing his terminal wealth, or the growth thereof over the holding period, which is equivalent to selecting an optimal portfolio from a set of possible security portfolios with a maximal rate of return on the portfolio. However, as the author himself notes, "the hypothesis (or maxim) that the investor does (or should) maximize discounted return must be rejected"[6], since the rational investor also wants the *expected* returns from his investments to be as certain as possible. This means that the investor seeks two conflicting objectives – to maximize expected return and minimize uncertainty or *risk* – which must be balanced against each other when making the investment decision. Markowitz suggests that "there is a portfolio which gives both maximum expected return and minimum variance"[7] and commends this portfolio to the investor.

The rate of return on a portfolio is calculated as the increase in investor wealth over the holding period:

$$r_p = \frac{W_1 - W_0}{W_0},$$

where W_0 denotes the aggregate purchase price of the portfolio at $t=0$, or the investor's initial wealth, and W_1 denotes the aggregate market value of these securities at the end of the holding period $t=1$, which essentially represents the investor's terminal wealth.

The rate of return on the investment portfolio in this model is not known precisely, and thus should be viewed as a random variable characterized by its first two moments: *(i)* the mean or the expected value and *(ii)* the standard deviation (or, as used from time to time, the variance). The former represents a quantitative measure of possible gains from the investment, while the latter measures the extent of risk typical for that investment. Markowitz asserts that the investor, in making a decision on which portfolio to invest in, should estimate the expected return and standard deviation of each portfolio and then choose the one that gives a "best" combination of the two (maximum return and minimum risk).

It should be noted that there is significant criticism of the use of the variance or the standard deviation (the square root of the variance) as a measure of risk. First, it considers the possibility of returns both below and above the expected return, whereas investors to not view the latter as an unfavorable outcome and do not associate those with risk. Further, variance is only one measure of how the returns vary around the expected return and does not provide for accurate results when the probability distribution for a portfolio's returns is not symmetrical. However, in cases when this

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[5] *See* H. Markowitz, "Portfolio Selection," *Journal of Finance* vol. 7, no. 1, March 1952; p. 77-91.
[6] Op. cit., p. 77.
[7] Op. cit., p. 79.

probability distribution can be approximated by a normal distribution (which is most often the case when analyzing returns on diversified portfolios with not very long investment horizons), both of these problems are solved.[8]

For any portfolio comprising *N* securities with expected returns of r_i *(i=1,...,N)* and standard deviations of σ_i *(i=1,...,N)*, the expected return and standard deviation of the portfolio are given by

$$r_p = \sum_{i=1}^{N} X_i r_i \text{ and}$$

$$\sigma_p = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} X_i X_j \sigma_{ij}} \ ,$$

where X_i denotes the weight of the *i*-th security in the portfolio, and σ_{ij} represents the covariance of the returns between security *i* and security *j*. This is a statistical measure of the interrelationship between the two random variables and essentially shows how the returns on two different securities move "together", in tandem. It can also be given through the correlation between two return variables, ρ_{ij}:

$$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij} \ .$$

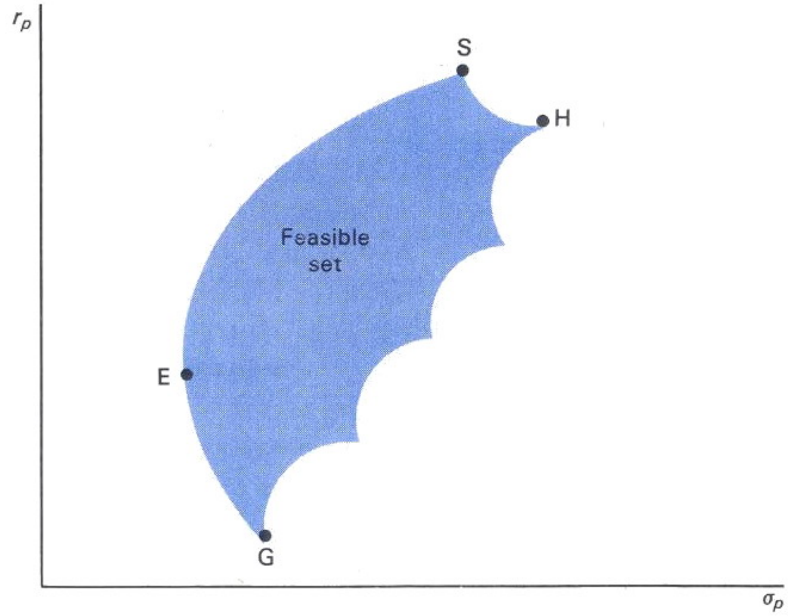
It appears from the above that, according to this model, it is *ceteris paribus* beneficial to combine in the portfolio securities which have significant negative correlation, in that it reduces the σ of the overall portfolio, and thus its risk. Evidently, minimum risk can be obtained in such case when $\rho_{ij}=-1$ at its possible minimum. Using the above formulae, we can construct a variance-covariance matrix for any portfolio of securities, where the entry in cell (*i,j*) will denote the covariance between security *i* and security *j*, and where *i=j* we obtain the variance of each security comprising the portfolio.

The Markowitz model then goes further to construct the set of all attainable portfolio combinations with their expected returns and standard deviations, which make the feasible set, or the opportunity set for the investor (see *Figure 1*). All possible portfolios that could be formed from the N securities lie either on or within the boundary of the feasible set (the points denoted by Latin letters in the figure are examples of such). In general, this set will have an umbrella-type shape, though differ in proportions and location on the plane depending on the particular securities involved. Further, the *efficient set theorem* states that:

> An investor will choose his or her optimal portfolio from the set of portfolios that:
> 1. Offer maximum expected return for varying levels of risk; and
> 2. Offer minimum risk for varying levels of expected return.
>
> The set of portfolios meeting these two conditions is known as the *efficient set,* or the *efficient frontier.*[9]

[8] *For greater detail on this issue see, for example,* F. J. Fabozzi, Financial Instuments. John Wiley & Sons, NJ USA 2002; W. F. Sharpe, G. J. Alexander, Investments (4th ed.). Prentice-Hall, Inc., NJ USA 1990.

[9] W. F. Sharpe, G. J. Alexander, Investments (4th ed.). Prentice-Hall, Inc., NJ USA 1990; p. 155.

