

# THE REAL BUSINESS CYCLE MODEL FOR ARMENIA

Hovsep Patvakanyan  
Central European University

## Abstract

*This contribution develops the traditional RBC model of indivisible labor proposed by Hansen and further introduce labor exports. The model is parameterized in the way to match core features of the Armenian economy. In the framework of the model, foreign component, particularly construction-workers in Russia, is introduced as a part of the labor supply along with the domestic component. Given the fact that net foreign factor income comprises around 73%<sup>1</sup> of the Armenian GDP and private consumption, on average, exceeds domestic output shocks in the foreign wages plays a significant role in the way domestic economy operates.*

*The results of the model suggest that as a response to positive foreign wage shock the consumption tends to increase while the domestic output declines due to the switch of the domestic labor supply to its foreign counterpart, moreover even though the remittances sent to Armenia from construction workers increase it is mostly absorbed through consumption and not increase in the domestic productivity.*

**Key words:** Migration, real business cycle model, foreign wages, labor exports

## INTRODUCTION

Human history is a history of migration; today there are even countries where the migrants make up more than half of the population<sup>2</sup>. Collapse of the USSR and creation of 21 new countries with the economies viable to even slightest external shocks has fuelled the process considerably. The Russian Federation has become by far the most popular destination for the labor migrants from post-Soviet countries due to common language, existence of the diasporas from recipient countries and a big wage differential. The wave of the labor migration from Armenia to Russia has been triggered from the early stage of its independence fuelled also by the war with the neighboring Azerbaijan and two strong earthquakes. Step by step Armenia was

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<sup>1</sup> Calculated based on the data of Armenian Statistical Office // [www.armstat.am](http://www.armstat.am)

<sup>2</sup> In Qatar 86,5 %, UAE - 70 %, Kuwait – 68,8 % // <http://data.worldbank.org/indicator/SM.POP.TOTL.ZS>

getting out of the deep crisis and turning into a new spot for investments especially from Diaspora Armenians who were denied this opportunity during the years of Soviet Occupation. Nevertheless, currently labor migration to Russia has accelerated immensely with the majority of labor migrants employed in the construction sector. Taking into account the fact that remittances from Russia and net foreign factor income makes up huge portion of GDP it is vital to analyze how the exogenous shock in the foreign markets influence the Armenian economy. The goal of this paper is to pin down the effect of a positive shock in the foreign wages key macroeconomic variables for the domestic economy with the application of the real business cycle with the indivisible labor and labor exports.

The outline of the paper is as follows. In Section 1, I review the materials that cover some theoretical aspects of the model (Hansen 1985), (Tlelima 2005) and discuss the selected economic indicators that are to be matched in the model . In Section 2, the model for an economy with labor exports is introduced, considered in the aggregate terms and further solved. In Section 3, the model is calibrated in the way to match selected macroeconomic indicators discussed in the Section 1. Section 4, followed by conclusion, discusses the results and interpretation of the impulse-response functions to the foreign wage and productivity shocks.

## **1 REVIEW OF EMPIRICAL LITERATURE**

Among the theories aiming to explain economic fluctuations caused by disturbances economy is hit by RBC model is considered to be the benchmark that helps to build up respective propagation mechanisms of the shocks and reveal their consequences for the real economy. Nevertheless, the basic RBC model performs poorly in matching the variability of hours worked. It was fairly noted by Hansen(1985) that in the job market workers mostly enter fixed hours contract and do not choose how long they want to work. Fixed hours contract particularly imply that at a given period there will be employed and unemployed households, hence the labor indivisibility assumption introduces the unemployment into the RBC model, making it more realistic for Armenian Economy where the official unemployment rate is 27,5 % ( ILO report, 2010). It should be noted thought that this assumption imply that agents have a non-convex consumption set which precludes us from getting a mapping of Pareto-optimal allocation. To overcome this issue Hansen introduce “lotteries” about whether or not each individual works thus effectively convexifying the consumption set.

The labor indivisibility assumption is justifiable for the case of the Armenian economy given the fact that majority of Armenian workers are employed on the basis of the fixed hours contract in the public, construction and manufacturing sectors both in Armenia and in Russia. Therefore, I will be applying Hansen's approach (1985) modified by Tlelima (2009), where households choose the probabilities and not hours of working for both Armenian and foreign labor markets. We note  $q_{1t}$ ,  $q_{2t}$  as the probabilities of being employed at home or abroad, respectively, and  $h_0^d, h_0^f$  as hours stipulated in the contracts in period  $t$ , this implies that the probability of non-wage employment<sup>3</sup> will be  $1 - q_{1t} - q_{2t}$ . The equilibrium per capita hours of labor thus will be defined by:

$$h_t^f + h_t^d = q_{1t}h_0^d + q_{2t}h_0^f, \quad (1)$$

where  $h_t^d$  - demanded hours of labor domestically;  
 $h_t^f$  - demanded hours of labor abroad.

The expected labor income will respectively be defined by:

$$E_t\{h_t w_t\} = w_t^d h_t^d + w_t^f h_t^f, \quad (2)$$

where  $w_t^d$  - domestic wages;  
 $w_t^f$  - foreign wages.

To enable the replication of the Armenian economy through the proposed model I will be matching the selected economic indicators given the actual data the same way as done by Tlelima (2009).

*Table1: Selected Macroeconomic Indicators for Armenia: 2000Q1 -2007Q4<sup>4</sup>*

<b>Variable</b>	<b>Ratio of GDP</b>
Output	<b>1.00</b>
Net foreign factor income	<b>0.73</b>
Consumption	<b>1.20</b>
Investment	<b>0.17</b>
Exports	<b>0.26</b>
Imports	<b>1.28</b>

In our case, though, the summary provided in the Table1 is based on the quarterly data from 2000-2007. It should be mentioned that here the government spending component is

<sup>3</sup> Non-wage employment is the conventionally known unemployment.

