A model with indivisible investments in different environments

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Abstract

In this paper we study an infinite horizon dynastic (stochastic) model, when at first credit markets are missing. We provide comparative statics results considering different scenarios for intergenerational mobility. After, we allow interaction among families in credit markets within (i) the same income classes and (ii) all families in an economy. Welfare analysis shows that when frictionless markets are available, a Pareto superior outcome is feasible. We then introduce benevolent government which taxes families purely on redistributive purposes and compare the policy implied outcome with one associated with available credit markets. In our model framework the government cannot replicate competitive allocation.

Keywords: Credit markets, welfare analysis, redistributive policies.

JEL code: C61, E44.

1 Introduction

It is well known that the solution to economic models with market imperfection often is not Pareto optimal. Different marginal products in the long run induce arbitrage opportunities for the individuals, if credit markets are available. In some model economies, this will lead to a higher income for all the individuals both in transition and in the long run, in other economies it will have only a transitional impact. In a stochastic environment, for instance, one described in Loury (1981), where the stochastic factor ability is not known, when education investment decision is undertaken, and each dynasty operates on its own technology, interaction among households in credit markets will lead to a higher income for each household.

The stochastic nature of that economy is crucial, since if the world is deterministic, then individuals, heterogeneous in accumulated factor, but identical in utilities and technologies, which satisfy neoclassical assumptions, reach to a unique steady state, if there is one[1]. Opening credit markets in the deterministic economy with convex technologies will not lead to Pareto improvement in the long, since all family reach to the unique steady state and need not trade with each other. However, if such markets exist each period, then households will equate their marginal products each period via trade. Since they know about the availability of credit markets and have perfect foresight, consumption and investment choices are accordingly adjusted to sustain optimality conditions, and transition path will be less painful for each dynasty. Bertola et al. (2006), (pp. $155-156$) shows how in a simple dynastic economy, where the agents have warm-glow utility and there is an integrated capital market with concave technology, the aggregate output is lower when credit markets are not available. The central result is when the economy is far from the steady state, inefficiencies stemming from the absence of capital markets are the largest. Also, lower inequality makes self financing constraint less binding and increases output. Main qualitative results are likely

[1]If agents are also heterogeneous in non-accumulated factor, then they reach to a conditional steady state, that is, the economy converges to a (k/l) distribution (see Bertola et al. (2006), pp.$150-152$).

to hold in the infinite horizon setting.

We may let the deterministic nature of the economy unchanged and assume indivisibility in investments in human capital as in Galor and Zeira (1993) (henceforth GZ). Then credit market imperfections imply an inefficient solution in the long run - multiple equilibria can be observed and the agents with lower capital stock will stay there forever. If we assume no imperfection in credit markets then in GZ's model a unique steady state will be reached. Then an interesting question is what happens, when we open credit markets in the economy, where physical capital technology is concave instead of linear as in Galor and Zeira (1993), and we allow individuals differ not only in capital, but also in abilities.

Indivisibility feature of investment when credit markets are imperfect has been intensively studied in different economic environments since early 90's. Amongst the seminal papers are Galor and Zeira (1993), Banerjee and Newman (1993), Aghion and Bolton (1997), Matsuyama (2000), Mookherjee and Ray (2002), Mookherjee and Ray (2003), Ray (2006)). The closest to our setting is Galor and Zeira (1993), who study an overlapping generation (OLG) model with bequest (warm-glow) motives. Our model, with the same family structure and timing, has an infinite horizon standard dynamic structure, and the corresponding techniques from that theory can be applied. In my analysis credit market imperfection takes its simplest form - these markets are not available[2]. Keeping returns from human capital exogenous simplifies much our analysis (together with separability of two technologies, namely, physical and human capital), since endogenous inequality, in the spirit of Mookherjee and Ray (2002), Mookherjee and Ray (2003), does not arise.

In this paper we discuss a model, which is a kind of hybrid of those in Loury (1981) and Galor and Zeira (1993). In fact, we use the same economic structure and timing as in Loury - the family consists of two individuals, the parent and the child, all decisions are made by the parent, the offspring's innate ability is disclosed only after the decision is made.

[2]I follow to the strand of the literature that for simplicity assumes no credit markets. See e.g., Maoz and Moav (1999), Mookherjee and Ray (2002), Mookherjee and Ray (2003), Ray (2006), Mokherjee and Napel (2006).

However, there are crucial differences, concerning the stochastic process and technologies. In Loury (1981) the descendants' abilities are independently and identically distributed random variable, while in our model abilities follow to a first order Markov process. Second difference is that the only engine of Loury's economy is human capital, the stochastic component incorporated, while we have a deterministic physical capital technology, similar to Galor and Zeira (1993)'s model, but with concave technology instead of linear. The main feature peculiar to the GZ's model that we use is the indivisible nature of human capital investment. In both economies credit markets do not exist[3], and we preserve this property for our model as well.

At some point, however, we allow interaction among the families in credit markets with no friction, study how the wealth is redistributed and describe main qualitative features of Pareto improving allocation. Abilities are of stochastic nature - the parent, when choosing an educational plan for the child, does not know the ability of the child and has only prior believe about it, given by the distribution of child's ability, conditional on parent's realized ability. We will allow intratemporal trade opportunities among agents each period, after abilities have been realized and hence educational plans have been determined. Then households will trade with each other in order to retrieve some part of income that they have wasted when investing in education and facing to a lower ability. We will see that these individuals do not have any incentive to trade with ones in the same income class, but with higher ability (hence who have obtained education), since they will have the same marginal product. Instead, both will trade with individuals who did not invest in education and might be in the same income class[4]. The optimal investment and consumption policies of households are subject to correction once trading opportunities are introduced. Each period all individuals will be better off after they trade. Having this knowledge, in average individuals will spend more on consumption, which contracts the capital stock in aggregate. On the other hand,

[3]In Galor and Zeira (1993) agents interact to each other in imperfect credit market, however, all relevant results are unchanged, when credits markets are totally absent.

[4]We will discuss in detail, how we may have dynasties that find optimal to save the same amount at some period, but some of them do invest in human capital and the others do not.

at each period the individual's production frontier is enlarged due to trading opportunities and hence higher capital stock induces *much* higher capital stock in the next period, which in its turn implies higher consumption.

After we again shut down credit markets and instead introduce a benevolent government with certain instruments to redistribute income generated by dynasties in order to lower inequality and sustain more egalitarian society. We also answer the question whether the government, using lump sum tax instrument can replicate competitive allocation, the one, obtained after we open credit markets. As the government will introduce non-distortionary taxes (lump sum) and the policy will have a purely redistributive purpose (no government expenditures), in principle we cannot rule out a positive answer. In fact, we continue to assume complete information in the model - everybody knows everything. Then, since the households are rational and knows the nature of the government's policy, which will be time consistent, the solution will be a rational expectation equilibrium tax schedule.

The paper has the following structure. In Section 1 we describe our economic environment and the model. In the next section we provide some results of comparative statics nature in order to characterize incentives of mature individuals when drawing optimal policies. Section 3 discusses how the model works, when credit markets are available. In Section 5 we introduce a benevolent government a provide welfare analysis discussing different allocations. Section 6 concludes the paper.

2 The economic environment and the model

Our economy consists of continuum of dynasties normalized to one and dynasties operate in the infinite time horizon. They produce a single type of perishable goods, which can be transformed into investments (physical and human) with no cost. Each period a dynasty is represented by a family consisting of two members, the parent and the heir. Every individual lives two periods, in the first period as a heir (child, descendant, offspring) and the second period as a parent (mature). The parent takes all decisions relevant to time t on behalf of the whole dynasty. That is, the altruistic motives go beyond the child and the highest rationality in terms of time horizon, over which decision is made, is assumed. As a child becomes a parent who will care about his offspring's lifetime utility, by iteration it is easy to see that we obtain an infinite horizon dynamic program[5].

The dynastic structure of the model economy, which reduces to a standard infinite horizon dynamic program, seems more realistic. Our two-period overlapping generation model, with purely altruistic (non-paternalistic) preferences can be modified into an infinite horizon program (Barro (1974)). There are at least two other alternative preference specifications, (i) in which the mature cares only about the child's wealth (so called a "warm-glow" utility introduced by Andreoni (1989)), or (ii) parents derive utility only from the bequest size that they give to their descendants. Here, though altruistic motives are much weaker, it has been extensively used in the literature[6].