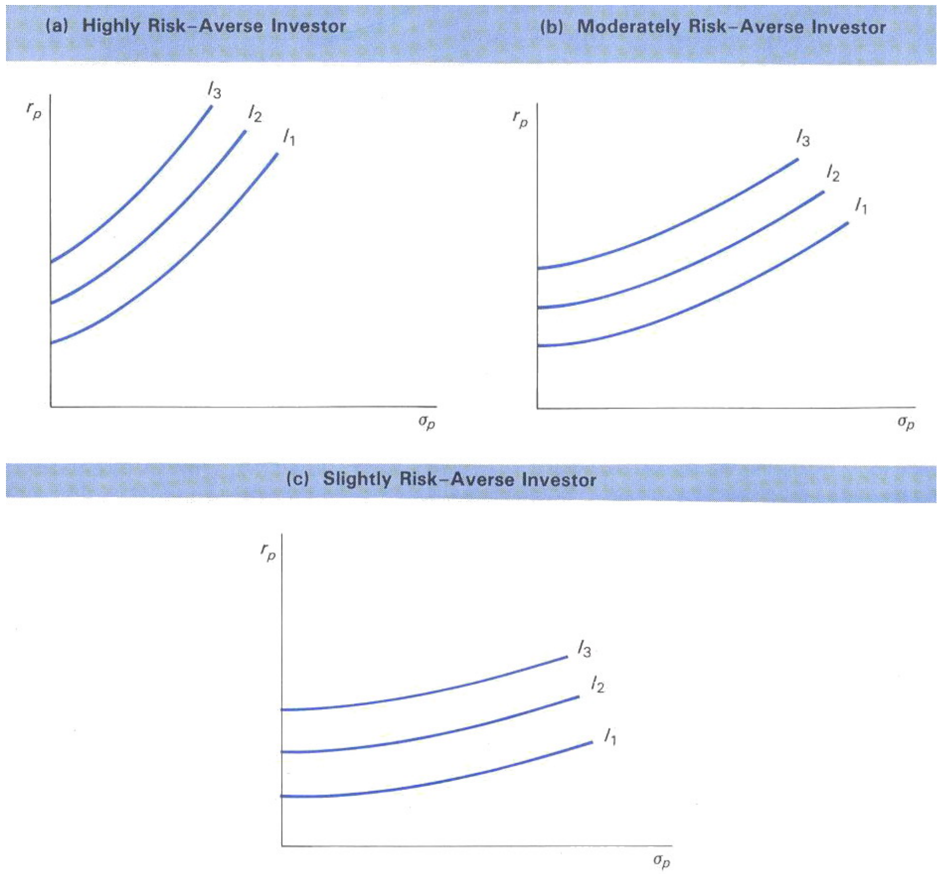


Source: W. F. Sharpe, G. J. Alexander, Investments (4th ed.). Prentice-Hall, Inc., NJ USA 1990

Figure 1. Markowitz model feasible and efficient sets

On the figure above, it can be seen that the efficient frontier is the upper left boundary of the feasible set lying between points *E* and *S*. It should be noted here that portfolio *E* is known as the minimum-variance portfolio and offers the minimal amount of risk among all possible portfolios. We derive an algorithm of identifying this minimum-variance portfolio on a specific example in the following section.

In order to find out which of the efficient portfolios will a specific investor choose as his optimal investment decision, we construct the investor's *indifference curves*. They represent a quantification of the investor's preferences in terms of the risk-return tradeoff. Each indifference curve indicates all risk-return combinations that would be equally desirable for the investor. The "utility" of the investor rises as his indifference curve moves further to the top left of the standard deviation – expected return plane. A few implications can be drawn about an investor's indifference curves at this point, which are utilized in the Markowitz model. First of all, indifference curves of an investor cannot intersect. Each indifference curve indicates a distinct level of "utility", and the utilities underlying two different curves cannot be equal. Further, we make the assumptions of non-satiation and risk-aversion, which state that all other things being equal, an investor would choose the portfolio with higher expected return or lower risk. Indifference curves of risk-averse investors are convex and upward sloping, with more risk-averse investors having more steeply sloped indifference curves (*Figure 2*). The indifference curves for risk-neutral investors are parallel to the standard deviation axis, whereas those of the risk-loving investor are downward sloping.



Source: W. F. Sharpe, G. J. Alexander, Investments (4th ed.). Prentice-Hall, Inc., NJ USA 1990

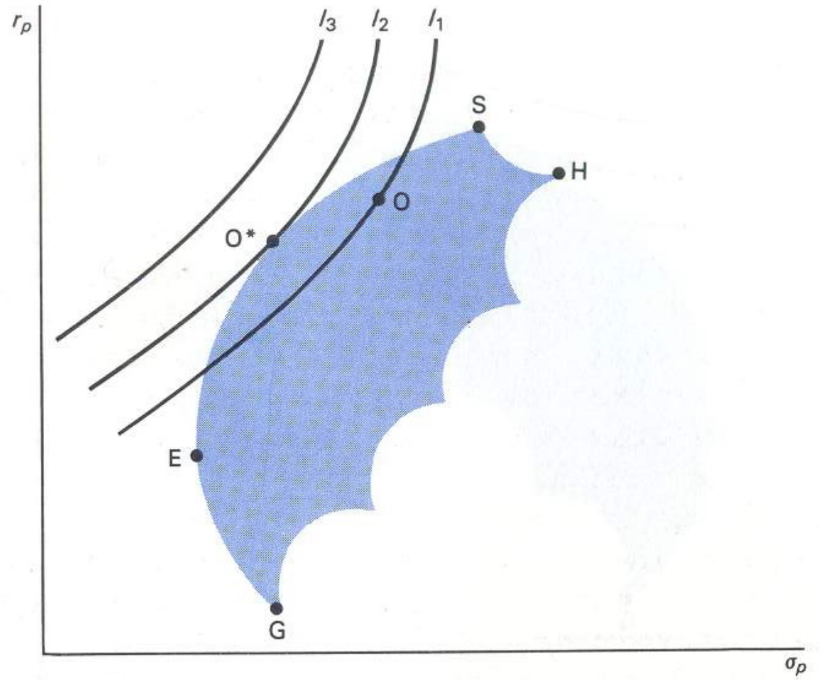
Figure 2. Indifference curves for different types of risk-averse investors

Combining the efficient set with an investor's indifference curve map, we obtain the optimal portfolio for the given investor. This portfolio will correspond to the point where an indifference curve (the highest possible) is tangent to the efficient frontier. In *Figure 3*, this corresponds to point O^* on indifference curve I_2. All portfolios lying on indifference curves below I_2, although attainable, are not desirable for a non-satiating investor, whereas those lying on indifference curves above I_2 cannot be attained with securities comprising the portfolio.

