Collateral versus Informed Screening During Banking Relationships*

Artashes Karapetyan†
and
Bogdan Stacescu‡

Abstract

We examine banks’ choice between two costly instruments that can be used to pick loan applicants: direct screening by acquiring borrower-specific information and collateral requirements. We show that with longer relationships the preference for initial screening increases, raising access to funding, while total welfare is enhanced as a result of more efficient selection. The model also rationalizes established empirical facts about lending relationships (such as reduced incidence of collateral, but not necessarily lower interest rates in later periods for safer borrowers). The results are stronger under bank competition. Our findings suggest that policies supporting loyal lending relationship can increase initial access to credit by reducing dependence on collateral.

Keywords: Collateral, screening, bank relationships

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†Norges Bank, Bankplassen 2, P.O. Box 1179 Sentrum, Norway. Tel: + 47 22 31 62 52, e-mail: Artashes.Karapetyan@norges-bank.no

‡BI Norwegian Business School, Nydalsveien 37, 0484 Oslo. Tel: +47 46 41 05 19, e-mail: bogdan.stacescu@bi.no
I  Introduction

Borrower selection is a key function of banks, one which has been emphasized by the recent financial crisis (Bolton et al. (2013)). One instrument that can be used to identify high-quality borrowers is collateral requirements.\(^1\) However, the use of collateral is exposed to liquidation losses in fire sales (Shleifer and Vishny (2011)). As an alternative, lenders can identify worthwhile projects by directly acquiring information about borrowers.\(^2\) In this paper, we examine the banks’ choice between alternative selection mechanisms.

In our model, banks can choose between directly collecting information (which we call screening for brevity) and requiring collateral to identify ex ante creditworthy borrowers. Screening involves bearing the cost of acquiring information about potential borrowers, while the use of collateral can lead to losses when borrowers default and the collateral is liquidated. We compare one-period and repeated lending, as well as initial-stage to later-stage lending.\(^3\) We find that the prospect of a long relationship increases the likelihood of screening in the initial stages of that relationship, that screening is more likely relative to collateral requirements in the later stages of a lending relationship, and that both results are stronger in a competitive banking system. Repeated interaction between lenders and borrowers can increase welfare through a more efficient use of selection mechanisms.

Our first main result is that borrowers who were initially required to post collateral in order to get a loan are likely to be just screened by their bank for subsequent loans. In contrast, borrowers that switch to competing banks are more likely to have to post collateral. Moreover, borrowers that can only get secured loans from their bank in later stages of lending relationships are more likely to be of low quality and also to switch to other providers of funds.

The intuition behind this finding is as follows. The cost of collateral comes from the liquidation losses in case of default, which are proportional to the amount of collateral required. This amount must be high enough to render bad projects unprofitable for loan applicants, and thus is a function of their success probability, but does not depend on the proportion of bad projects in the overall population. In contrast, screening depends on the average quality of the borrower pool: since they cannot distinguish ex ante between good and bad borrowers, banks have to screen all loan applicants, but may lend to only the high-quality group. This selection device becomes less expensive per borrower when the borrower pool has a higher quality. Because in subsequent lending banks learn more about their borrowers and are able to weed out the worst among

\(^1\)Theoretical work on the screening role of collateral dates back to Bester (1985). For recent empirical evidence on ex ante collateral-based selection see Berger, Frame and Ioannidou (2011), Berger et al. (2011), and Cerqueiro, Ongena and Roszbach (2014).

\(^2\)The importance of information collection by banks about their borrowers has been studied extensively: see Boot (2000) for a review.

\(^3\)Repeated lending is usually taken as an indicator of the strength of a bank relationship; see for instance Boot (2000), Bharath et al. (2007), Li, Lu and Srinivasan (2013).
them, the set of borrowers they focus on in later stages is of relatively higher quality, and the per-borrower cost of screening goes down. Conversely, borrowers who are trying to switch to competing banks are of relatively lower quality, and will be required to post collateral by those banks.

We also show that unlike collateral requirements the interest rate for repeat borrowers may actually go up in further lending. There are two reasons for this: on the one hand the hold-up problem (Sharpe (1990), von Thadden (2004)), which means that adverse selection allows inside (incumbent) banks to raise interest rates over time, and on the other hand the fact that in later periods banks are more likely to screen rather than require collateral. Screening is associated with higher interest rates since banks do not receive the post-liquidation value of collateral in case of default.

Our second main result is that when the potential length of the banking relationship increases, the preference for screening in the initial stages of the relationship is more pronounced. The explanation behind this finding is linked to the classical hold-up problem in lending. In long-term relationships borrowers anticipate that banks will extract information rents in the later stages, and competition between banks will push them to offer discounts in the initial period. A lower interest rate offered in initial lending will however increase collateral requirements to prevent low-quality borrowers from getting a loan. This will raise the expected liquidation costs. In contrast, screening does not present such a disadvantage: banks can simply lower interest rates and screen at the same cost per borrower. As a result, a higher probability of repeated lending decreases the incidence of collateral in initial lending.