If the parent's utility function includes the lifetime utility of the child and the child does so when becomes a parent and so on, if fact each individual as a parent cares about all the dynasty and simply represent an infinitely lived representative household of a dynasty at time t. I believe that in many societies, the individuals' care for their progeny is beyond the first generation (the children), as they know they will spend much time with their grandchildren and often experience their success as mature. In our model we may assume a third period for individuals in which they are inactive in terms of decision makings and simply have their share from the current consumption, which is a family involved activity. That is, the intergenerational link is very strong and deriving utility from descendants' utility is more natural.

The next point worth discussing is the indivisibility nature of investments in education.

[5]Things are different when the parent's utility function contains only the bequest that the parent lets for the child. In that case the intergenerational link is not strong enough to ensure a Pareto optimal allocation, a fact that has been already documented by Diamond (1965).

[6]The warm-glow utility form is usually very handy to obtain analytical results. For instance, with a log utility and regular production function first order conditions yield closed form policy functions.

The literature studying several aspects of indivisibilities in educational investments has been developed enormously fast, which proves the relevance of this phenomenon in the real life. We confront indivisibilities in educational investment in our everyday life. Educational organization from elementary schools to PhD studies always provide educational programs, indivisible both in time and qualification aspects. One can list only a very limited number of honors, diplomas or certificates that an individual can pass through in his or her life. Moreover each level has its peculiar time interval and the price. Not only one has to spend certain amount of hours in the college to get qualifications, but also he or she has to pass exams, which assumes complete devotion to the educational program that one commits. Typically, the educational program as any other good in the market has its price, and an institution may provide only a limited number of degrees (usual ones are bachelor, master and PhD). Families hence confront to a discrete choice - either invest to the child's education and to pay the required amount or not to invest at all (no education). From our trivial discussion it follows that education is both forgone consumption and leisure. As any kind of investment it sacrifices on part of consumption, and not only that. Investment in human capital is an educational investment, which also requires time to dedicate to education - that is the forgone leisure. Here we do not model leisure choice, instead we assume inelastically supplied labor.

We have already noted that only investment in education does not guaranty that the heir will have success. Students must process their abilities to learn and pass compulsory exams. That is to say, ability matters. Educational decision is undertaken before education process takes place - this sounds tautology - but has important implication for our novelty. When the parent pays the tuition fee, he or she may have only believes about the talent of the offspring. Believes are based on two information sources: cultural and social background of the child, which is being formed within the family and genetic information that is transmitted from one generation to another (see e.g. Becker and Tomes (1979)). Abilities are assumed to have stochastic nature, but there is a positive correlation between parent's and offspring abilities. Imperfect correlation preserves the stochastic nature of abilities and importantly, correlation coefficients do not vary from time to time. That is, we model ability as a first order Markov process, characterized by the initial distribution and the transition matrix. We assume only two type of abilities for simplicity (high and low) and hence the transition matrix Π is 2×2 matrix with the generic element $\pi(z_i|z_j)$, $i, j = High, Low$. Information embraced in the stochastic matrix may or may not be available to a third part. In reality it can be only partially observed, but it is possible to improve the knowledge about ability processing observing the path the economy has passed[7]. Here we do not assume learning process from parties, we discuss only one case, when transition probabilities, together with all other relevant information are known to each party the government included (perfect information assumption).

In general, if financial markets were available, they would increase educational investment opportunities. Families would find optimal to borrow relevant resources in these markets and invest in their offspring' education. Intertemporal borrowing technology not only helps poor families to catch up with the reach ones in the probabilistic sense, but also serves as an alternative consumption smoothing instrument. Intratemporal trade in capital markets enlarges feasible production set and expands consumption each period. The fundamental argument in favor of market imperfection when educational investment opportunities take place, is that no collateral can be offered to lenders for financing education, and moreover, the returns of that investment belong to the descendant, who is the next period representative of the family and has no legal commitment to pay the debt incurred by the parent in the previous period. When there is no trade opportunity, the impact of the distribution of parents'income on the efficiency how educational resources are allocated across children is high. The impact is even magnified, when there are indivisible investments, as a minimum capital stock is required to ensure feasibility of acquiring education for offspring. On the other hand, this adverse impact will be mitigated due to the correlation between the parent's

[7]Law of large numbers will ultimately recover the form of the transition matrix.

and the offspring's abilities, but only to some extent as individuals are heterogeneous not only in abilities but also in capital stock.

Each dynasty maximizes time separable discounted stream of utilities. Labor is supplied inelastically and we do not model the consumption - leisure choice. At time zero each dynasty through its t-representatives (the mature of the family each time t) chooses the sequence of consumption levels that maximizes the lifetime utility of the dynasty:

$$\max_{\{c_t\}} \mathbf{E} \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1}$$

where $\mathbf{E}$ is the expectation operator and captures the stochastic nature of the problem. Abilities follow to a first order Markov process, and at each time period the history of the realized abilities is given by $z^t = (z_0, ..., z_t)$. We run the histories in our model in an implicit way having in mind, for instance, $c_t(z^t)$, when writing c_t.

Each period the family of the dynasty is restricted by a resource constraint, which is given by inherited physical capital and educational investment choices from the previous period and time invariant corresponding technologies (physical and human). Then the head of the family optimally chooses t-period family consumption, bequest to the child in the physical capital stock form and investment into the child's education. The constraint takes the following form:

$$c_t + k_{t+1} + x_{t+1} = f(k_t) + W(x_t, z_t), t = 0, 1, ..., \tag{2}$$

where c_t k_t and x_t are consumption, investment in physical capital and investment in human capital, respectively. f is the physical capital technology, with $f(0) = 0$, $f'(k) > 0$, $f''(k) < 0$ for all $k > 0$ and $f'(0) = \infty$, $f'(\infty) = 0$.

W is the wage function (human capital technology) and obtains two distinct values. As we assume that there are two types of abilities, high and low, denoting z_H and z_L, respectively, a certain amount must be invested to ensure skilled wage if high ability type is disclosed, otherwise, irrespective the human capital investment choice, an individual obtains the unskilled wage. Formally, the wage function takes the form of

$$W(x,z) = \begin{cases} w_L, & \text{if } x = 0, \\ w_L, & \text{if } z = z_L, \\ w_H, & \text{if } x = \bar{x} \text{ and } z = z_H; \end{cases} \tag{3}$$

where w_L, w_H and $\bar{x}$ are unskilled, skilled wages and investment in education, respectively. We restrict these values to be $0 < w_L < w_H$ and $\bar{x} < w_H - w_L$ so that the model makes sense.

We choose technologies to be additively separable, that is, we assume that two input factors are perfect substitutes. Exogenous nature of technologies implies that the optimal policies are time invariant and in particular does not depend on characteristics outside the control of a dynasty. As we see the wage function consists of two distinct points and the standard first order condition argument cannot be applied. In the second chapter of the thesis we obtain the optimal physical and human capital investment policies numerically and qualitative features of those policies are invariant to a large range of sensible parameter joint configuration. We use logarithmic utility due to its handiness in numerical solutions, but we believe the results do not depend on the log form.

In the next section we provide some comparative statics, which will help to understand the decision making process for individuals through studying how they respond to some changes in fundamentals, in particular to a transition matrix Π.