It can be shown that the efficient set is generally concave due to less than perfect correlations between securities comprising the portfolio.[10] This feature, coupled with the convexity of the indifference curve of the risk-averse investor, provides for a *single* tangency point between the two curves.

At the present level of analysis, we omit such clarifications and improvements to the Markowitz mean-variance optimal portfolio selection model as short sales restrictions, prospects of risk-free borrowing and risk-free investment opportunities, etc. The consideration of these would require a much larger scope of research and more liberal timeframes. They should, however, become a topic of further research in the near future to allow for more realistic and practical portfolio selection solutions in today's sophisticated world of financial markets.

[10] *See* Z. Bodie et al, Investments (6th ed.). McGraw-Hill, NY USA 2005; p. 231-233; W. F. Sharpe, G. J. Alexander, Investments (4th ed.). Prentice-Hall, Inc., NJ USA 1990; p. 158-166.



Source: W. F. Sharpe, G. J. Alexander, Investments (4th ed.). Prentice-Hall, Inc., NJ USA 1990

Figure 3. The Markowitz mean-variance optimal portfolio

DATA AND RESULTS

In constructing the data set for the present research, we have deployed our selection criteria from the Central Bank of Armenia's reserve management and investment decision making strategies, as well as constraints on its investment activity dictated by internal regulation and state legislation, in particular the Armenian Law on the Central Bank. This law limits the universe of financial instruments available to the CBA for investment activities to "bonds, as well as forward or repurchase agreements expressed in foreign currency issued and secured by central or other outstanding banks of foreign states or international financial organizations."[11] The International Reserve Management Strategy of CBA explicitly states that the Bank has adopted a conservative approach in its reserve management policies, and that the prime objective of its investment activities is the security and liquidity of investments.

Financial instruments utilized in CBA's investment operations include government debt securities from the USA and other OECD countries, agency securities, fixed-income securities of international financial institutions, swap operations. For the purpose of the present analysis, we have chosen a total number of ten financial instruments eligible for CBA in its investment operations, which include:

- US Government debt securities with maturities of: 3 months, 6 months, 1 year, 3 years and 5 years (composite indices);
- 3-month and 6-month deposit instruments (USD LIBOR); and
- 1-, 3- and 5-year USD swap operations (semi-annual rates).

[11] The Republic of Armenia Law on The Central Bank of the Republic of Armenia (adopted 30.06.1996, amended and revised), Article 52.1(e).

The observation timeframe stretches from March 2006 back to September 2002, in monthly increments (a total of 43 observations). The historical reach of the time series data was limited to this due to the unavailability of earlier data on most selected securities, although analysis and previous experience proves the data scope to be sufficient. We have chosen the MID prices of the selected instruments as basis for historical return calculations. The time series data is presented in Appendix 1 below. In constructing the model, we have used MS Office Excel 2003, which provides the basic range of mathematical functions and solving capabilities needed. Even though some minor limitations of the model can be attributed to the insufficient technical capabilities of the utilized software, this package nonetheless does meet our technical requirements at the present level and scope of analysis.

Based on this data, we then calculate historical monthly returns for the observation periods, as well as mean returns for each security and the excess returns to be used in the calculation of the variance/covariance matrix (see Appendix 2 for details). We use a simple formula for the calculation of returns,

$$r_t = \frac{p_t}{p_{t-1}} - 1 \;, \qquad t = \overline{1,T}$$

where T denotes the number of time periods (in our case $T=42$, one less than the number of observations). The excess return matrix has T rows and 10 columns and is calculated as follows:

$$ER = \begin{pmatrix} r_{A1} - \bar{r}_A & r_{B1} - \bar{r}_B & \dots & r_{J1} - \bar{r}_J \\ r_{A2} - \bar{r}_A & r_{B2} - \bar{r}_B & \dots & r_{J2} - \bar{r}_J \\ \dots & \dots & \dots & \dots \\ r_{AT} - \bar{r}_A & r_{BT} - \bar{r}_B & \dots & r_{JT} - \bar{r}_J \end{pmatrix}.$$

Here $\bar{r}$ denotes the mean returns of separate securities over the observation period, defined as the simple average of observed monthly returns. The variance/covariance matrix of returns is calculated using the excess return matrix and its transposed matrix and has the following form:

$$S_{10x10} = \frac{ER^T \cdot ER}{T} = \begin{pmatrix} \sigma_{rA}^2 & \text{cov}(r_A, r_B) & \dots & \text{cov}(r_A, r_J) \\ \text{cov}(r_A, r_B) & \sigma_{rB}^2 & \dots & \text{cov}(r_B, r_J) \\ \dots & \dots & \dots & \dots \\ \text{cov}(r_A, r_J) & \text{cov}(r_B, r_J) & \dots & \sigma_{rJ}^2 \end{pmatrix}$$

We note here that the variance/covariance matrix is symmetric in that for any M and N random variables,

$$\text{cov}(M, N) = \text{cov}(N, M).$$

We further note that:

$$\text{cov}(M, M) = \sigma_M^2 \;.$$

Next, we must construct the efficient frontier of portfolios. For this purpose, Harry Markowitz in his work[12] suggests what is known as the *critical-line method* involving the use of a quadratic programming algorithm. We use the concept of envelope portfolios deriving from this algorithm (see Appendix 3). An envelope portfolio *w* is such that:

$$z_{10x1} = S^{-1}_{10x10} \cdot [R_{10x1} - c]$$
$$w_i = \frac{z_i}{\sum_{i=1}^{10} z_i}, i = 1,...,10 \quad .$$

Any portfolio *w* defined in this manner will lie on the Markowitz efficient frontier. In the formulae above, *R* denotes the column vector of expected returns of separate securities, *W* the column vector of portfolio weights, and *c* is any constant. By changing this constant, we can obtain two envelope portfolios. The expected return and variance of any such portfolio, as well as the covariance between two envelope portfolios, are given by:

$$E(r_p) = W^T \cdot R$$
$$\sigma_p^2 = W^T \cdot S \cdot W$$
$$\text{cov}(P_1, P_2) = W_{P1}^T \cdot S \cdot W_{P2}$$