<sup>4</sup> Armenian Statistical Office: <http://www.armstat.am/en/?nid=82>

excluded from GDP to ensure the plausible matching since the model used doesn't take into account governmental interference. As can be seen in the table, households' average consumption exceed the output with the ratio of 1.2 which is consistent with the exponentially increasing trade deficit. Net foreign factor income, which makes up 73% of GDP, can be deemed as the main way of its financing. The last point is discussed thoroughly by Baruah, Kumar(2009) with the emphasis on the ways remittances of the Armenian construction-workers have become vital for the domestic economic activity.

## 2 A MODEL FOR AN ECONOMY WITH LABOR EXPORTS

As discussed in the review of the literature this contribution will be using the Hansen's model with a feature of labor exports (Tlelima 2009) as benchmark framework for our analysis.

### 2.1 HOUSEHOLDS

We consider the economy to consist of a continuum identical households  $i=\{0,1\}$  which will help us to further introduce our model in aggregate terms with the purpose of analyzing the way aggregate variables behave. Unlike Tlelima (2009) the general form of the utility function is used in the current model, where household is choosing consumption and probability. Thus, the full period's utility function is:

$$U(c_t, q_{1t}, q_{2t}) = q_{1t} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + A \frac{(1-h_0^d)^{1-\vartheta} - 1}{1-\vartheta} \right\} + q_{2t} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + A \frac{(1-h_0^f)^{1-\vartheta} - 1}{1-\vartheta} \right\} + (1 - q_{2t} - q_{1t}) \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} \right\} = \frac{c_t^{1-\gamma}}{1-\gamma} + q_{1t} A \frac{(1-h_0^d)^{1-\vartheta} - 1}{1-\vartheta} + q_{2t} A \frac{(1-h_0^f)^{1-\vartheta} - 1}{1-\vartheta}, \quad (3)$$

where  $h_0 = h_0^d + h_0^f$ , with normalization of time endowment to unity:  $1 - h_0$  is leisure in  $t$ .

From the equation 1, we can express the respective probabilities in the following way:  $q_{1t} = \frac{h_t^d}{h_0^d}$

and  $q_{2t} = \frac{h_t^f}{h_0^f}$ . As a result, the equation (3) will have the form of:

$$U(c_t, q_{1t}, q_{2t}) = \frac{c_t^{1-\gamma}}{1-\gamma} + h_t^d \alpha_1 + h_t^f \alpha_2, \quad (4)$$

where  $\alpha_1 = A \frac{(1-h_0^d)^{1-\vartheta} - 1}{(1-\vartheta)h_0^d}$ ,  $\alpha_2 = A \frac{(1-h_0^f)^{1-\vartheta} - 1}{(1-\vartheta)h_0^f}$

$$A > 0 \text{ and } 0 < h^j < 1, j = d, f$$

In order to moderate volatility of the investment and be able to separate foreign bonds and capital after log-linearization capital adjustment costs are introduced in our model of small open economy as proposed by Schmitt- Grohe and Uribe (2003). Thus, the law of motion for capital will be as follows:

$$k_{t+1} + \frac{1}{2} \xi (k_{t+1} - k_t)^2 = (1 - \delta)k_t + i_t, \quad (5)$$

where  $k_t$  is capital stock owned by household at time t,  
 $\delta$  is quarterly depreciation rate,  
 $i_t$  is investment expenditure at time t,  
 $\xi(\bullet)$  is a function of net investment.

In addition, we assume that households have the possibility to hold foreign bonds as well as borrow from international markets. As argued by McCandless (2008), when a household is a net debtor he faces the higher interest rate on the international markets whereas it is a decreasing function of the household's savings. McCandless also points out the importance of considering interest rate as a function of the country's risk premium since it ensures existence of the steady state around which the log-linearization is to be done. Thus, all above-mentioned is described by:

$$r_t^f = r^* - \Phi B_t, \quad (6)$$

where  $B_t$  is country's total stock of foreign bonds,  
 $r_t^f$  is the rate at which households are borrowing from international markets,  
 $r^*$  is the fixed world interest rate,  
 $\Phi$  is country's risk premium and is  $> 0$

Summarizing all the points the household's problem can be described as choosing a sequence of  $\{b_t, c_t, h_t^d, h_t^f, k_{t+1}\}_{t=0}^{\infty}$  in order to maximize:

$$E_t \sum_{t=0}^{\infty} \left( \frac{c_t^{1-\gamma}}{1-\gamma} + h_t^d \alpha_1 + h_t^f \alpha_2 \right), \quad (7)$$

subject to the budget constraint:

$$b_t + c_t + k_{t+1} + \frac{1}{2} \xi (k_{t+1} - k_t)^2 = w_t^d h_t^d + w_t^f h_t^f + r_t k_t + (1 - \delta)k_t + (1 + r_{t-1}^f) b_{t-1} \quad (8)$$

as well as labor market clearing condition:

$$h_t^f + h_t^d = \bar{h} \quad (9)$$

With the purpose of preventing the ever growing consumption financed through the foreign debt no-Ponzi game condition is added:

$$\lim_{t \rightarrow \infty} \frac{b_t}{(1+r_t^f)^t} = 0 \quad (10)$$

Denoting  $\beta \in (0; 1)$  as the discount factor of the household and  $\lambda_t, \mu_t$  as the Lagrange multipliers on (8) and (9) respectively, the optimization problem takes the following form:

$$L = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + h_t^d \alpha_1 + h_t^f \alpha_2 - \mu_t \left( b_t + c_t + k_{t+1} + \frac{1}{2} \xi (k_{t+1} - k_t)^2 - w_t^d h_t^d - w_t^f h_t^f + r_t k_t - (1 - \delta) k_t - (1 + r_{t-1}^f) b_{t-1} \right) + \lambda_t (-h_t^f - h_t^d + \bar{h}) \right\} \quad (11)$$