3 Comparative statics

The stochastic nature of the model complicates the optimality rules that individuals must respect when splitting their income into three parts, consumption and two types of investment. Let $\tilde{k}(k,x,z)$ and $\tilde{x}(k,x,z)$ be physical and human capital optimal policies,

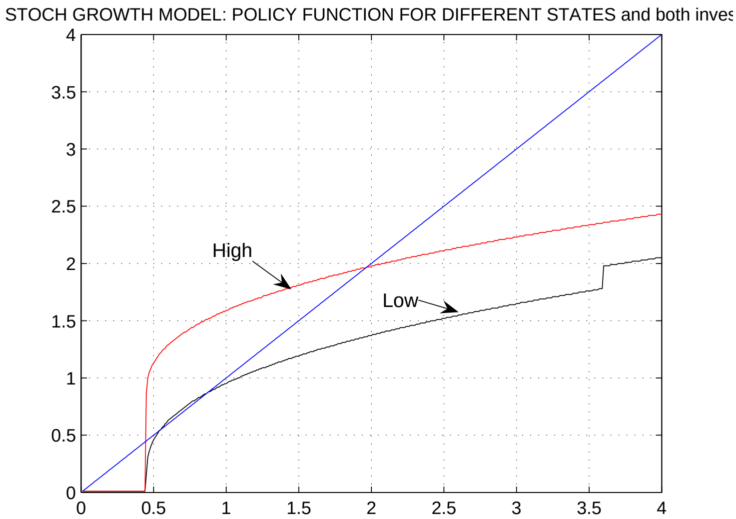

Figure 1: Low intergenerational mobility

respectively, and let $\tilde{a}(k, x, z) = \tilde{k}(k, x, z) + \tilde{x}(k, x, z)$ be the aggregate investment policy[8]. Incidentally note, that we may have two different policies, say, $(\tilde{k}_1, \tilde{x}_1)$ and $(\tilde{k}_2, \tilde{x}_2)$, that sum up to the same total capital, $\tilde{k}_1 + \tilde{x}_1 = \tilde{k}_2 + \tilde{x}_2 = \tilde{a}$.

Any choice of $\tilde{x} \in \{0, \bar{x}\}$ cannot guaranty a skilled wage for the offspring by 100%, but it does by some probability less than 100%, provided that $\tilde{a} \geq \bar{x}$. That probablity is conditional on the parent's realized ability, which means that individuals indentical in income, or even more, identical in savings, but different in abilities may choose different optimal human capital investment plans. Different choices may happen, as these individuals have different believes about their offspring's ability or, in other words, their children confront to different ability distributions, which are given by the corresponding rows in the transiton matrix Π. Thus the extent of this difference in particular depends on the form of the transition matrix. We observe this phenomenon, when looking at the optimal policies from our simulated model. We plot the aggegated optimal polices, when abilities are different, but they both invested in human capital[9].

[8]We suppress time indexes, since the optimal investment policy vector $(\tilde{k}, \tilde{x})$ is time invariant and tildes are used for the next period variables throughout the paper.

[9]Alternatively, we could plot the picture when they both did not invest in education. The important point is to have the individuals heterogeneous only in realized abilities.

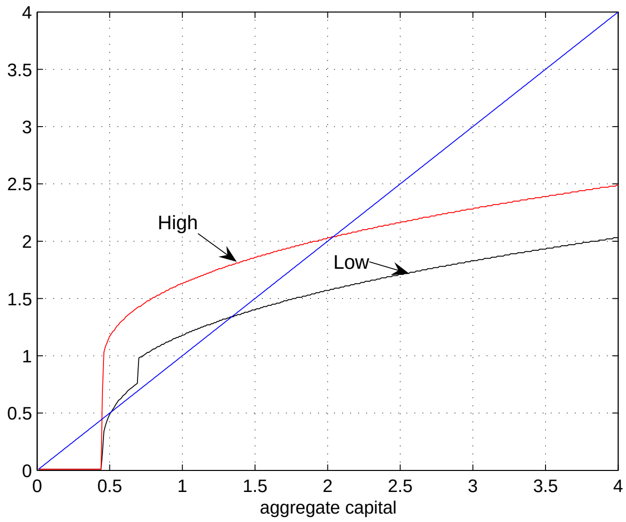

Figure 2: High intergenerational mobility

In Figure 1 we have the model with low intergenerational mobility (numbers in the diagonal in the transition matrix Π are close to one), while in Figure 2 we allow a higher extent of mobility[10]. We observe that in the low mobility case the capital threshold for the low skilled individual to invest in education is much higher than that in the higher mobility case[11], and the difference in thresholds, at which the low and high ability individuals invest in education is higher. On the contrary, when there is higher mobility, that difference is smaller. We document this observation below.

Proposition 1 *Cetirus paribus, the smaller is the intergenerational mobility (that is, the higher are diagonal numbers in* Π*) the larger are differences in total investment levels, at which the individuals with different abilities find optimal to invest in educaiton.*

In other words, if a high ability individual is expecting to have a child of high ability with high probability, then he will be eager to invest to the child's education, while at the same

[10]Figure 1 and Figure 2 correspond to the models with $\Pi = [0.6, 0.4; 0.3, 0.7]$ and $\Pi = [0.6, 0.4; 0.55, 0.45]$, respectively.

[11]The points in the graphs at which the policies jump correspond to a switch in human capital investment regime, from zero to $\bar{x}$ investment. From that level of aggregate capital households find optimal always to invest in education.

level of savings a low ability individual will find optimal not to invest, especially if he expects to have a low ability child with high probability.

Looking at the optimal investment policy functions from the previous chapter, we also observe that individuals with low ability, but heterogeneous in investment in education, implement the same optimal investment policy, that is $\tilde{a}(k, \bar{x}, z_L) = \tilde{a}(k, 0, z_L)$, for all $k > 0$. This is due to (i) the fact that the aggregate aggregate technology function is identical for low ability individuals, $f(k) + w$, and it is invariant to educational investment choices, and (ii) the assumption that transitional probabilities are exogenously given and time invariant - households do not update their beliefs (no learning) and/or they cannot increase the probability of higher ability occurrence for the child through appropriate investment choices[12]. Individuals endowed with high ability, derive different optimal capital policies when they are heterogeneous in inherited investment in education, since the ones with investment in education receive skilled wages contrary to the others who receive unskilled wages. That is, the first argument, noted for low ability individuals, drops.

We notice that existence of a unique capital threshold in switching from zero to $\bar{x}$ educational investments, associated to a certain ability type of an individual (a mature of a family), has been proved for the linear utility function in the previous chapter (Proposition 2.1). That there is no a single threshold for all the individuals, is due to the difference in conditional probabilities for abilities (nonidentical rows of the transition matrix Π), as discussed in Section 2.1, in the previous chapter. When a broader class of utility functions is assumed, then a total capital threshold that household should reach to invest in education depends not only on conditional distributions abilities, but also on the curvature of the utility function.

When a family invest in education and offspring turns to be unlucky (low ability is disclosed) the amount invested is lost. A good question is whether the households being isolated from each other, still have tools in terms of technologies available to mitigate a possible loss

[12]For instance, Konrad (2001) allows probabilities be an increasing function of effort.

incurred by educational investments. Clearly, after they invest, they are aware of the fact that their descendants cannot take any action to (partially) recover the wasted investment. Hence, mature individuals are the only ones who can take any action translated in optimal policies that will lead to mitigation of losses that actually bear their offspring. Thus, the functional form that optimal policies take, contain all the information that we are to extract to understand the "insurance" mechanism that dynasties are using. Families, as current representatives of dynasties, form expectations which cover the infinite horizon. However, each current family, knowing that the future families will choose optimal consumption stream, given its optimal current consumption choice, confronts to a two period (intertemporal) investment choice problem, "treating as given" future optimal choices.

In order to complete our understanding what the intertemporal problem is that each family solves, taking into account possible losses due to disclosed low ability for the offspring when invested in education, we use first order condition representation for physical capital[13]. Define the aggregate technology $\phi(k, x, z)$ to be equal to the left hand side of (2). Given the states (k_t, x_t, z_t) at time t, the family picks the optimal policies $k^*_{t+1} = \tilde{k}(k_t, x_t, z_t)$ and $x^*_{t+1} = \tilde{x}(k_t, x_t, z_t)$. Then the first order condition takes the following form:

$$u'(c_t) = \beta \mathbf{E}_{z_t}[\frac{\partial \phi}{\partial k_{t+1}}(k_{t+1}, x_{t+1}, z_{t+1})u'(c_{t+1})]. \tag{4}$$

Noting that the partial derivative with respect to physical capital in the right hand side is a constant number, since the physical capital technology is deterministic and two technologies are separable, (4) reduces to

$$u'(c_t) = \beta f'(k_{t+1})\mathbf{E}_{z_t}[u'(c_{t+1})], \tag{5}$$

[13]Despite the irregular form of the human capital technology, one can proof that the first order condition with respect to physical capital necessarily holds, since the optimal k_{t+1} is always in interior due to the condition $f'(0) = \infty$.

and it can be written in a more tractable form:

$$\beta f'(k_{t+1}) = \frac{u'(c_t)}{\mathbf{E}_{z_t}[u'(c_{t+1})]}. \tag{6}$$

We know that when an individual has an access to a risky technology, say, an asset that provides a certain payoff for each state to be realized with some probability, then she is ready to pay a relatively high price for an asset that guaranties a relatively high return in states bad for consumption. Such stochastic technologies might be available in the environment, where there is no interaction among families as well. Our task is to understand whether our stochastic technology, namely, human capital technology, provides such kind of insurance. From (6), we observe the expectation operation only over the next period consumption. With two states, two next period consumption values (optimally chosen) are associated with other relevant choices. Suppose $x_t = \bar{x}$, then $c_{t+1}(z_H)$ is expected to be higher than $c_{t+1}(z_L)$ as additional resources are available amounted to $W_H - W_L$, and it is expected that this differences will not completely absorbed by $t+1$ optimal investment choices[14], (k^*_{t+1}, x^*_{t+1}). Good states in consumption are the ones when investment in human capital is transformed into a positive return. That is, human capital technology per *se* does not provide insurance against bad states, instead moves to the same direction with consumption. As already discussed, two optimal policies associated with two different realized states are different[15].