The efficient frontier can be constructed through linear combinations of the two envelope portfolios. This means that by allowing α in the formulae below to vary, we can construct the complete efficient portfolio:

$$E(r_{EP}) = \alpha E(r_{P1}) + (1-\alpha) E(r_{P2})$$
$$\sigma_{EP} = \sqrt{\alpha^2 \sigma_{P1}^2 + (1-\alpha)\sigma_{P2}^2 + 2\alpha(1-\alpha)\,\text{cov}(P_1, P_2)}$$

For purposes of conceptual consistency, we also determine the minimum-variance portfolio using the abovementioned two envelope portfolios, which will serve as the lower left endpoint of our efficient frontier. In general, it can be shown that minimum risk from the combination of two portfolios is attained when the weights for these portfolios are given by:

$$w_{P1} = \frac{\sigma_{P2}^2 - \text{cov}(P_1, P_2)}{\sigma_{P1}^2 + \sigma_{P2}^2 - 2\,\text{cov}(P_1, P_2)} \qquad w_{P2} = 1 - w_{P1}$$

The characteristics of the minimum-variance portfolio in our specific case are brought in Appendix 3.

Having completed the construction of the efficient frontier, we now turn to the indifference curves of the CBA as an individual investor. Of key importance here is the issue of the investor's risk-aversion, which determines the shape and form of the indifference curve and thus the choice of the optimal portfolio for a given efficient frontier. A large number of studies have addressed the issue of estimating an individual investor's risk-aversion, but there is no single acceptable methodology that could be used in estimating the risk-aversion of the CBA at this point. As some studies point out, "the

[12] *See* H. Markowitz, "The Optimization of a Quadratic Function Subject to Linear Constraints," *Naval Research Logistics Quarterly* vol. 3, no. 1-2, March-June 1956.

determination of individuals' risk-taking attitudes and behavior may be so complex that it is not possible to characterize [them] as exhibiting a particular rate of risk aversion"[13]. Further:

> "there is little consensus as to the index of risk aversion for … investors, with … estimates ranging from 0.3 to numbers in excess of 6. Asset demand studies … by Friend and Blume (1975), Friend and Hasbrouck (1982), and Grossman and Shiller (1981) support estimates of 2 to 6. In a similar study, Frankel (1983) attempts to estimate this parameter, but after obtaining an estimate in excess of 100, he set the value at 2."[14]

We deploy our analysis from the analytic form of the investor's indifference curve given by:

$$U = E(r) - A \cdot \sigma^2 \ ,$$

where we take the risk aversion parameter, *A*, to be equal to 6. Unfortunately, this assumption cannot be backed by any econometric reasoning or concrete studies. This choice has only been made intuitively in light of the literature review and merely to provide a base for the methodology we propose in identifying the optimal investment portfolio. The estimation of the rate of risk aversion for an individual investor (especially one like CBA), being somewhat questionable even in its meaningfulness, requires a separate study and conceptually different methodologies in taking into account the goals and constraints of the Bank. At this point it can only be stated that this choice can be consistent with CBA's expected high risk aversion. Graphically, as shown in Appendix 4, this means that the optimal choice for an excessively risk-averse investor such as the CBA should be located near the minimum-variance portfolio, which is the case.

We now have the (hypothetical) indifference function of the Central Bank of Armenia,

$$U=E(r) - 6\,\sigma^2,$$

and a 6-degree polynomial trendline estimation of the efficient frontier:

$$E(r) = -193042\,\sigma^6 + 125499\,\sigma^5 - 32962\,\sigma^4 + 4466.6\,\sigma^3 - 329.08\,\sigma^2 + 13.02\,\sigma - 0.1861$$
$$R^2 = 1.00$$

Combining the two and solving for *max{U}*, we obtain for the optimal mean-variance portfolio:

return	$E(r) = 0.0323$
standard deviation	$\sigma = 0.0524$
maximum attainable investor utility	$U = 0.0158$

This combination is achieved when securities comprising the portfolio have the following weights:

Security A	Security B	Security C	Security D	Security E	Security F	Security G	Security H	Security I	Security J
0.254	0.107	-0.450	0.655	1.953	1.492	-1.433	0.989	-0.822	-1.744

[13] D. Schooley, D. Worden "Risk Aversion Measures: Comparing Attitudes and Asset Allocation," *Financial Services Review* 5(2), 1996; p. 98.
[14] R. Pindyck, "Risk aversion and determinants of stock market behavior", NBER Working Paper No. 1921, May 1986.

CONCLUSION: SHORTCOMINGS OF THE MODEL AND DIRECTIONS OF IMPROVEMENT

The above analysis provides us with a concrete selection of the optimal investment portfolio according to Markowitz's mean-variance portfolio selection model for a given investor and a given universe of investment instruments. However, the present scope of research leaves out a number of noteworthy issues which have not been accounted for but which strongly influence investment decisions in the real financial markets. In concluding, we should point out some of the main shortcomings of the above analysis with the aim of concentrating further research in these directions and endeavor to improve the practical qualities of the model.

First of all, we have not considered in the above model restrictions on short sales of investment instruments, which most often apply to such investors as the Central Bank of Armenia. It appears that this problem may be solved by introducing non-negativity constraints on weights in the phase of constructing the envelope portfolios, which should allow us to further construct the efficient frontier from only those portfolios which do not assume short sales. However, this is only a hypothesis which needs to be tested.

Further, the model as discussed fails to take into consideration risk-free investment alternatives and non-risk borrowing. Even though the latter seldom applies to investors such as the CBA, and even though the theory of finance suggests there is virtually no risk-free investment, some short-term fixed-income instruments (e.g., 3-month US T-bills) can be assumed to be risk-free for the purposes of the model. In this case we would have an additional straight line connecting the risk-free rate to a unique "tangency portfolio" (or an optimal risky portfolio) on the efficient frontier, and the optimal choice of the investor could be located on that line combining an optimal risky portfolio with some share of risk-free investment.

Technical issues may arise with the trendline estimation of the efficient frontier. Even though the accuracy of a 6-degree polynomial estimation proved to be sufficient in this case, more precise methods may be used if technical capabilities allow. Further, the estimation may change if we alter the upper right boundary of the efficient frontier. Nonetheless, we believe that such adjustments would be small and should not significantly alter the results.