The respective first order conditions of the problem would be:

$$\frac{\partial L}{\partial c_t}: c_t^{-\gamma} - \mu_t = 0 \quad (12a)$$

$$\frac{\partial L}{\partial h_t^d}: \alpha_1 + \mu_t w_t^d - \lambda_t = 0 \quad (12b)$$

$$\frac{\partial L}{\partial h_t^f}: \alpha_2 + \mu_t w_t^f - \lambda_t = 0 \quad (12c)$$

$$\frac{\partial L}{\partial k_{t+1}}: -\mu_t (1 + \xi(k_{t+1} - k_t)) + \beta E_t \mu_{t+1} (\xi(k_{t+2} - k_{t+1}) + r_{t+1} + (1 - \delta)) = 0 \quad (12d)$$

$$\frac{\partial L}{\partial b_t}: -\mu_t + \beta E_t \mu_{t+1} (1 + r_t^f) = 0 \quad (12e)$$

$$\frac{\partial L}{\partial \mu_t}: b_t + c_t + k_{t+1} + \frac{1}{2} \xi (k_{t+1} - k_t)^2 = w_t^d h_t^d + w_t^f h_t^f + r_t k_t + (1 - \delta) k_t + (1 + r_{t-1}^f) b_{t-1} \quad (12f)$$

$$\frac{\partial L}{\partial \lambda_t}: h_t^f + h_t^d = \bar{h} \quad (12g)$$

and the no-Ponzi game condition (10).

## 2.2 FIRMS

For the given sequence of  $\{r_t, w_t^d\}_{t=0}^{\infty}$  and factor productivity  $z_t$  firm exploits capital  $k_t$  and labor  $h_{d,t}$  to produce goods and services based on the standard Cobb- Douglas production function:

$$y_t = z_t k_t^\theta h_{d,t}^{1-\theta}, \quad (13)$$

where  $\theta \in (0; 1)$  and productivity is defined by the AR(1) process as follows :

$$\begin{aligned} \log z_t &= (1 - \rho^z) \log \bar{z} + \rho^z \log z_{t-1} + \varepsilon_t^z \\ \rho^z &\in (-1; 1), \varepsilon_t^z \sim i. i. d. N(0, \sigma_z^2) \end{aligned} \quad (14)$$

Thus, the profit maximization problem of the representative firm would be:

$$\max_{h_{d,t}, k_t} \{ z_t k_t^\theta h_{d,t}^{1-\theta} - w_t^d h_{d,t} - r_t k_t \} \quad (15)$$

As a result, the first order conditions are:

$$\theta z_t k_t^{\theta-1} h_{d,t}^{1-\theta} = r_t \quad (16a)$$

$$(1 - \theta) z_t k_t^\theta h_{d,t}^{-\theta} = w_t^d \quad (16b)$$

(16a) and (16b) imply that under neoclassical assumption production factors are paid the marginal products.

## 2.3 MARKET CLEARING CONDITIONS

The capital letters in this section will stand for aggregate variables. As already mentioned we have assumed that households are identical to each other and defined on the unit interval, thus in equilibrium we would expect the aggregate variables to follow the behavior of the individual counterparts:

$$B_t = b_t$$

$$K_t = k_t$$

$$C_t = c_t$$

$$H_t = h_t$$

$$I_t = i_t$$

The resource constraint of the economy is derived through the aggregation of (8):

$$B_t + C_t + I_t = w_t^d H_t^d + r_t K_t + w_t^f H_t^f + (1 + r_{t-1}^f) B_{t-1} \quad (17a)$$

or 
$$(B_t - B_{t-1}) + C_t + I_t = Y_t + w_t^f H_t^f + r_{t-1}^f B_{t-1}, \quad (17b)$$

where  $I_t = K_{t+1} + \frac{1}{2} \xi (K_{t+1} - K_t)^2 - (1 - \delta) K_t$  is aggregate investment expenditure.

From (17b) we can derive the balance of payments condition that is also needed to ensure clearing of the foreign exchange market:

$$B_t - B_{t-1} = Y_t + w_t^f H_t^f + r_{t-1}^f B_{t-1} - C_t - I_t \quad (18)$$

By applying the national income accounting identity  $Y_t = X_t + C_t + I_t$  to equation (18) we are going to have the BOP ( balance of payments condition):

$$B_t - B_{t-1} = X_t + w_t^f H_t^f + r_{t-1}^f B_{t-1} \quad (19)$$

The right side of the equation stands for the current account surplus or deficit which will be expressed through the change in net foreign assets. To conclude the market clearing conditions we also need to add labor market clearing condition, given by:

$$H_t^f + H_t^d = \bar{H} \quad (20)$$

## 2.4 THE MODEL IN AGGREGATE TERMS

After rearrangement and substitution of the given equations we have eliminated the Langrange multipliers as well as foreign interest rate. As a result we have a system of 11 equations, the model comprises 9 variables and 2 stochastic processes:  $C_t, B_t, K_{t+1}, r_t, H_t^d, H_t^f, w_t^d, X_t, Y_t, z_t, w_t^f$ .