In what follows from the above discussion, we are better to look at the behavior of optimal investments in deterministic (physical) technology, since households, apart from to decide at what total capital accumulation level to invest education, they also must decide how much to save at all. For instance, the parameter configuration and functional forms that we use to simulate the model, it turns out that when educational investment has been undertaken and

[14]This is indeed so - when looking at the optimal policies, the jumps associated to investment in education amounts only a half of the required investment amount $\bar{x}$ and much less than $W_H - W_L$, since $W_H - W_L > \bar{x}$.

[15]If no educational investment is undertaken ($x^*_{t+1} = 0$), then the unskilled wage is realized in both states, but still the investment policy schedules for high an low abilities in $t+2$ differ, as they are different in when educational investments take place (see Figure 1 and 2). This is, again, due to the Markov property that for abilities, as a the stochastic process, is assumed.

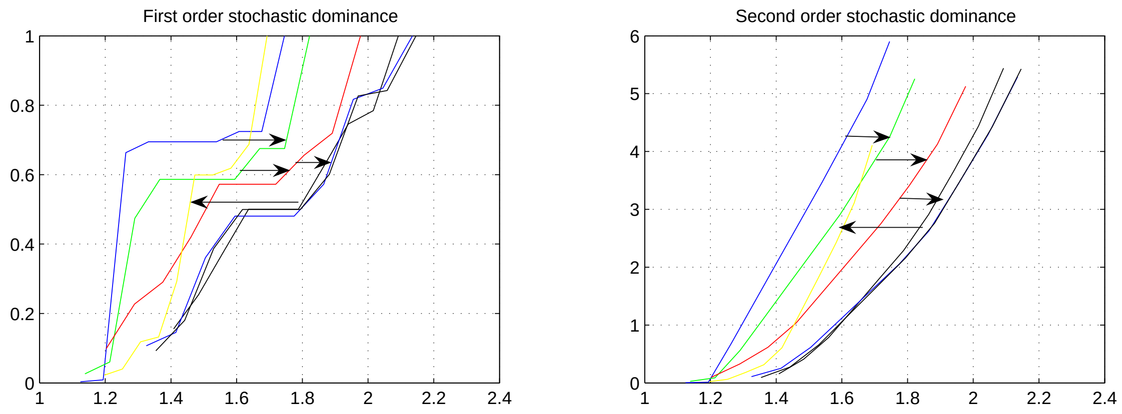

Figure 3: Stochastic dominance, (a-b)

the realized state is high, then it is optimal to invest in education already at a very low level of capital accumulation (a little more than $\bar{x}$), and since the input in the physical technology can be chosen from the interval (contrary to discrete choices for investment in education), we think of a possible insurance tool to be the physical capital rather than human capital technology. In this context we in particular study what are the incentives that drive optimal policies when different intergenerational mobilities are assumed.

Before doing that, we answer the following trivial question - how the ergodic income distribution responds to a certain change in intergeneration mobility. In our model-economy we take different levels of intergenerational mobility, from very low to very high and look at the corresponding ergodic distributions to check for first and second order stochastic dominance. Pareto improvement is associated to first order stochastic dominance, while a good measure of inequality in the economy the second order stochastic dominance, which is equivalent to the generalized Lorenz curve. Any order of stochastic dominance implies all higher order stochastic dominance, but the converse does not hold. That is, inequality improvement in terms of second order stochastic dominance is a weaker criterion of welfare improvement than the Pareto improvement in the first order stochastic dominance sense.

In Figure 3, a-b, we plot cumulative distributions against for each level of immobility. The very left schedules in both figures (a) and (b) correspond to very low extent of mobility, that is, the diagonals in the transition matrix Π are close to one. Then the arrows indicate the corresponding schedules that are drawn for each next higher level of intergenerational mobility. The very right (a cluster) of graphs, dominant to all other graphs correspond to the perfect mobility (all entries in Π are 0.5), very high mobility (off diagonal numbers are bigger than diagonals, but not much) and mixed mobility (on one state there is a high mobility, on the other not). Finally, the schedule that goes back and is dominated by moderate and perfect mobility schedules, corresponds to the extreme mobility (off-diagonal numbers are very close to one). The main conclusion is that dynasties prefer higher, rather than low mobility. Uncertainty allows to escape from a poverty trap that is usually generated when both technologies are deterministic (back to Galor and Zeira (1993)). However, dynasties do not like extremely volatile status, since investment in education in the current period will be wasted in the next period. The closer is the intergenerational mobility to the one defined perfect, the higher is welfare for dynasties (in distributional sense) and hence in average the higher optimal policy schedules are undertaken.

Turning to the previous discussion we continue our comparative statics on the mobility looking at the optimal policies. For instance, with very low intergenerational mobility high ability individuals have little incentives to invest in the deterministic technology, as the extent of uncertainty they confront, is low. Instead, they will invest in human capital technology at the earlier stage of capital accumulation. The individuals, who have bequeathed positive human capital investment, but happened to be endowed with low ability, will have the opposite strategy of ones who have high ability (and they parents invested in human capital). They will invest much in physical capital and will find optimal to invest in human capital only at higher stage of capital accumulation. That is, when an economy is characterized by low intergenerational mobility, that it is difficult to change the status, high ability class is becoming richer, while low ability is becoming poorer, and self reinforcement mechanism leads poor dynasties with low abilities to a poverty trap with some (high) probability. The corresponding behavior is peculiar to the households who did not invest in education. With

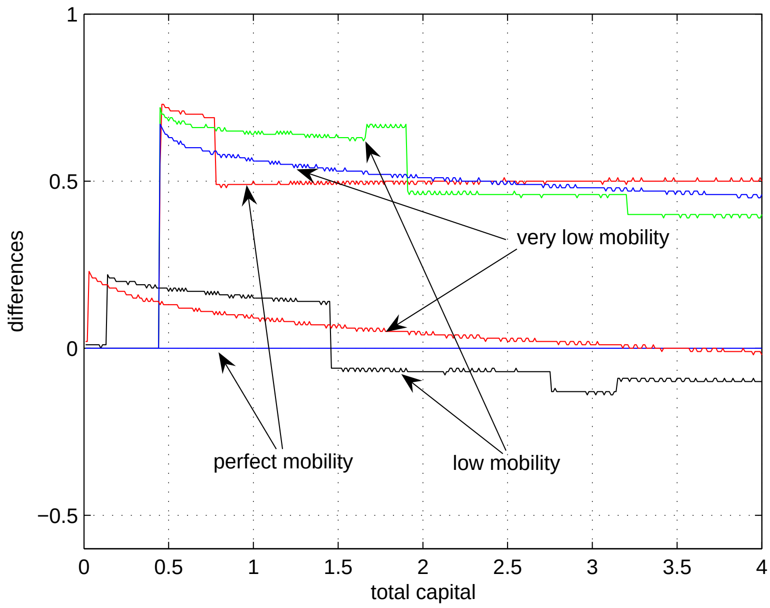

Figure 4: Differences in total investments

low intergenerational mobility the individuals with high ability will invest in education rather than in physical capital and vice versa for the low ability individuals.