Finally, as mentioned above, the present analysis does not include an economically justified estimation of the rate of risk aversion of the individual investor. This is one of the major methodological gaps in utilizing the Markowitz model in practice. The estimation of an individual investor's risk aversion requires significant further insight and will undoubtedly prove a strong input in boosting the practicality of this model. Nonetheless, it should be noted at this point that the notion of risk aversion for an individual investor, more so for a central bank taking into account its non-commercial objectives and political constraints, has not been vastly analyzed in the literature and still requires significant clarification. It is yet a disputable question on whether a risk-averse utility function defined over the investment returns is the best suitable representation of a central bank's investment preferences.

It is this ambiguity that has brought about the usage of other types of subjective constraints defining a central bank's investment preferences, which are especially typical in light of such investment operations being part of reserve management policies. Those may include, in relative order of priority, capital preservation requirements, e.g. non-negativity constraints on portfolio returns over a certain time horizon (or at least a statistically significant probability that losses incurred will not exceed a fixed sum); requirements on minimal return which will at least cover operational expenses; risk-minimization requirements; strict lists of eligible assets; etc.

Overall it should be admitted that this study can claim to be nothing more than a first experimental step in applying a well-developed theoretical model to the practical investment activities of an investor as peculiar and unique as a central bank. Among other concrete results the above analysis implies that any single model cannot be used for practical decision-making in isolation from other similar models and alternative approaches. The latter do not contradict, but rather come to complement any single model developed by the theory of finance, and it is their combination with some extent of subjective reasoning that provides better results.

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APPENDIX 1

DATA: MID Prices

Date	Security A	Security B	Security C	Security D	Security E	Security F	Security G	Security H	Security I	Security J
3/31/2006	4.6195	4.8708	4.877	4.8217	4.8246	5	5.14	5.286	5.266	5.304
2/28/2006	4.6304	4.7255	4.7319	4.6726	4.6287	4.8225	4.99	5.117	5.068	5.0675
1/31/2006	4.5016	4.5682	4.5749	4.5017	4.4645	4.68	4.81	4.95	4.925	4.9565
12/30/2005	4.2045	4.4572	4.4074	4.3851	4.3778	4.53625	4.7	4.847	4.84	4.877
11/30/2005	3.9852	4.3322	4.4536	4.4109	4.436	4.42	4.60063	4.819	4.865	4.9255
10/31/2005	3.8694	4.2051	4.3578	4.4482	4.47	4.26	4.46625	4.718	4.868	4.9385
9/30/2005	3.5309	3.9211	4.1011	4.2014	4.2239	4.065	4.23063	4.49	4.6005	4.653
8/31/2005	3.4548	3.6848	3.766	3.8275	3.9054	3.87	4.055	4.112	4.219	4.2815
7/29/2005	3.38	3.6957	3.9053	4.08	4.1621	3.7	3.92375	4.218	4.4635	4.563
6/30/2005	3.061	3.359	3.5238	3.6708	3.7557	3.51625	3.71	3.911	4.0365	4.13
5/31/2005	2.9126	3.1118	3.3906	3.6836	3.8402	3.3375	3.5375	3.769	4.03	4.149
4/29/2005	2.89	3.15	3.38	3.76	3.96	3.21	3.40875	3.731	4.1725	4.35
3/31/2005	2.8092	3.1363	3.4802	4.0112	4.2766	3.12	3.4	3.808	4.3935	4.618
2/28/2005	2.7409	2.9547	3.289	3.8277	4.1062	2.92	3.16	3.566	4.164	4.415
1/31/2005	2.4617	2.6955	2.9536	3.4851	3.7792	2.75	2.96	3.282	3.807	4.075
12/31/2004	2.2126	2.4796	2.7742	3.3089	3.7039	2.56438	2.78063	3.096	3.664	4.025
11/30/2004	2.2044	2.3554	2.6516	3.325	3.7927	2.41	2.635	2.973	3.656	4.1045
10/29/2004	1.9146	2.0524	2.1958	2.8799	3.3812	2.17	2.3125	2.542	3.213	3.708
9/30/2004	1.7249	1.9088	2.1786	2.9546	3.4864	2.02	2.19625	2.488	3.285	3.805
8/31/2004	1.6277	1.6818	1.9553	2.8188	3.4115	1.8	1.99	2.271	3.155	3.754
7/30/2004	1.4677	1.7072	2.046	3.1516	3.7917	1.7	1.98	2.383	3.544	4.177
6/30/2004	1.2953	1.6341	2.0081	3.2054	3.8125	1.61	1.94	2.375	3.594	4.259
5/31/2004	1.0404	1.3137	1.7741	3.1297	3.8465	1.315	1.5775	2.119	3.516	4.295
4/30/2004	1.004	1.14	1.5858	2.9047	3.657	1.18	1.38	1.816	3.293	4.0825
3/31/2004	0.9479	0.9946	1.1119	2.0341	2.8144	1.11	1.16	1.342	2.39	3.165
2/27/2004	0.9555	0.9711	1.1195	2.176	2.978	1.12	1.17	1.369	2.525	3.354
1/30/2004	0.9323	0.9763	1.2375	2.3616	3.1953	1.13	1.21375	1.463	2.741	3.545
12/31/2003	0.9021	0.972	1.2519	2.4231	3.2986	1.15188	1.22	1.458	2.761	3.665
11/28/2003	0.89	0.9908	1.4698	2.6737	3.4963	1.17188	1.25875	1.601	2.973	3.755
10/31/2003	0.9181	0.9751	1.1915	2.4104	3.3959	1.16938	1.23	1.4725	2.786	3.676
9/30/2003	0.9433	1.0075	1.0787	1.9848	3.058	1.16	1.18	1.2375	2.3415	3.245
8/29/2003	0.9379	0.9623	1.134	2.4939	3.6306	1.14	1.1975	1.4175	2.965	3.935
7/31/2003	0.9194	1.0328	1.22	2.3449	3.5147	1.11438	1.14625	1.3901	2.73	3.805
6/30/2003	0.8622	0.9753	1.0321	1.7105	2.6274	1.11625	1.11938	1.1575	1.952	2.7365
5/30/2003	1.1031	1.0762	1.1047	1.6364	2.3784	1.28	1.21375	1.2175	1.873	2.611
4/30/2003	1.1157	1.1523	1.1377	2.0021	2.866	1.31	1.29	1.2803	2.255	3.115
3/31/2003	1.1462	1.1329	1.1497	1.9981	2.8108	1.27875	1.23125	1.2525	2.305	3.145
2/28/2003	1.1809	1.1645	1.2124	1.953	2.7209	1.34	1.34	1.365	2.285	3.055
1/31/2003	1.1236	1.1416	1.2395	2.2391	3.0517	1.35	1.34875	1.4287	2.574	3.385
12/31/2002	1.0947	1.1035	1.1667	2.0899	2.83	1.38	1.38	1.4075	2.395	3.155
11/29/2002	1.1952	1.2396	1.4884	2.5675	3.3985	1.425	1.46875	1.6925	3.025	3.745
10/31/2002	1.4144	1.3209	1.3407	2.1205	2.8094	1.68625	1.6	1.5875	2.58	3.33
9/30/2002	1.5595	1.5142	1.4405	2.087	2.6728	1.79	1.71	1.6425	2.555	3.23