$$\alpha_1 - \alpha_2 = w_t^f C_t^{-\gamma} - w_t^d C_t^{-\gamma} \quad (21a)$$

$$\frac{1}{\beta} \left( 1 + \xi (K_{t+1} - K_t) \right) = E_t \left( \frac{C_t}{C_{t+1}} \right)^\gamma \{ \xi (K_{t+2} - K_{t+1}) + r_{t+1} + 1 - \delta \} \quad (21b)$$

$$E_t \left( \frac{C_{t+1}}{C_t} \right)^\gamma = \beta (1 + r^* - \phi B_t) \quad (21c)$$

$$\theta z_t K_t^{\theta-1} H_{d,t}^{1-\theta} = r_t \quad (21d)$$

$$(1 - \theta) z_t K_t^\theta H_{d,t}^{-\theta} = w_t^d \quad (21e)$$



$$B_t + C_t + K_{t+1} + \frac{1}{2} \xi (K_{t+1} - K_t)^2 = Y_t + w_t^f H_t^f + (1 - \delta)K_t + (1 + r^* - \phi B_{t-1})B_{t-1} \quad (21f)$$

$$B_t - B_{t-1} = X_t + w_t^f H_t^f + (r^* - \phi B_{t-1})B_{t-1} \quad (21g)$$

$$Y_t = z_t K_t^\theta H_{d,t}^{1-\theta} \quad (21h)$$

$$H_t^f + H_t^d = \bar{H} \quad (21i)$$

$$\lim_{t \rightarrow \infty} \frac{B_t}{(1+r_t^f)^t} = 0 \quad (21j)$$

$$\log w_t^f = (1 - \rho^w) \log \bar{w} + \rho^w \log w_{t-1} + \varepsilon_t^w \quad (21k)$$

$$\rho^w \in (-1; 1), \varepsilon_t^w \sim i. i. d. N(0, \sigma_w^2)$$

$$\log z_t = (1 - \rho^z) \log \bar{z} + \rho^z \log z_{t-1} + \varepsilon_t^z \quad (21l)$$

$$\rho^z \in (-1; 1), \varepsilon_t^z \sim i. i. d. N(0, \sigma_z^2)$$

Most of the equations above are standard for the RBC model of the small open economy but some of them enter the system stemming from the specification of the model with labor exports. For instance, equation (21a) equates marginal benefits of working domestically and on the foreign labor market. In this equation,  $\alpha_1$  and  $\alpha_2$  are the marginal disutilities of labor, implying equality in case there is no wage differential between domestic and foreign labor market. As Baruah, Kumar (2009) argue, marginal disutility from foreign labor supply is higher than that of domestic work due to bad working conditions in the Russian construction sector with no health insurance and not developed unions. Rationality assumption informs that agents will tend to work abroad in case the costs associated with it are compensated by the high wages on the foreign labor markets.

Equations 21f and 21g imply that it may happen that the economy will run a trade deficit along with accumulation of foreign assets which will depend on the size of the foreign labor income, which is in line with the empirical observation of a high net foreign factor income ratio to the GDP for the Armenian economy.

## 2.5 LOGLINEAR APPROXIMATION

### 2.5.1 STEADY STATE

The steady state solution of the model is as follows:

$$\alpha_1 - \alpha_2 = \overline{w^f} \overline{C}^{-\gamma} - \overline{w^d} \overline{C}^{-\gamma} \quad (22a)$$

$$\frac{1}{\beta} = \bar{r} + 1 - \delta \quad (22b)$$

$$\frac{1}{\beta} = 1 + r^* - \phi \bar{B} \quad (22c)$$

$$\bar{r} = \theta \bar{z} \left( \frac{\overline{H^d}}{\bar{K}} \right)^{1-\theta} \quad (22d)$$

$$\overline{w^d} = (1 - \theta) \bar{z} \left( \frac{\bar{K}}{\overline{H^d}} \right)^\theta \quad (22e)$$

$$\bar{C} + \delta \bar{K} = \bar{Y} + \overline{w^f} \overline{H^f} + (r^* - \phi \bar{B}) \bar{B} \quad (22f)$$

$$\bar{X} + \overline{w^f} \overline{H^f} + (r^* - \phi \bar{B}) \bar{B} = 0 \quad (22g)$$

$$\bar{Y} = \bar{z} \bar{K}^\theta \overline{H^d}^{1-\theta} \quad (22h)$$

$$\bar{H} = \overline{H^d} + \overline{H^f} \quad (22i)$$

where  $\bar{S} = S_t = S_{t+i}$  for any  $S_t$  and all  $i \in Z$

From the 22b and 22c the  $\bar{r}$  and  $\bar{B}$  can be determined as:

$$\bar{r} = \frac{1}{\beta} - (1 - \delta) \quad (23)$$

$$\bar{B} = \frac{(1 - \frac{1}{\beta} + r^*)}{\phi} \quad (24)$$

Equation 22g will determine  $\bar{X}$ , given  $\overline{w^f}$  and  $\overline{H^f}$ .

From 22d and 22e the steady state level of domestic wage will be:

$$\overline{w^d} = (1 - \theta) \left( \frac{\theta}{\bar{r}} \right)^{\frac{\theta}{1-\theta}} \quad (25)$$

Given 25, the capital steady state value can be determined as follows:

$$\bar{K} = \left( \frac{\overline{w^d}}{1-\theta} \right)^{\frac{1}{\theta}} \overline{H^d} \quad (26)$$

Consistent with the labor indivisibility assumption, one third of time endowment makes up the aggregate hours worked:

$$\overline{H^d} + \overline{H^f} = 0.333 \quad (27)$$

$\overline{H^d}, \overline{H^f}$  are further determined based on the empirical data to replicate long run average shares of domestic and “foreign” employment. Given this, the rest of the steady state values can be found from the respective equations.