However, as we speak about certain capital type but not the total one $(k + x)$, when plotting optimal total policies, these effects are not obvious, since the incentives in investing in different technologies are confronting to each other, and the positive effect is likely to dominate in most cases of parameters. In fact, all policies different in statuses in investment in education and abilities are positively correlated to the extent of the intergenerational mobility - the higher is mobility in the economy, the higher is the optimal total investments at each level of total capital stock, when the same educational investment policy is taken. This result in the end leads to the first order stochastic dominance of income distributions with higher mobility over the income distributions with lower mobility. Increments are however different, and in order to justify our arguments, we must look at the differences in policies under different mobility regimes rather than the levels.

In Figure 4 we have plotted the differences in optimal (total) investment policies for high and low individuals, when they are identical in human capital investments. The upper three graphs corresponds to difference of optimal total capital, when invested in human capital, that is $\tilde{a}(k, \bar{x}, z_H) - \tilde{a}(k, \bar{x}, z_L)$, and the lower three graphs are the case when no human capital investment took place, $\tilde{a}(k, 0, z_H) - \tilde{a}(k, 0, z_L)$. We immediately note, that differences are higher with educational investments than without. That is, educational investments, as a source of income, amplifies differences in income between low and high ability individuals, and this is translated into the optimal policies.

We reiterate our argument putting differently:

Proposition 2 *The higher is mobility, the higher (lower) is investment in physical capital for high (low) ability individual, and investment in human capital is undertaken at the higher (lower) level of capital accumulation.*

In Figure 1 and 2 we actually observe differences in capital accumulation. For the low ability individuals it is obvious, and for the high ability ones it is less obvious, but there *are* different thresholds and difference can be easily made bigger specifying higher investment level, required for education, $\bar{x}$. Somewhat too high level of investment in physical capital is expected for the high ability type, as she wants to account for the next period adverse shock (with high probability) on income and hence on consumption. The physical capital technology, as a zero risk asset, takes crucial role in consumption smoothing when a status is very likely to change from period to period. We observe in Figure 4 that with perfect mobility[16] the difference in optimal total capital when invested in human capital, $\tilde{a}(k, \bar{x}, z_H) - \tilde{a}(k, \bar{x}, z_L)$, is the highest for the same level of next period human capital ($\tilde{x} = 0$ or $\tilde{x} = \bar{x}$). The area at which the differences are higher with no perfect mobility, corresponds to different next period human capital investments. For instance, when low mobility takes, the households having optimal policy $\tilde{a}(k, \bar{x}, z_L)$ invest in human capital only when they have around 1.8 unit of capital and hence the total optimal capital jumps up at that point[17], while households with $\tilde{a}(k, \bar{x}, z_H)$ invest in human capital when they own a little more than $\bar{x}$ unit

[16] Again, perfect mobility corresponds to $\Pi = [0.5, 0.5; 0.5, 0.5]$.

[17] The upward jump in fact is less than the level of human capital investment, $\bar{x}$, that is, some amount of investment in physical capital is sacrificed in order not to have an $\bar{x}$ distortion in consumption.

of capital. Thus, the downward jump around the point 1.8, corresponds to the lower bound of capital, at which individuals, identical in investment in human capital, but heterogeneous in abilities, find optimal to invest in human capital. Then we observe that in the levels of capital higher that 1.8 the differences in optimal total investments are higher with perfect mobility. We in fact do not observe a jump in differences for the very low mobility, as individuals with low ability (but invested in human capital in the next period) needs higher units of capital stock than 4. Independent from the jump, differences with perfect mobility are higher than ones with very low mobility for the levels capital, higher than around 2.5. After the jump occurs with very low mobility, that difference becomes the smallest (It is out of the domain we have plotted, but the difference is basically preserved - we see this from two other graphs, even if it has diminishing feature, its extent is negligible).

In the lower group of graphs in Figure 4, we observe zero differences under perfect intergenerational mobility. That is, when conditional distributions are the same and are uniform, there is are no differences in policies $\tilde{a}(k, 0, z_H)$ and $\tilde{a}(k, 0, z_L)$, as no human capital investments has been undertaken and differences in abilities are not sufficient to take different human capital investment policies (both are restricted not to invest in human capital, as they bequeathed zero human capital investment). Similar to differences when invested in human capital, the difference in perfect mobility case will be the highest. In the end, the individuals who turns to have high ability, but they are heterogeneous in bequeathed human capital, form the same expectations based on the extent of the intergenerational mobility, and their policies are identical in reacting changes in the mobility extent. A lower, even negative differences are observed, as the total production is the same in both cases $f(k)+w$ and at the low level of capital, when investment in education is taken for high ability individuals, consumption is less distorted and the difference, $\tilde{a}(k, 0, z_H) - \tilde{a}(k, 0, z_L)$ is not that high, given the mobility in the economy. Low ability individuals take human capital investment scheme at a higher capital stock[18] and more consumption is sacrificed, which in end lead to

[18]Note that what we call perfect mobility, is just a lower bound of a positive extent of immobility - we in fact do not allow off diagonal numbers, higher than off diagonal. Analysis similar to above can be carried.

negative differences.

4 Opening credit markets versus redistributive tax policies

When loan markets are available, households are willing to trade with each other, in order to improve their wealth positions. We assume a loan market without any friction (no transaction costs and a unique rate for both lenders and borrowers). Markets are opened after a state of nature has been realized so that outcomes of investment decisions are known. We discuss two kinds of credit markets. First, when trade is possible within the same capital class. That is, households which are identical in total capital stock at time t, $a_t = k_t + x_t$, are trading with each other. Then we allow interaction among all households, to identify the Pareto superior outcome for the economy.

Somewhat surprisingly, households identical in investments in physical and human capital, but different in realized abilities do not trade with each other. It is easy to see this point when recalling the two corresponding technologies, $f(a - \bar{x}) + w + d$ (high ability is realized) and $f(a - \bar{x}) + w$ (low ability is realized). Then, since the marginal productivities are the same, there are no incentives for these households to trade. Instead, there will be trade between households in the some capital class, which differ in investment in education. Recall, that two policies in the previous period in total can be the same, $a_1 = a_2$, but we may have different policies in educational investment[19] due to difference in realized abilities in the previous period. Then we have the following technologies, $f(a) + w$ and $f(a - \bar{x}) + w(+d)$. Marginal productivities are different and households will trade which will lead to a unique price of capital (interest) rate and new income distribution.

At each period t, the i type household, in the same income class $a \geq \bar{x}$, solves the

[19] For instance we may have $k_1 = a_1$ and $k_2 + \bar{x}_2 = a_2$.

following maximization problem, when having credit (borrowing or lending) opportunities:

$$\max_{a_i} f(a_i - x_i) + R(a)(a - a_i) + W(x_i, z_i), x_i \in \{0, \bar{x}\}, i = 1, 2; \tag{7}$$

subject to

$$p_1(a)(a - a_1) + p_2(a)(a - a_2) = 0; \tag{8}$$

where $p_1(a)$ and $p_2(a)$ are the proportions of households invested and not invested in education, respectively. Note that here households are distinguished only by investment in education and by the realized state. For type 1 and 2 $x_1 = \bar{x}$ and $x_2 = 0$, respectively, and $z_i \in \{z_H, z_L\}$, $i = 1, 2$. The problem is to be solved each period, but for convenience we suppress time index in formulas and also to indicate the static nature of the problem. The interest rate $R(a)$ is given to each household in capital class a and it is the same both lenders and borrowers. First order conditions are necessary and sufficient[20] and yield no arbitrage condition:

$$f'(a_1 - \bar{x}) = R(a) = f'(a_2) \Rightarrow a_1 = a_2 + \bar{x}. \tag{9}$$

Using the constraint in (8) we obtain formulas for a_1 and a_2 as a function of capital stock a, educational investment $\bar{x}$ and the weights $(p_1(a), p_2(a))$. We recall that the following relations hold each period:

$$a_1 = a + \frac{p_1(a)\bar{x}}{p_1(a) + p_2(a)}, a_2 = a - \frac{p_2(a)\bar{x}}{p_1(a) + p_2(a)}. \tag{10}$$

For each $a \geq \bar{x}$, the type one household has higher marginal utility than the type two, and, when equating marginals, the type one increases its capital stock while pays an interest for the borrowed resources to the type 2, which decreases its capital stock and receives interest payments. Both types are better off after trade, that is, post trade total income is

[20]The problem for both types can be transformed into a maximization of strictly concave function over the compact set, and hence solution is unique and interior. For type 1 we can write the program as $\max_{a_1 > a} f(a_1 - \bar{x}) + w + d - R(a)(a_1 - a)$ and for type 2 we have $\max_{a_2 < a} f(a_2) + w - R(a)(a_2 - a)$. Then a_1 and a_2 must satisfy the constraint in (8).