Concerning welfare we find that the possibility of repeated lending can enhance it by improving the choice of selection instruments. In later periods, selection costs will be reduced by screening higher quality borrower pools (and requiring collateral from low-quality pools). Compared to one-period relationships, borrower surplus is significantly higher in initial periods, and can be marginally higher in later ones. As a result, policies that encourage stable lending relationships, for instance by strengthening bank balance sheets or promoting local banks, may be welfare-enhancing.

We further analyze the difference between collateral and screening in terms of the reusability of information. When a borrower posts collateral in order to receive a loan, this indicates that her current project is of relatively high quality. However, that may say little about the possible quality of the future projects of that particular borrower. In contrast, screening their potential borrowers provides banks with information that can be used both to assess the quality of their current projects, and to get a signal about the quality of their future projects. We show that this difference makes screening even more useful in the case of repeated interaction between banks and borrowers.

Our results contrasting screening and collateral are stronger under bank competition than under a bank monopoly. The reason is that without competition banks can charge higher interest
rates, and that reduces the amount of collateral that is needed to separate high- and low-quality borrowers. The lower amount of collateral in turn implies that liquidation costs are lower.

Our work contributes to the literature on arm’s length (transactional) vs. relationship lending. We find that relationship lending can be welfare-enhancing because it allows borrower selection at a lower cost. This finding suggests that promoting durable relationships may be good policy. For instance, regulations that strengthen bank balance sheets and safeguard lending relationships even in the event of financial crises may be useful. In contrast, our findings indicate a disadvantage of policies reducing customer loyalty, such as facilitating competition from arm’s length lenders and borrower switching.

While the role of collateral in reducing adverse selection is well-established in both the theoretical and the empirical literature, there is little research concerning banks’ choice between various borrower selection technologies (Steijvers and Voordeckers (2009)). Manove et al. (2001) show that under some circumstances banks can be “lazy”, that is, they may screen too little when borrowers can post collateral. This is because high-type borrowers unaware of the actual quality of their current project may choose to pledge collateral, leading to lending losses. We examine one of the possible circumstances that can work in the opposite direction: longer-term relationships may make screening more attractive and reduce the potentially damaging use of collateral. This can help render banks more “active” and enhance information acquisition in the economy.

We restrict our attention to the role of collateral in reducing adverse selection (Bester (1985), Besanko and Thakor (1987a, b)). Collateral can also be a device used to reduce agency issues either on the borrower’s (Boot, Thakor and Udell (1991)) or on the lender’s side (Inderst and Mueller (2007), Rajan and Winton (1995)). While the agency-reducing role of collateral is well documented, its use in reducing adverse selection is also important. Several recent empirical papers have been able to bring rigorous evidence supporting the use of collateral to reduce ex ante adverse selection (Jiménez, Salas-Fumás, and Saurina (2006), Berger, Frame and Ioannidou

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4Bank relationships vary across countries and local markets (Ongena and Smith (2000), Degryse and Ongena (2005)). Over the last several decades banks have come under increased competitive pressure as a result of deregulation, cross-border banking, the emergence of alternative sources of finance and increasing number of non-traditional banks, as well as advances in information sharing technologies and policies facilitating customer switching such as account number portability. (For instance, in September 2013, a major account switching service was launched in the UK, that would facilitate current account switching and reduce necessary time from one month to a week. See https://www.gov.uk/government/news/bank-account-switching-service-set-to-launch. A long-term overview of bank relationships can be found in Braggion and Ongena (2014).) As a result, banks may find it harder to retain customers and the potential length of borrower relationship may decrease. Moreover, arm’s length lending by foreign banks (Beck, Ioannidou and Schäfer (2014)), banks from outside the region (Bofondi and Gobbi (2006)), large banks (Berger et al. (2005)), non-bank institutional investors (Nini (2013)), as well as funding via bond markets coexist with relationship lending, which tends to be more concentrated in local, small banks.

5Recent empirical evidence (Kapan and Minoiu (2013), Diewel, Frey and Lipponer (2011)) shows that better capitalized banks were more likely to maintain lending relationships during the 2008 financial crisis.
Our main focus is on the choice between direct screening and requiring collateral. Just like in our paper, there are results in the empirical literature suggesting banks face tradeoffs when deciding between the two. (Jiménez, Salas-Fumás, and Saurina (2006)), and that bank relationships and collateral are substitutes under asymmetric information (Jiménez, Salas-Fumás, and Saurina (2011)).