For instance, the value of steady state consumption will be:

$$\overline{C^{-\gamma}} = \frac{\overline{w^f - w^d}}{a_1 - a_2} \quad (28)$$

### 2.5.2 LOG-LINEARISATION

We denote  $\widehat{s}_t = \log \frac{S_t}{\bar{S}}$ , where  $\bar{S}$  is the steady state value of  $S_t$ . Thus the model is now expressed in variables that stand for the log deviation from their steady state:  $\widehat{c}_t, \widehat{k}_{t+1}, \widehat{b}_t, \widehat{r}_t, \widehat{h}_t^d, \widehat{h}_t^f, \widehat{w}_t^d, \widehat{x}_t, \widehat{y}_t$ .

$$\widehat{c}_t = \frac{\overline{w^f} \widehat{w}_t^f - \widehat{w}_t^d \overline{w^d}}{(\overline{w^f} - \overline{w^d})\gamma} \quad (29a)$$

$$\bar{K}(1 + \beta)\xi \widehat{k}_{t+1} = \gamma \widehat{c}_t - \gamma E_t \widehat{c}_{t+1} + \bar{K}\xi \widehat{k}_t + \beta\xi \bar{K} E_t \widehat{k}_{t+2} + \beta\bar{r} E_t \widehat{r}_{t+1} \quad (29b)$$

$$\beta\phi\bar{B}\widehat{b}_t = \gamma \widehat{c}_t - \gamma E_t \widehat{c}_{t+1} \quad (29c)$$

$$\widehat{r}_t = \widehat{z}_t + (\theta - 1)\widehat{k}_t + (1 - \theta)\widehat{h}_t^d \quad (29d)$$

$$\theta \widehat{h}_t^d = \widehat{z}_t + \theta\widehat{k}_t - \widehat{w}_t^d \quad (29e)$$

$$\bar{B}\widehat{b}_t + \bar{C}\widehat{c}_t + \bar{K}(\widehat{k}_{t+1} - (1 - \delta)\widehat{k}_t) = \bar{Y}\widehat{y}_t + \overline{w^f H^f} (\widehat{w}_t^f + \widehat{h}_t^f) + \widehat{b}_{t-1}((1 + r^*)\bar{B} - 2\phi\bar{B}^2) \quad (29f)$$

$$\bar{B}\widehat{b}_t = \bar{X}\widehat{x}_t + \overline{w^f H^f} (\widehat{w}_t^f + \widehat{h}_t^f) + \widehat{b}_{t-1}((1 + r^*)\bar{B} - 2\phi\bar{B}^2) \quad (29g)$$

$$\widehat{y}_t = \widehat{z}_t + \theta\widehat{k}_t + (1 - \theta)\widehat{h}_t^d \quad (29h)$$

$$\overline{H^d} \widehat{h}_t^d + \overline{H^f} \widehat{h}_t^f = 0 \quad (29i)$$

And the 2 stochastic processes:

$$\widehat{w}_t^f = \rho^w \widehat{w}_{t-1}^f + \varepsilon_t^w \quad (30a)$$

$$\widehat{z}_t = \rho^z \widehat{z}_{t-1} + \varepsilon_t^z \quad (30b)$$

where the distributions of  $\varepsilon_t^w$  and  $\varepsilon_t^z$  are given by 21k and 21 l.

### 3 CALIBRATION OF THE MODEL

The parameterization strategy used in the paper is to match steady state values of the theoretical model to the long-run values of the selected macroeconomic indicators for the Armenian economy as stipulated in the first section. As mentioned in the first section, the ratios calculated and presented in the table 1 exclude the presence of the government to ensure plausible matching with the theoretical model. In this way, taking into account the significance of the income earned by Armenian construction-workers in Russia the model's parameters are selected to be consistent with Armenia's average net foreign factor income for the period of 2000Q1-2007Q4, which is 73% of GDP, after excluding government. Obtaining the parameter values and steady states consist of 2 sets: one is determined based on the empirical data and literature while the other is solved given the information from the first set.

I start consideration of the first set from the average quarterly real interest rate  $\bar{r}$ , which is 4.34 % based on the data that covers 2000Q1-2007Q4 period for Armenia. The US quarterly real interest rate is taken as the proxy for the world interest rate,  $r^*$  and is equal to 1.4 %. Both indicators are obtained from the database of the World Bank. The values of  $\overline{H^f}$  and  $\overline{H^d}$  are obtained from the reports of OSCE on labor migration from Armenia and country study of International Labor Organization on migration and development in Armenia. Authors argue, that the share of Armenian construction-workers in Russia makes up around 25 % of the total labor force in Armenia, which implies that 75% of the workers are employed domestically. In this way, the values for  $\overline{H^f}$  and  $\overline{H^d}$  are as follows:  $\overline{H^f} = 0.333 * 0.25 = 0.0832, \overline{H^d} = 0.333 * 0.75 = 0.2497$  . The most frequently used value of the capital share,  $\theta$ , is 0.4 in the literature. But it should be noted that majority of authors refer to this value when considering advanced economies. As fairly argued by Mendoza (1995) the value for the developing and emerging markets should be smaller; for our purposes we will be using the value of 0.38 to ensure the ratio of foreign to domestic wages to be  $3.8^5$ .

The value for the household's discount factor  $\beta = 0.97$  (from equation 23) is chosen in a way that given  $\bar{r}$ , the quarterly rate of depreciation for the Armenian economy will be around 1.75 % . The latter is based on the calibration by Easterly and Rebelo (1993) with yearly depreciation rate of 7 % . As they argue, the depreciation rates in the developing countries are

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<sup>5</sup> The value is consistent with the observations in the country study by International Labor Organization.

lower than that of developed countries due to the lower efficiency of the investment projects and more pronounced corruption in the former ones. Following Uribe (2002) and Mendoza(1991) I set the parameters of debt elasticity of interest rate premium,  $\phi$ , and capital adjustment cost,  $\xi$ , to be 0.01 and 0.028 respectively. Value for the persistence of the productivity shock  $\rho^z$  is chosen to be 0.41 in line with Mendoza's average persistence for developing countries,  $\sigma_z^2$  is set 0.04. To pin down the values for  $\rho^w$  and  $\sigma_w^2$  I run the OLS regression based on the seasonally adjusted, logged, detrended data on the wages of the Armenian construction-workers in Russia. The results in the Table 2 suggest to assign the values of 0.724 to persistence of the foreign wage shock and 0.0075 to  $\sigma_w^2$ .