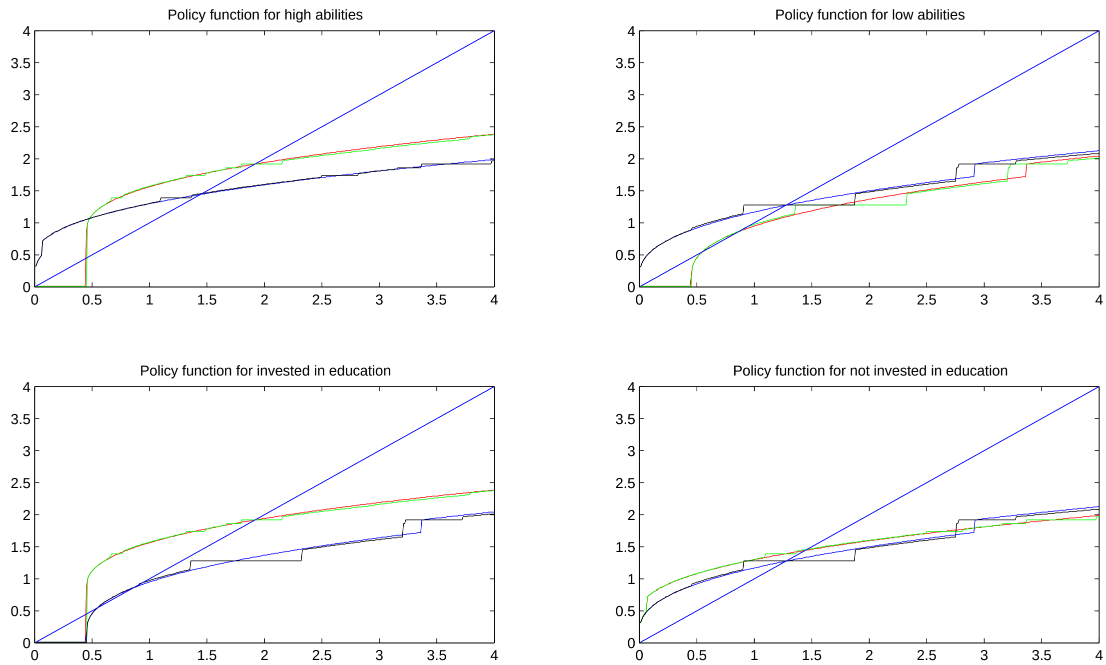

Figure 5: Optimal policies

higher than that before the trade.

As a consequence of the trade within the same the capital class, households find themselves in other class of income and this is of course taken into account when deriving optimal policies in each period. Higher income that they expect after the each period trade induces higher consumption, as a normal good (income effect). On the other side, households are motivated to save and invest more (substitution effect), as the higher is the next period capital stock, the higher will be income after the trade. These two effects are shaped in a way that the optimal consumption path respects intertemporal optimality conditions[21].

In Figure 5 we demonstrate optimal policies when loan markets are available in the class of families with identical savings versus the policies when markets are absent. Schedules that are in some way deviating (they fall short) from the regular ones correspond to the optimal policies with credit markets available. For instance, in the upper left figure, corresponding to

[21]First order condition with respect to capital in (6) must hold, with the left hand side $\beta(f'(a_i - x_i) - R)$.

the choices of high ability individuals that differ in investment in education (higher schedule is for invested), differences are less significant than in the upper right figure, corresponding to the choices of low ability individuals (again, higher schedule is for invested). A trivial observation is that for low ability individuals, loan markets are of more importance and they account for their existence more heavily. We evidence this importance in the other two graphs as well, where low ability individuals' policies are the lower schedules. (In the lower-right graph, however, we see that for sufficiently higher savings low ability individuals draw optimal saving policies higher than the policies corresponding to the high ability individuals, when credit markets are available).

We also provide corresponding schedules of ergodic distributions. In Figure 6 bold graphs correspond to the economy with credit markets within the same saving class available. The main observation is quite interesting: When loan markets are available, dynasties account for their existence choosing lower level capital in the long run. That is, in the long run income effect is dominating over the substitution effect. Clearly, to have such result in the long run, optimal policies with markets should fall short from the once without markets each period as well, that is, this dominance should be evidenced in the short run as well. This can be observed (in most cases) when looking at the graphs in Figure 5 more carefully. Each period lower savings means higher consumption, and the trade in credit markets compensates dissavings in the previous period enabling to sustain consumption each period higher than consumption when there are no markets. We plot ergodic consumption distributions and observe that the consumption distribution corresponding to the economy with markets second order stochastically dominates to the consumption distribution of the economy with no markets.

What happens if we allow interaction among all households. Most aspects of above analysis remains valid, in particular, the incentives of households when correcting the optimal policies for the credit opportunities. It is convenient to keep the physical capital as a variable, rather than the total capital. As before, the market is open after the state of nature is

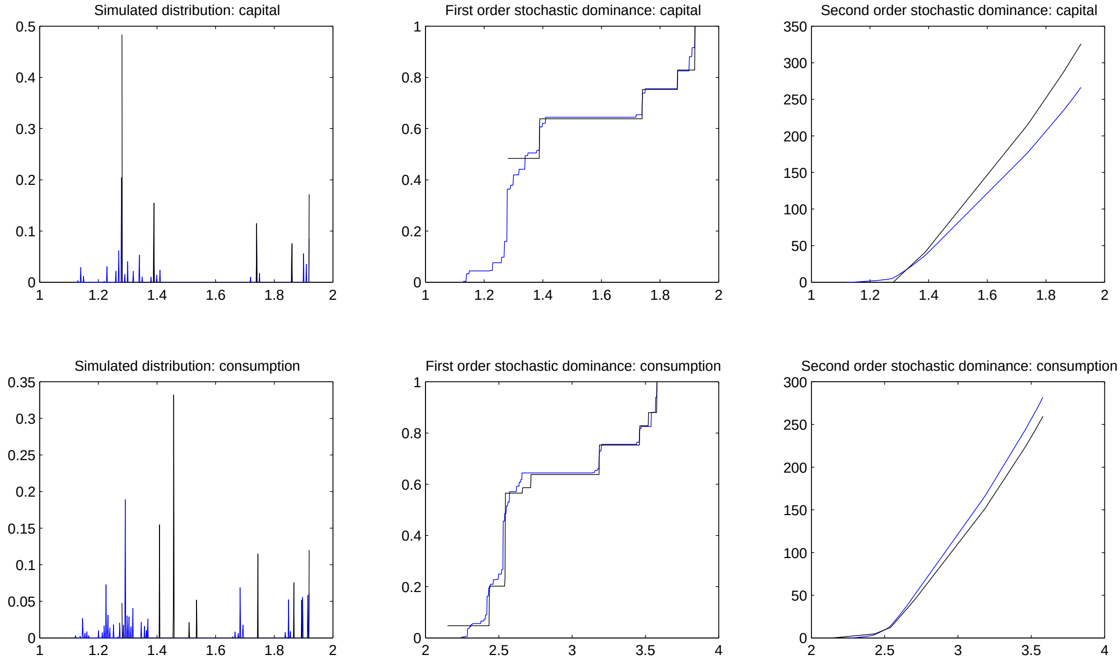

Figure 6: Stochastic dominances

realized and hence wages become known. We have already argued, that households differ in marginal productivities if they have different investment levels in physical capital technology and investment in education and the realized ability have no any impact on marginals. Thus we distinguish households only in terms of marginal productivities (equivalently, in terms physical capital stock). Households, inherited physical capital stock k_i, each period solves the following reduced program:

$$\max_{\tilde{k}\geq 0} f(\tilde{k}_i) + R(k_i - \tilde{k}_i), i = [0,1]; \tag{11}$$

subject to

$$\int_{j\in[0,1]} (\tilde{k}_j - k_j) dP(k_j) = 0, \tag{12}$$

where $\tilde{k}$ is the capital stock after the trade and P is the current probability distribution.