Source: Bloomberg network

APPENDIX 2

Monthly Returns

Date	Security A	Security B	Security C	Security D	Security E	Security F	Security G	Security H	Security I	Security J
3/31/2006	-0.002	0.031	0.031	0.032	0.042	0.037	0.030	0.033	0.039	0.047
2/28/2006	0.029	0.034	0.034	0.038	0.037	0.030	0.037	0.034	0.029	0.022
1/31/2006	0.071	0.025	0.038	0.027	0.020	0.032	0.023	0.021	0.018	0.016
12/30/2005	0.055	0.029	-0.010	-0.006	-0.013	0.026	0.022	0.006	-0.005	-0.010
11/30/2005	0.030	0.030	0.022	-0.008	-0.008	0.038	0.030	0.021	-0.001	-0.003
10/31/2005	0.096	0.072	0.063	0.059	0.058	0.048	0.056	0.051	0.058	0.061
9/30/2005	0.022	0.064	0.089	0.098	0.082	0.050	0.043	0.092	0.090	0.087
8/31/2005	0.022	-0.003	-0.036	-0.062	-0.062	0.046	0.033	-0.025	-0.055	-0.062
7/29/2005	0.104	0.100	0.108	0.111	0.108	0.052	0.058	0.078	0.106	0.105
6/30/2005	0.051	0.079	0.039	-0.003	-0.022	0.054	0.049	0.038	0.002	-0.005
5/31/2005	0.008	-0.012	0.003	-0.020	-0.030	0.040	0.038	0.010	-0.034	-0.046
4/29/2005	0.029	0.004	-0.029	-0.063	-0.074	0.029	0.003	-0.020	-0.050	-0.058
3/31/2005	0.025	0.061	0.058	0.048	0.041	0.068	0.076	0.068	0.055	0.046
2/28/2005	0.113	0.096	0.114	0.098	0.087	0.062	0.068	0.087	0.094	0.083
1/31/2005	0.113	0.087	0.065	0.053	0.020	0.072	0.065	0.060	0.039	0.012
12/31/2004	0.004	0.053	0.046	-0.005	-0.023	0.064	0.055	0.041	0.002	-0.019
11/30/2004	0.151	0.148	0.208	0.155	0.122	0.111	0.139	0.170	0.138	0.107
10/29/2004	0.110	0.075	0.008	-0.025	-0.030	0.074	0.053	0.022	-0.022	-0.025
9/30/2004	0.060	0.135	0.114	0.048	0.022	0.122	0.104	0.096	0.041	0.014
8/31/2004	0.109	-0.015	-0.044	-0.106	-0.100	0.059	0.005	-0.047	-0.110	-0.101
7/30/2004	0.133	0.045	0.019	-0.017	-0.005	0.056	0.021	0.003	-0.014	-0.019
6/30/2004	0.245	0.244	0.132	0.024	-0.009	0.224	0.230	0.121	0.022	-0.008
5/31/2004	0.036	0.152	0.119	0.077	0.052	0.114	0.143	0.167	0.068	0.052
4/30/2004	0.059	0.146	0.426	0.428	0.299	0.063	0.190	0.353	0.378	0.290
3/31/2004	-0.008	0.024	-0.007	-0.065	-0.055	-0.009	-0.009	-0.020	-0.053	-0.056
2/27/2004	0.025	-0.005	-0.095	-0.079	-0.068	-0.009	-0.036	-0.064	-0.079	-0.054
1/30/2004	0.033	0.004	-0.012	-0.025	-0.031	-0.019	-0.005	0.003	-0.007	-0.033
12/31/2003	0.014	-0.019	-0.148	-0.094	-0.057	-0.017	-0.031	-0.089	-0.071	-0.024
11/28/2003	-0.031	0.016	0.234	0.109	0.030	0.002	0.023	0.087	0.067	0.021
10/31/2003	-0.027	-0.032	0.105	0.214	0.110	0.008	0.042	0.190	0.190	0.133
9/30/2003	0.006	0.047	-0.049	-0.204	-0.158	0.018	-0.015	-0.127	-0.210	-0.175
8/29/2003	0.020	-0.068	-0.070	0.064	0.033	0.023	0.045	0.020	0.086	0.034
7/31/2003	0.066	0.059	0.182	0.371	0.338	-0.002	0.024	0.201	0.399	0.390
6/30/2003	-0.218	-0.094	-0.066	0.045	0.105	-0.128	-0.078	-0.049	0.042	0.048
5/30/2003	-0.011	-0.066	-0.029	-0.183	-0.170	-0.023	-0.059	-0.049	-0.169	-0.162
4/30/2003	-0.027	0.017	-0.010	0.002	0.020	0.024	0.048	0.022	-0.022	-0.010
3/31/2003	-0.029	-0.027	-0.052	0.023	0.033	-0.046	-0.081	-0.082	0.009	0.029
2/28/2003	0.051	0.020	-0.022	-0.128	-0.108	-0.007	-0.006	-0.045	-0.112	-0.097
1/31/2003	0.026	0.035	0.062	0.071	0.078	-0.022	-0.023	0.015	0.075	0.073
12/31/2002	-0.084	-0.110	-0.216	-0.186	-0.167	-0.032	-0.060	-0.168	-0.208	-0.158
11/29/2002	-0.155	-0.062	0.110	0.211	0.210	-0.155	-0.082	0.066	0.172	0.125
10/31/2002	-0.093	-0.128	-0.069	0.016	0.051	-0.058	-0.064	-0.033	0.010	0.031