Dependent variable:  $\widehat{w}_t^f$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$\widehat{w}_{t-1}^f$	0.724099	0.125896	5.751555	0.0000
R-squared	0.524411	Mean dependent var		0.000323
Adjusted R-squared	0.524411	S.D. dependent var		0.100769
S.E. of regression	0.069494	Akaike info criterion		-2.463438
Sum squared resid	0.144881	Schwarz criterion		-2.417180
Log likelihood	39.18329	Hannan-Quinn criter.		-2.448359
Durbin-Watson stat	1.847932			

Table2. Results of the OLS regression based on the data 2000Q1-2007Q2

The inter-temporal elasticity of substitution,  $\gamma$ , is chosen to be 1.002 which is used by Mendoza (1991), Uribe and Yue(2006) and Aguiar, Gopinath (2007).

After determining the values for the first set of parameters and steady states I use it to solve endogenously for the second set. In this way, given  $\beta$ ,  $r^*$ ,  $\phi$  from equation 24  $\bar{B} = -1.692$  and is negative, implying that the country is net debtor. This is consistent with the fact that net rate of return on capital  $\bar{r} - \delta = 0.0259$  and is bigger than the world interest rate,  $r^* = 0.014$ . It follows, that in equilibrium, residents will dissave in foreign markets to make the domestic rates equal.

The domestic wage rate can be found from equation 25 given the values of  $\theta$ ,  $\bar{r}:\bar{w}^d = 2.34$  and further  $\bar{K} = 8.26$  is obtained from equation 26. Now enough information is provided to find the value of the state aggregate output from 22h,  $\bar{Y} = 0.943$ . This means that  $\frac{\bar{l}}{\bar{y}} = 0.1532$ , which is consistent with the empirical data we analyzed in the first section. Knowing that

$\frac{\text{Net foreign factor income}}{\text{GDP (without gov.)}} = 0.73$ , the following is stated:  $\frac{\overline{w^f H^f} + (r^* - \phi \bar{B}) \bar{B}}{\bar{Y}} = 0.73 \Rightarrow \overline{w^f H^f} + (r^* - \phi \bar{B}) \bar{B} = 0.73 * 0.943 = 0.69$ , from this  $\overline{w^f} = 8.9$ , thus the foreign wages are about 3.8 higher than domestic wages which is in line with the conjunctures of the Armenian labor market. From equation 22f,  $\bar{C} = 1.48$  and the ratio of consumption to GDP, excluding government, is  $\frac{\bar{C}}{\bar{Y}} = 1.57$ . The latter is slightly higher than the value given in the Table1 of the section 1, nevertheless the fact that consumption expenditures exceed output by 57 % is taken to be sufficient to approximate the Armenian Economy. Ultimately, to pin down the values for the  $a_1$  and  $a_2$  I refer to the equation 28 from where  $a_1 - a_2 = \bar{C}^\gamma (\overline{w^f} - \overline{w^d}) = 4.4$ . Following Hansen (1985) and Tlelima(2009), I set the value of  $a_1 = -2$  and then  $a_2 = -6.4$ , which ensure the ratio between the disutilities to be 3.2 making it reasonably approximate with the ratio of the foreign and domestic wages. The summary of the parameter and steady state values are given in below presented tables 3 and 4, respectively.

*Table 2. Parameters*

<b>Parameter</b>	<b>Value</b>	<b>Description</b>
$a_1$	-2.0	disutility of working in Armenia
$a_2$	-6.4	disutility of working in Russian construction sector
$\phi$	0.01	debt elasticity of interest rate premium
$\theta$	0.38	capital share in the output
$\delta$	0.0175	quarterly rate of depreciation
$\beta$	0.97	discount factor
$\xi$	0.028	capital adjustment cost parameter
$r^*$	0.014	fixed world interest rate
$\gamma$	1.002	Intertemporal elasticity of substitution
$\rho^w$	0.724	persistence of the foreign wage shock
$\rho^z$	0.41	Persistence of the productivity shock
$\sigma_w^2$	0.007	variance of $\varepsilon_t^w$
$\sigma_z^2$	0.04	Variance of $\varepsilon_t^z$