First order conditions yield

$$f'(\tilde{k}_i) = R = f'(\tilde{k}_j) \Rightarrow k^* = (f')^{-1}(R). \tag{13}$$

Then plugging the unique capital stock k^*, optimal for each household, into the constraint in (12), we obtain

$$\int_{j\in[0,1]} (k^* - k_j)dP(k_j) = 0 \Rightarrow k^* = \int_{j\in[0,1]} k_j dP(k_j) = \bar{k}. \tag{14}$$

That is, each household's physical production size is $f(\bar{k})$ and if had more than $\bar{k}$ units of physical capital is a lender and a borrower if has less than $\bar{k}$. Capital is traded by the price R.

Thus heterogeneity in physical capital invested in the production technology is completely removed, however households continue to be different not only in investment in human capital and ability, but also in income from deterministic technology which includes the term $R(\tilde{k}_i - k_i)$, a consequence of the trade. Cetirus paribus, each household is better off and unrestricted trade (no capital or income classes, within which only trade is allows), Pareto improving outcome is reached. As already mentioned, optimal policies are subject to change, which means corresponding changes in the transition matrix of the stochastic joint process (a, z) and hence corresponding change in the ergodic distribution. Given the fundamentals (parameters and preferences and technologies), trade opportunities without any restriction induces first best outcome not only in the transition, buy also in the long term.

We may even improve welfare of dynasties, if we allow interatemporal borrowing, instead of or in addition to the intratemporal one. This will allow agents to go short and invest in education always, if educational return is high enough. We do not do this exercise, since we are basically interested in a government's policy of intratemporal nature and we want to compare two economies, in which incomes are redistributed within time period through either credit markets or a tax policy.

5 Welfare analysis

In the final step of our analysis we introduce a benevolent government that taxes some households and subsidize to others in order to sustain more egalitarian society. The question that we raise is whether the redistributive tax policy can replicate the allocation obtained by assuming availability of credit markets. The benevolence nature of the government implies that one cannot exclude such possibility, and thus it is worth answering this question. In our economy the government and dynasties agree on a tax policy, which is a rational expectation equilibrium outcome. (Neither of agent types (the households and the government) violate the rule they agree on.) There is a perfect foresight, all agents are rational, everybody knows everything (perfect information) and all the decisions are taken at time zero.

Each period the family confronts to a tax/subsidy as a deterministic, time invariant function of (k_t, x_t, z_t), defined as $T(k_t + x_t, z_t)$ where k_t, x_t, and z_t are physical capital, investment in human capital and the ability, respectively. As we have noted, the government's redistribution policy concerns a class of agents identical in total investments and hence the tax T is a function of the total capital $k_t + x_t = a_t$, rather than a function of k_t and a_t.

The program of each dynasty in the economy is as follows:

$$\max_{\{c_t, k_t, x_t\}} \mathbf{E} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{15}$$

subject to

$$c_t + k_{t+1} + x_{t+1} = f(k_t) + W(x_t, z_t) - T(k_t + x_t, z_t), \tag{16}$$

$$c_t, k_t, x_t \geq 0, \forall t, \tag{17}$$

where f is strictly concave and strictly increasing with $f(0) = 0$ and W is the wage function given by (3).

For an equilibrium tax function T, we denote the household's optimal policy

$$g(k, x, z, T(k+x, z)) = g_T(k, x, z). \tag{18}$$

We incorporate the constraint (16) into the utility function of a family and call it one period return function:

$$F(k, x, z, g_T(k, x, z), T(k+x, z)) = U(f(k) + W(x, z) - T(k+x, z) - \tilde{k} - \tilde{z}) \equiv U_T(c_t); \tag{19}$$

where $g_T(k, x, z) = g(k, x, z, T(k, x, z)) = (\tilde{k}, \tilde{x})$ is the vector of the policy function when the equilibrium tax policy is applied.

Note that in the same class of capital ideally we should distinguish 4 subclasses of agents: (i) families who invested in human capital and get a high ability child, (ii) families who did not invest in human capital and get a high ability child, (ii) families who invested in human capital and get a low ability child, (iv) families who did not invest in human capital and get a low ability child. We have argued that in the available loan market case families identical in human capital investment choices do not trade, otherwise they do irrespective of disclosed ability types. Here the rule of interaction among families is the solution to the social planner's objective function - the social welfare function, which we introduce next.

As we distinguish for subclasses within each income class in the society, we need to have corresponding weights, which are time invariant, $(w_{H,0}, w_{H,\bar{x}}, w_{L,0}, w_{L,\bar{x}})$, respectively. The social planner may have certain incentives when choosing these weights. As we will see soon, differences in two allocations are not related with weights, used in the social welfare function.

The social planner each period chooses the tax policy $(T_{t,H,0}, T_{t,H,\bar{x}}, T_{t,L,0}, T_{t,L,\bar{x}})$, to maximize the infinite sum of the following stream of utilities:

$$\max \sum_{t=0}^{\infty} \sum_{i=H,L} \sum_{x \in \{0,\bar{x}\}} \beta^t w_{i,x} U_T(c_{t,i,x}). \tag{20}$$

As already noted, for each class of agents owning a certain amount of total capital, there is a redistribution channel that allows to transfer some resources from one ability type to another within the class. No transfer is possible from one income class to another. This is somewhat restrictive assumption but can be reasoned as follows: we define the primary purpose of redistribution to mitigate the adverse effects coming from the stochastic nature of the model. If some dynasty has a high initial capital and happens to have many talented members, it will only pays for being lucky having more talents than others, but not for being initially reach. We also recall that the primary goal, we follow, is to compare two allocations, namely competitive one and the social planner's allocation, when interaction is possible within the same income class.

We have the following constraint for the government:

$$\sum_{i=H,L} \sum_{x=0,\bar{x}} N_{t,i,x} T_{t,i,x} = 0, t = 0, 1..., \tag{21}$$

where $N_{t,i,x}$-s are the realized numbers for high and low ability individuals when human capital investment is zero, $x = 0$, or positive, $x = \bar{x}$. Since optimal investment policies are known to the government, which are the once when the optimal tax schedules are implemented, the government's program is in fact of static nature - each period the government solves the following problem: The program of the government is

$$\max \sum_{i=H,L} \sum_{x=0,\bar{x}} w_{i,x} U(c_{t,i,x}), t = 0, 1, ..., \tag{22}$$

subject to (21).

Though we do not impose any restriction on the social planner's program - given that redistribution is possible only within classes - the nature of the program leads to a resource reallocation necessarily different from the competitive one. In principle, we have a proper comparison base, since we potentially allow the government to redistribute resources among all above mentioned 4 types of agents within the same capital class.[22]

To see the crucial difference of two allocation, suppose the optimal tax policy is such that small perturbation of it it makes educational investment scheme unchanged. That is, at the optimum, the partial differential of $\tilde{x}_{i,x}$ with respect to $T_{i,x}$ (time index is suppressed) is well defined (equal to zero). Moreover, suppose we have the same picture for any two couples of states, say, for $(H, \bar{x})$ and $(L, 0)$. Then, solving the Lagrangian of (21) - (22), for these two cases, we get the following ration of marginal utilities:

$$\frac{U'(c_{H,\bar{x}})}{U'(c_{L,0})} = \frac{w_{L,0}}{w_{H,\bar{x}}} \frac{N_{H,\bar{x}}}{N_{L,0}} \left[\frac{1 + \partial\tilde{k}/\partial T_{L,0}}{1 + \partial\tilde{k}/\partial T_{H,\bar{x}}} \right]. \tag{23}$$

The current ratio of marginal utilities is a function of the ratio of current numbers of corresponding families, the inverse ratio of the time invariant weights and the inverse ratio of infinitesimal changes in consumption, when taxes/subsidies are perturbed. If the actual ratio of number of two types of families happens to be equal the ratio of the weights, then the government simply equates marginal utilities according to the inverse ratio of consumption derivatives.

Clearly, the government policy implied allocation differs from the competitive one in that in the latter case there is no trade between high and low individuals in the same income class and the families' criterion is to equalize marginal productivities, while in the former case redistribution is between all possible four groups and the government's criterion is to optimal ratios of marginal utilities. Even if we allow families to trade after they are taxed, the end period outcome will differ from the one in the loan market environment, as we have argued in the last footnote. Eventually, these two different objectives in our setting will lead to different allocations. The main conclusion is that the redistribution tax policy implemented by the benevolent government cannot replicate the allocation of resources when

[22] Alternatively, the government may identify weights only by distinguishing ability types, and then let the families trade to exploit arbitrage opportunities stemmed from differences in the previous period human capital investment choices. This again leads to a different allocation, since, as we have seen, trade occurs between families different in educational investment plans, while the planner first redistributes resources within ability types.

credit markets are available.