Mean Returns

Security A	Security B	Security C	Security D	Security E	Security F	Security G	Security H	Security I	Security J
0.029	0.031	0.035	0.027	0.019	0.027	0.029	0.032	0.024	0.017

Excess Returns

Date	Security A	Security B	Security C	Security D	Security E	Security F	Security G	Security H	Security I	Security J
3/31/2006	-0.032	0.000	-0.004	0.005	0.023	0.010	0.001	0.001	0.015	0.030
2/28/2006	-0.001	0.004	-0.001	0.011	0.018	0.004	0.009	0.001	0.005	0.006
1/31/2006	0.041	-0.006	0.003	-0.001	0.001	0.005	-0.005	-0.011	-0.006	0.000
12/30/2005	0.026	-0.002	-0.045	-0.033	-0.032	0.000	-0.007	-0.026	-0.029	-0.027
11/30/2005	0.001	-0.001	-0.013	-0.036	-0.027	0.011	0.001	-0.011	-0.025	-0.019
10/31/2005	0.067	0.042	0.028	0.031	0.039	0.021	0.027	0.018	0.034	0.045
9/30/2005	-0.007	0.033	0.054	0.070	0.062	0.024	0.015	0.060	0.067	0.070
8/31/2005	-0.007	-0.034	-0.071	-0.089	-0.081	0.019	0.005	-0.057	-0.079	-0.078
7/29/2005	0.075	0.069	0.073	0.084	0.089	0.026	0.029	0.046	0.082	0.088
6/30/2005	0.022	0.049	0.004	-0.031	-0.041	0.027	0.020	0.005	-0.022	-0.021
5/31/2005	-0.021	-0.043	-0.032	-0.048	-0.049	0.013	0.009	-0.022	-0.058	-0.063
4/29/2005	-0.001	-0.026	-0.064	-0.090	-0.093	0.002	-0.026	-0.053	-0.074	-0.075
3/31/2005	-0.004	0.031	0.023	0.021	0.022	0.042	0.047	0.036	0.031	0.029
2/28/2005	0.084	0.065	0.079	0.071	0.067	0.035	0.039	0.054	0.070	0.067
1/31/2005	0.083	0.056	0.030	0.026	0.001	0.046	0.036	0.028	0.015	-0.004
12/31/2004	-0.026	0.022	0.011	-0.032	-0.043	0.037	0.027	0.009	-0.022	-0.036
11/30/2004	0.122	0.117	0.173	0.127	0.103	0.084	0.111	0.137	0.114	0.090
10/29/2004	0.081	0.044	-0.027	-0.053	-0.049	0.048	0.024	-0.011	-0.046	-0.042
9/30/2004	0.030	0.104	0.079	0.021	0.003	0.096	0.075	0.063	0.017	-0.003
8/31/2004	0.080	-0.046	-0.079	-0.133	-0.119	0.032	-0.024	-0.079	-0.134	-0.118
7/30/2004	0.104	0.014	-0.016	-0.044	-0.025	0.029	-0.008	-0.029	-0.038	-0.036
6/30/2004	0.216	0.213	0.097	-0.003	-0.028	0.198	0.201	0.089	-0.002	-0.025
5/31/2004	0.007	0.122	0.084	0.050	0.033	0.088	0.114	0.135	0.044	0.035
4/30/2004	0.030	0.115	0.391	0.401	0.280	0.036	0.161	0.321	0.354	0.273
3/31/2004	-0.037	-0.007	-0.042	-0.092	-0.074	-0.036	-0.037	-0.052	-0.077	-0.073
2/27/2004	-0.004	-0.036	-0.130	-0.106	-0.087	-0.036	-0.065	-0.097	-0.103	-0.071
1/30/2004	0.004	-0.026	-0.046	-0.053	-0.051	-0.046	-0.034	-0.029	-0.031	-0.049
12/31/2003	-0.016	-0.050	-0.183	-0.121	-0.076	-0.044	-0.059	-0.122	-0.095	-0.041
11/28/2003	-0.060	-0.015	0.199	0.082	0.010	-0.025	-0.005	0.055	0.043	0.005
10/31/2003	-0.056	-0.063	0.070	0.187	0.091	-0.019	0.014	0.158	0.166	0.116
9/30/2003	-0.024	0.016	-0.084	-0.231	-0.177	-0.009	-0.043	-0.159	-0.234	-0.192
8/29/2003	-0.009	-0.099	-0.105	0.036	0.014	-0.004	0.016	-0.013	0.062	0.017
7/31/2003	0.037	0.028	0.147	0.344	0.319	-0.028	-0.005	0.169	0.375	0.374
6/30/2003	-0.248	-0.125	-0.101	0.018	0.085	-0.155	-0.106	-0.082	0.018	0.031
5/30/2003	-0.041	-0.097	-0.064	-0.210	-0.189	-0.050	-0.088	-0.081	-0.193	-0.179
4/30/2003	-0.056	-0.014	-0.045	-0.025	0.000	-0.002	0.019	-0.010	-0.046	-0.026
3/31/2003	-0.059	-0.058	-0.087	-0.004	0.014	-0.072	-0.110	-0.115	-0.015	0.013
2/28/2003	0.022	-0.011	-0.057	-0.155	-0.128	-0.034	-0.035	-0.077	-0.136	-0.114
1/31/2003	-0.003	0.004	0.028	0.044	0.059	-0.048	-0.051	-0.017	0.051	0.056
12/31/2002	-0.113	-0.141	-0.251	-0.213	-0.186	-0.058	-0.089	-0.201	-0.232	-0.174
11/29/2002	-0.184	-0.092	0.075	0.184	0.190	-0.182	-0.111	0.034	0.149	0.108
10/31/2002	-0.122	-0.158	-0.104	-0.011	0.032	-0.085	-0.093	-0.066	-0.014	0.014