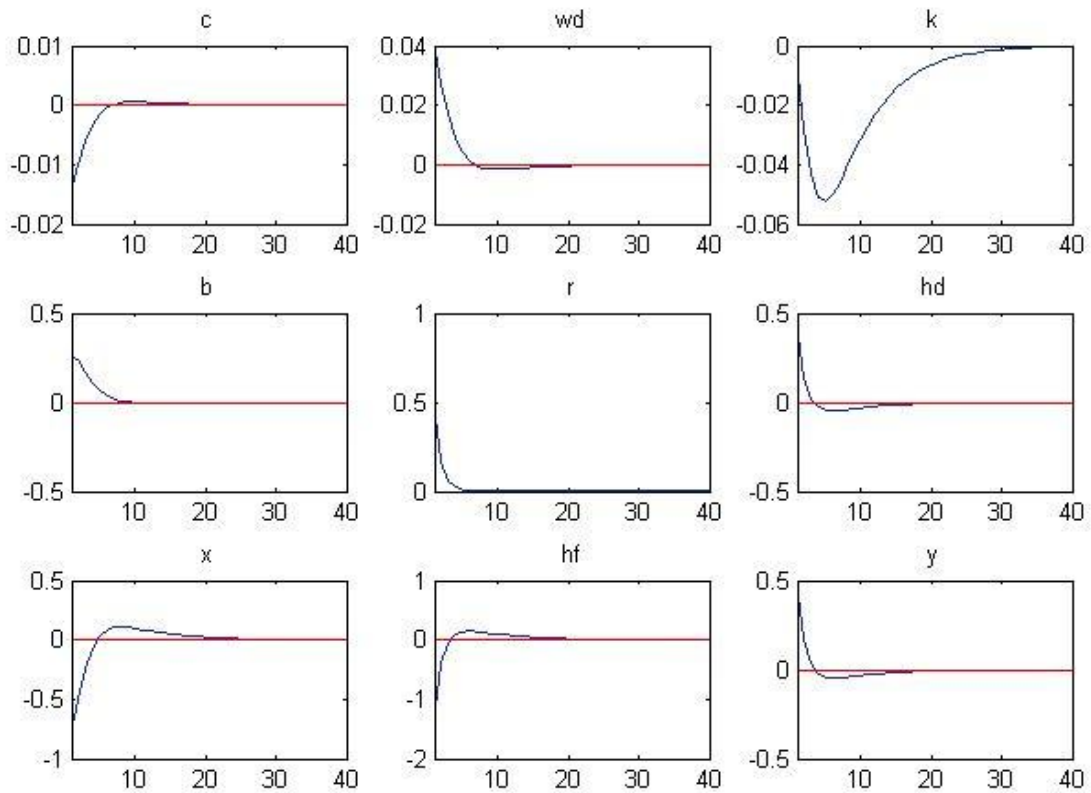
Table 3. Steady State Values

$\bar{B}$	$\bar{C}$	$\bar{K}$	$\bar{H}^d$	$\bar{H}^f$	$\bar{H}$	$\bar{X}$	$\bar{Y}$	$\bar{w}^d$	$\bar{w}^f$	$\bar{r}$
-1.692	1.48	8.26	0.2497	0.0832	0.333	-0.69	0.943	2.343	8.9	0.0434

## 4 RESULTS

### 4.1 IMPULSE RESPONSES TO TOTAL FACTOR PRODUCTIVITY (TFP) SHOCKS

Figure 1. Impulse Responses to Factor Productivity Shocks



The model used in the paper doesn't aim to mimic all the empirical features of the Armenian economy given the simplifying assumption we are making. Nevertheless, the qualitative results obtained can be applied to analyze the ways Armenian economy behaves given its strong dependence on Russian economy.

Before considering the effect foreign wage shock has on the Armenian economy I will analyze firstly the impact of the productivity shocks on the real economy. Figure 1 show the

impulse responses of the model's variables to a standard deviation shock to TFP. As we can see a positive shock translates into a higher output and also raises the marginal product of labor,  $\widehat{w}_t^d$ . The substitution effect of real wage increase dominates over income effect thus resulting in an increased domestically supplied labor. The latter leads to reduction of foreign labor supply  $\widehat{h}_t^f$  given the assumption of fixed total hours of labor. We can see also that a positive TFP shock entails trade deficit which is explained by the fact that Armenian economy is very much import-oriented and a large portion of its imports covers inputs that are further used in the domestic production.

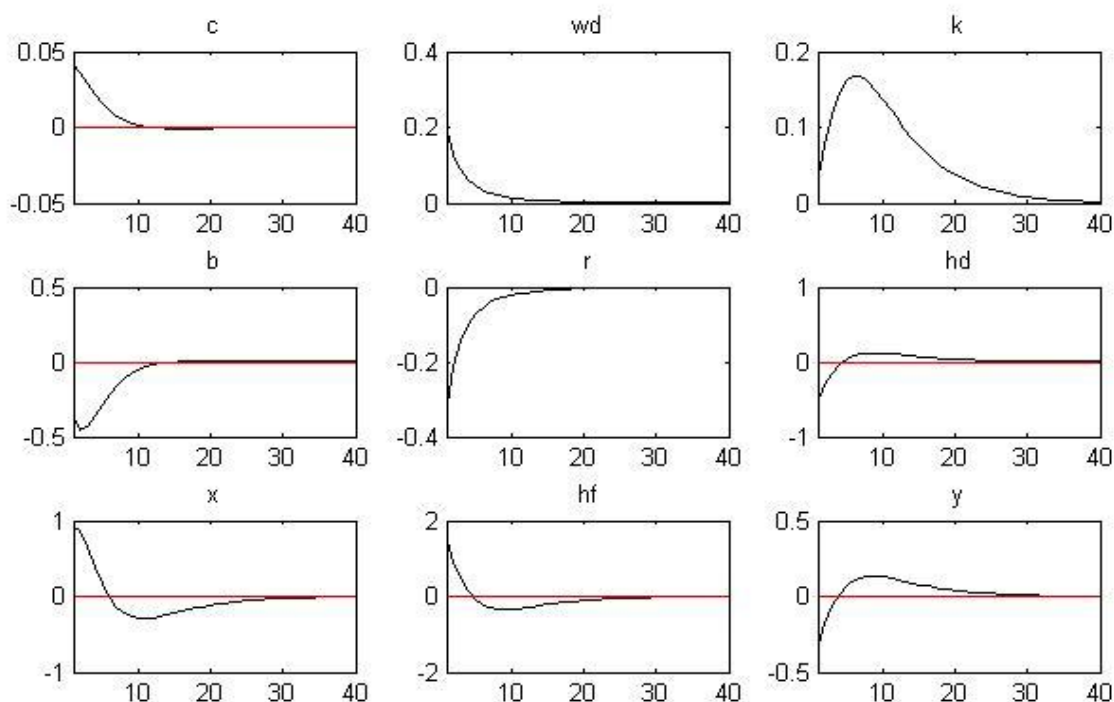
The pattern of the positive response of the foreign asset holdings,  $\widehat{b}_t$ , is in line with theoretical expectations, particularly an increased income makes agents to raise their domestic and foreign savings to insure themselves against adverse effect during the economic downturn. The pronounced effect of the positive TFP shock on the savings can also be noticed from the decreased consumption. For the conditions of Armenian economy people respond to even a slightest positive shock with a considerable cushion of safety. In this way, the increased real interest rate are perceived as higher price for today's consumption triggering high savings. In addition, the increase in the interest rate is perceived as purely temporary and bearing in mind that it will fall in the future people tend to disinvest, which explain the negative response of the capital to a positive TFP shock.