The last issue worth discussing is the relation between efficiency and equality. When credit markets are available, families in fact become price takers and there are no externalities that could deviate market allocation from the optimal. Thus, the First Welfare Theorem ensures that our competitive equilibrium leads to Pareto efficient allocation, in the sense, there is no other allocation, in which no agent can be better off without making at least one agent worse off. However, the First Welfare Theorem remains silent about equity consideration. Any resource allocation that exhausts all resources in an economy is Pareto efficient, but by far not egalitarian. The social planner's allocation, as we have seen, cannot replicate the competitive allocation, and moreover, it is generally not Pareto efficient. To see the last point, we notice that after the social planner's allocation, if we open credit markets, families will have incentives to trade that make every body better off, since the planner's allocation does not equalize marginal products, but instead pursues to sustain more egalitarian society, the extent of which conditions on weights that the planner chooses.

The Second Welfare Theorem concerns redistributive issues, but it has a limited use. It tells that, udder suitable conditions, which is translated in economic terms into no market failures, competitive equilibrium can be achieved, after properly redistributing initial resources. But then, apart from market failures, the fundamental issue is to find a proper way to redistribute the wealth without influencing the process of leading competitive market outcome. Thus the weights, the social planner is to choose, become crucial in order not to distort the efficient allocation, eventually achieved when the market does its job.

Concerning the issue of market failures, the economic theory has provided solutions to proper interventions for many situations, like uncertainty, externalities, market frictions due to collaterals, which corrects resource allocation making all agents better off. Greenwald and Stiglitz (1986) argue that, at least theoretically, there are always such interventions, which lead to Pareto improvement. For the deterministic case of our model, proper redistributive policies are trivial - taxing the rich and subsidizing the poor who otherwise would be locked in the poverty region, enables the poor to escape from the poverty. But notice, that such intratemporal redistributions are not Pareto optimal, contrary to the allocation, when credit markets are available (the competitive one). In the deterministic case, however, it is easy to see that these interventions do not affect on the final outcome negatively.

What happens, when a stochastic factor is incorporated into the model? The social planner's redistribution with proper weights, in fact accelerates the process of equalization of families, and hence poor families escape from the poverty trap with higher probability and sooner. The equalizing power is preserved in the long run as well. The ergodic distribution, achieved under the competitive environment with no interventions, will necessarily have a larger variance. But to say that government interventions in favor of poor once will not have negative influence on the processing to the long run efficient allocation, will be wrong. Within each period as well, the market allocation after the planner's redistributing policy almost necessarily differs from the market allocation without planner's intervention. The intuition is the following: as we have shown, social planner cannot replicate the competitive allocation, and, since the new distribution is different from the original one as well, we cannot hope that, when markets are available, two outcomes, namely, market allocation after the initial redistribution by the social planner (we call it two-stage allocation) and the competitive allocation, can be the same.

Thus, both in the transition and long run period the two-stage allocation at the first instance accounts for equality while the competitive allocation is Pareto efficient by definition. From the structure of the model it then follows that the long run two-stage allocation first order stochastically dominates the long run competitive allocation, since in both cases long run distribution depends crucially on the physical technology and the transition matrix, and we expect to have a mean preserving distribution with a higher variance for the competitive case.

In the language of income distribution theory, we thus rank the two-stage allocation in the first place since it is Pareto optimal in the sense of first (or second) order stochastic dominance. We importantly notice that in this allocation the fundamental trade off between equity and efficiency is minimized, since we completely account for equality and efficiency is achieved in the long run. It is then not trivial what is the second best allocation, since the two allocations, namely the social planner's and the competitive one, are not compatible in the sense that the first only accounts for equality, while the second solves the efficiency problem.

A possible extension on the topic

It can be quite interesting to allow some asymmetry in information, that is, when the private and public sectors have different access to information relevant for the policies in both sectors. An interesting case is when a part of private information is not available to the government. Then the government confronts to the self selection problem. For instance, if there is complete information (no asymmetry in information), then a very simple redistributive policy leading to complete egalitarian society, is to tax everybody by 100% and redistribute equally. When, however, some private information, for instance ability type, is not available to government, then the agents anticipating such a radical redstributive policy, will perhaps pretend to be low skilled and inefficient plans will be undertaken in the economy. The government's policy should solve two different, often confronting issues - the tax system must insure people against skill shocks and at the same time provide incentives for higher skilled agents to produce more than the lower skilled ones. This strand of research has ben inititiated by Mirrlees (1971), and extended for an infinite horizon dynamic programs only recently[23].

What happens, if the government does not observe the ability type of an agent, and, for instance, commits to implement time consistent policy. In the similar setting, but with no discontinuities and for two periods, Konrad (2001) has ranked the social welfare for *lassiez faire*, complete and incomplete information cases. The striking result is that the

[23]The dynamic version of the Mirrlees (1971) model has been solved by Battaglini and Coate (2005). An essay by Kocherlakota (2005) on this literature gives an excellent view on the topic.

limit to the government's access to information leads to Pareto superior outcome, making all the individuals better off even if the governmental policy suffers from time-consistency problem[24]. One can solve our model in the spirit of Konrad (2001) and, in particular, see whether incomplete information leads to a similar (really surprising) result.

6 Conclusion

The main findings in the paper can be divided into two groups, (i) results from the comparative statics analysis and (ii) welfare analysis when considering available credit markets in its own and versus a redistributive tax policy.

In the first part we discuss how the investment policies respond to different levels of intergenerational mobility in th economy. Low mobility perpetuates differences of aggregate investment levels at which high and low ability individuals find optimal to invest in education. Also, the higher is mobility, the higher is the aggregate investment level for high ability individuals and relatively lower for low ability individuals, due to more risky educational investments. In general, all policies different in human capital investments and abilities are positively correlated to the extent of the intergenerational mobility - total investments at each level of total capital stock are associated to a higher extent of mobility in the economy, when the same educational investment policies are taken. This in the end leads to the first order stochastic dominance of income distributions with higher mobility over the income distributions with lower mobility. These results are interpreted in the context of investments in physical capital as an insurance tool that mitigates the adverse effects coming from the stochastic nature of human capital returns.

Another interesting observation is that both in short and long run income effect dominates over the substitution effect, when additional income is available from trade. The increase in consumption, due to the gains from the trade, amounts the income effect, while spending

[24]Time consistency problem in taxation has motivated considerable research initiated by Kydland and Prescott (1980). In many situations the government implements a policy different from the one ex ante committed. This usually distorts the policies in the private sector and leads Pareto inferior outcome.

additional resources on investment goods, determines the extent of substitution effect. As a consequence, the long run bequest distribution, corresponding to the competitive allocation, first order stochastically dominated by the long run bequest distribution, when markets are shut down. For the ergodic consumption distribution, as expected, we have the converse picture, since competitive allocation apparently makes everybody better off.

In the second part we study how our economy evolves, when interaction in the loan markets is available for families within the same income class and all families. Then the price of capital is endogenously determined as function of average capital stock in the given income class (in the economy), and the outcome is definitely Pareto improving, since we integrate capital markets with no friction and there are no distortions from external sources (government etc). After, we introduce a benevolent government and study its program aimed at redistributing resources to sustain more egalitarian society. We provide an answer to the question, whether the government's policy implied allocation is compatible with the one associated to the loan markets, and the answer is that these two allocations are different.

In the end of the welfare analysis we provide on efficiency and equality consideration. We argue that the competitive allocation is Pareto efficient, but need not account for equality in the society. An allocation that is reached (i) when the social planner redistributes resources pursuing equality incentives, and (ii) after families trade in credit markets to equalize marginal products (we call this "two-stage" allocation), is ranked as the first best, since it accounts for equality entirely, and efficiency is achieved in the long run.

In the very end of the paper, we motivate the main directions of possible extension of the model. The basic idea is to allow asymmetry in information available to parties in the model. In general, there are two sources of difficulties, namely, dynamic (stochastic) nature of the model and technological nonconvexities.

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