Var/Covar matrix

	Security A	Security B	Security C	Security D	Security E	Security F	Security G	Security H	Security I	Security J
Security A	0.006	0.004	0.003	0.001	0.000	0.004	0.004	0.002	0.001	0.001
Security B	0.004	0.005	0.005	0.003	0.002	0.004	0.004	0.004	0.003	0.002
Security C	0.003	0.005	0.012	0.011	0.008	0.003	0.005	0.009	0.010	0.008
Security D	0.001	0.003	0.011	0.015	0.012	0.000	0.003	0.010	0.015	0.012
Security E	0.000	0.002	0.008	0.012	0.011	0.000	0.002	0.008	0.012	0.010
Security F	0.004	0.004	0.003	0.000	0.000	0.004	0.004	0.003	0.001	0.000
Security G	0.004	0.004	0.005	0.003	0.002	0.004	0.004	0.005	0.003	0.002
Security H	0.002	0.004	0.009	0.010	0.008	0.003	0.005	0.009	0.010	0.008
Security I	0.001	0.003	0.010	0.015	0.012	0.001	0.003	0.010	0.014	0.012
Security J	0.001	0.002	0.008	0.012	0.010	0.000	0.002	0.008	0.012	0.010

APPENDIX 3

Efficient frontier using envelope portfolios: step 1

	First envelope portfolio, c=0		Second envelope portfolio, c=0.01	
	z	**port. Weights**	**z**	**port. Weights**
	2.932	0.266	2.720	0.955
	1.651	0.150	7.095	2.492
	-5.413	-0.491	-7.755	-2.723
	8.174	0.742	15.617	5.485
	21.807	1.979	9.731	3.418
	16.696	1.515	7.974	2.800
	-16.623	-1.509	-16.016	-5.625
	11.429	1.037	10.372	3.643
	-9.114	-0.827	-3.194	-1.122
	-20.520	-1.862	-23.697	-8.322
sum z	11.01863	1.000	2.847	1.000

	First port.	Second port.	I and II
Mean return	0.0333	0.0903	
Variance	0.0030	0.0282	
Sigma	0.0550	0.1679	
Covariance			0.0082
Correlation			0.8872

Efficient frontier using envelope portfolios: step 2

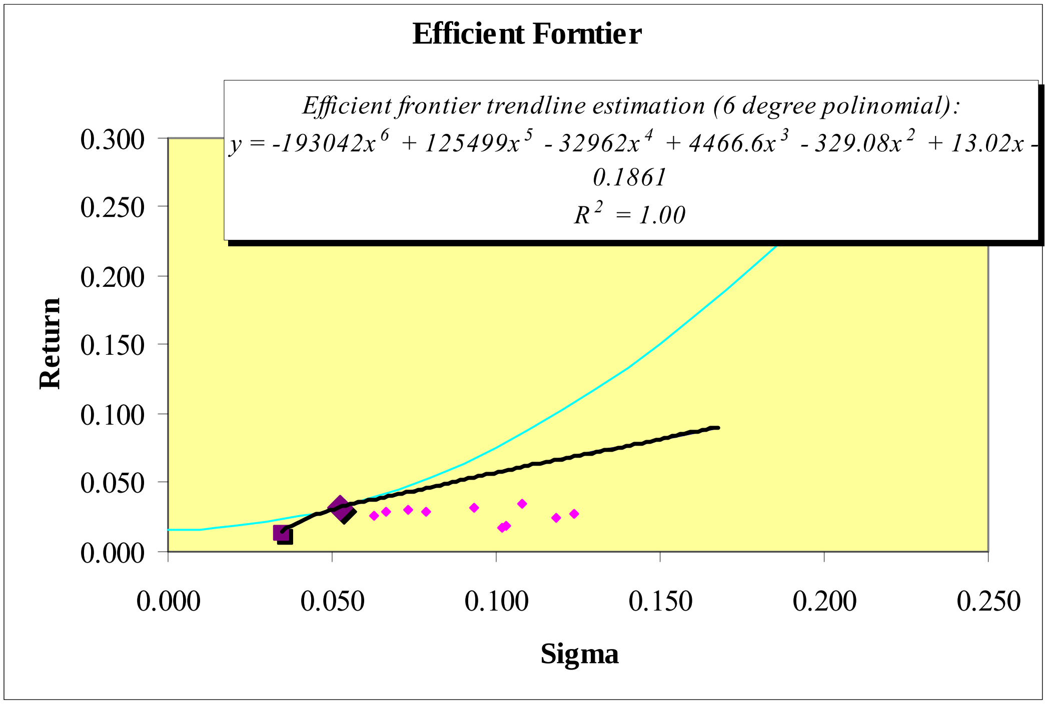
	Share of first portfolio	sigma	return
	0.00	0.168	0.090
	0.10	0.156	0.085
	0.20	0.144	0.079
	0.30	0.132	0.073
	0.40	0.121	0.067
	0.50	0.109	0.062
	0.60	0.098	0.056
	0.70	0.086	0.050
	0.80	0.075	0.045
	0.90	0.065	0.039
	1.00	0.055	0.033
	1.10	0.046	0.028
	1.20	0.039	0.022
	1.30	0.035	0.016
min-var port	**1.35**	0.035	0.013
	1.40	0.036	0.011
	1.50	0.040	0.005
	1.60	0.046	-0.001
	1.70	0.055	-0.007
	1.80	0.065	-0.012
	1.90	0.076	-0.018
	2.00	0.087	-0.024
	2.10	0.098	-0.029
	2.20	0.109	-0.035
	2.30	0.121	-0.041
	2.40	0.133	-0.046
	2.50	0.145	-0.052

minimum variance portfolio

share of I port	1.348467
MVP sigma	0.034983
MVP return	0.013485

APPENDIX 4

The efficient frontier and the indifference curve



	Sigma	Mean Return
Security A	0.0784	0.0293
Security B	0.0734	0.0308
Security C	0.1078	0.0348
Security D	0.1240	0.0272
Security E	0.1033	0.0192
Security F	0.0627	0.0267
Security G	0.0662	0.0286
Security H	0.0933	0.0323
Security I	0.1186	0.0239
Security J	0.1021	0.0167

weights	port. return	
0.254	0.007	
0.107	0.003	
-0.450	-0.016	
0.655	0.018	
1.953	0.037	
1.492	0.040	
-1.433	-0.041	
0.989	0.032	
-0.822	-0.020	
-1.744	-0.029	
1.000	**0.0323**	*0.0323 target*

portfolio variance	0.0028	
portfolio sigma	**0.0524**	*0.0524 target*

optimal portfolio

sigma	utility	return
0.0523861	0.015819137	0.032284927