### 3.2 IMPULSE RESPONSES TO FOREIGN WAGE SHOCKS

The fluctuations of the model variables around their steady state as a result of a foreign wage shock are represented in the Figure 2, from which we notice the strong positive response of the foreign supplied labor. And due to the switch from the domestic to foreign labor supply the output decreases. It should, however, be noted that after a 4<sup>th</sup> period it not only goes back to and soon exceed its steady state level, meaning that on that stage the increased net foreign factor income outweighs the drop in GDP. As already mentioned, remittances of the Armenian construction-workers makes up a great part of the country's GDP with the consumption expenditure channel as the main way of realization. Thus, following this logic a positive foreign wage shock entails an increase in consumption that lasts around 20 periods. At the same time, the positive pattern of the domestic wages can be explained by the decreased domestic labor supply.



Figure 2. Impulse Responses to Foreign Wage Shocks



The behavior and the relationship of real interest rate and the capital stock is in line theoretical expectations that stand for the negative relationship between the two. Considering the response of the net exports, the positive pattern can be noticed that is followed by going below the steady steady in the 10<sup>th</sup> period. As already discussed, Armenian economy being very much dependent on import of intermediate goods and productive inputs is very viable to external shocks, particularly, those imports might fall with output in the first periods and result in a positive response of net exports to a positive foreign wage shock. Nevertheless, taking into account the large import component in the consumption basket of an average Armenian, increasing consumption will pull up the imports. In our case, the correlation of imports with output is much stronger than that of consumption which might explain the large positive response of the net exports variable. As results suggest, the foreign savings,  $\hat{b}_t$  drop which can be connected with the “artificial” exploitation of foreign reserves in order to restore favorable environment to boost imports, especially taking into consideration that many policy makers in Armenia are a part of the importing business.

To conclude, we have seen that both shocks have considerable and not unambiguous effects on the real economy but the effect of a foreign wage shock was a way stronger and long-lasting in comparison with a TFP shock. The findings in the paper can be furthermore utilized to shape the policy targeted at forecasting the potential consequences of the foreign wage shocks on the Armenian economy and based on the latter propose a credible labor migration policy.

## REFERENCES

- Caporale Guglielmo Maria, Yannis Georgellis, Nicholass Tsitsianis and Yin Ya Ping.** 2007. "Income across Europe: do reference values matter?". Cesifo working paper # 2146.
- Borjas, James.** 1989. "Economic theory and international migration". *International Migration Review*, 23.
- Kumar Krishna, Baruah Nilim.** 2009. "Migration and Development: Country study, Armenia". International Labor Organization.
- Minasyan Anna, Poghosyan Alina, Gevorgyan Lilit, Chobanyan Haykanush.** 2008. "Return Migration to Armenia in 2002-2008: a study". Asoghik, OSCE report, 66pp.
- Tlelima Tanka.** 2009. "Real Business Cycle Model for Lesotho". University of Cape Town.
- Hansen, Gary.** 1985. "Indivisible labour and the business cycle", *Journal of Monetary Economics* 16, 309.327.
- King Robert, Rebelo Sergio.** 2000. "Resuscitating real business cycles", Working Paper 467, Rochester Center for Economic Research.
- McCandless, Gregory.** 2008. *The ABCs of RBCs: An Introduction to Dynamic Macroeconomic Models*, Harvard University Press.
- Mendoza, Enrique.** 1991. "Real business cycles in a small open economy", *The American Economic Review* 81(4): 797.818.
- Mendoza, Enrique.** (1995). "The terms of trade, the real exchange rate, and economic fluctuations", 36(1): 101.137.
- Schmitt-Grohe Stephanie, Uribe Martin.** 2003. "Closing small open economy models", *Journal of International Economics* 61: 163.185.

**Uribe, Martin.** 2002. "The price-consumption puzzle of currency pegs", Journal of Monetary Economics 49: 533-569.

**Uribe Martin, Yue Vivian.** 2006. "Country spreads and emerging countries: who drives whom?", Journal of International Economics 69 (2006) 6-36.

**Easterly William, Rebelo Sergio.** 1993. "Fiscal policy and economic growth: An empirical investigation", Journal of Monetary Economics 32 (1993) 417-458 North-Holland.

**Aguiar Mark, Gopinath Gita.** 2007. "Emerging Market Business Cycles: the cycle is the trend", University of Rochester.

**World Bank Database.** World Development Indicators.

Available at <http://data.worldbank.org/indicator/SM.POP.TOTL.ZS> .

**Armenian Statistical Office.** Quarterly data on macroeconomic indicators of Armenian economy. Available at <http://www.armstat.am/en/?nid=82>

**Russian Federation Federal State Statistics Service.** Data on migration and labor.

Available at: <http://www.gks.ru/wps/wcm/connect/rosstat/rosstatsite.eng/figures/labour/>