In our model, unobservably high-quality (low-risk) borrowers are more likely to pledge collateral than low-quality (high-risk) borrowers. Over time, high-quality borrowers will be less likely to pledge collateral, while low-quality borrowers will be more likely to do so. This pattern is consistent with empirical results. Berger, Frame and Ioannidou (2011) find that unobservably riskier borrowers are less likely to pledge collateral, consistent with the adverse selection role of collateral, but as the lending relationship progresses they become more likely to pledge collateral. The overall decline in collateral requirements as the lending relationship progresses has been consistently documented in empirical studies (Berger and Udell (1995), Chakraborty and Hu (2006), Bharath et al. (2011) and Jiménez, Salas-Fumás, and Saurina (2006)). We show that this can result can come from the switch from collateral to direct screening in later stages of the relationship.

In contrast, while collateral requirements tend to decrease over the length of lending relationships, interest rates change little (Petersen and Rajan (1994)) or may rise, especially in bank-based financial systems (Degryse and Cayseele (2000) and Ioannidou and Ongena (2010)). Our results indicate that interest rates charged to existing borrowers can increase over time as a result of the switch from requiring collateral to directly screening loan applications.

While some of our theoretical predictions are consistent with existing empirical results, others may provide inspiration for further rigorous empirical testing in the future. Our model indicates that the potential for repeated lending reduces the incidence of collateral in the initial stages of the lending relationship. Consistent with that, Fraser (2012) finds that collateral requirements are more likely for term loans than for overdrafts. Further evidence may be uncovered by comparing banks which differ in terms of balance sheet strength, size and lending technologies.

In what follows, section II presents the setup of the model. Section III presents the results under bank competition, and section IV outlines the findings in the monopoly case. Section V discusses borrower surplus, bank profits and overall welfare. Section VI concludes.

II The Model

A The Setup

Suppose there is a continuum of borrowers with total mass equal to 2. There are two types of projects available to entrepreneurs in our economy: good projects \((G)\) and bad projects \((B)\).
Both projects require an initial investment of one unit of capital. Good projects $G$ produce a cash flow $X > 0$ with probability $p_G$ and 0 with probability $1 - p_G$. Bad projects $B$ have a success probability $p_B < p_G$: they produce $X > 0$ with probability $p_B$ and 0 with probability $1 - p_B$.

There are two types of entrepreneurs in the economy: high-quality $H$ and untalented low-quality $L$ who represent proportions $\lambda$ and $1 - \lambda$ in the population respectively. Borrowers do not have any funds of their own and have to borrow the initial investment amount from at most one bank at a time. They do however have (illiquid) assets in place than can be pledged as collateral.

High-quality entrepreneurs have a probability $p_H = 1$ of having a good project, while the low-quality ones have a good project with probability $p_L < 1$, and a bad project with $1 - p_L$. Entrepreneurs do not know their own types, but they are aware of the quality of their current projects. Good projects are creditworthy: $p_G X - \bar{R} > 0$, while bad projects are not: $p_B X - \bar{R} < 0$. Since good projects are creditworthy, so are high-type borrowers. For simplicity, we assume the average borrower is not creditworthy, which also implies that on average low-type borrowers should not receive a loan: $(\lambda p_G + (1 - \lambda)(p_L p_G + (1 - p_L) p_B)) X < \bar{R}$. We assume that borrowers with a bad project derive non-monetary utility from being in the credit market, and they will apply for a loan even if they know they are not creditworthy.\footnote{This assumption is innocuous. Alternatively, we could assume that there is a positive probability of errors in screening, so that borrowers with bad projects will also have positive expected profits. Our results carry over to that case.}

We consider both the competitive case (which we model as competition between two banks), and the case in which we have just one monopolistic bank (which is still a price taker on the market for its own funds). Banks get funding at a (gross) cost of $\bar{R}$. When we look at repeated lending, we use a discount rate of zero across periods. Both banks and borrowers are risk-neutral.

We examine the distinct cases of one and two-period bank relationships. In the former borrowers live and require funding for just one period. In the latter borrowers live for two periods, have an investment project in each period and require funding for each project.\footnote{As usual in the literature, we assume that at the end of the first period successful borrowers consume their surplus and they need to borrow to finance their second-period projects.} The success probability of the investment projects is independent across periods for a given borrower. The borrower types remain unchanged across periods, however.

Banks cannot immediately distinguish between borrower types and projects of different quality. In order to tell apart good and bad projects banks and borrowers can use collateral and/or screening.

Since they have a lower probability of failure, borrowers with good projects can pledge collateral to separate themselves from borrowers with bad projects. When a project has been financed by a secured loan and the project fails, the collateral is liquidated by the bank and loses a fraction $\alpha$ ($0 \leq \alpha \leq 1$) of its value in that process.\footnote{The assumption of costly liquidation of collateral is well-established in the theoretical literature. The importance of conservative assumptions is discussed in detail in our section on collateral.} Screening involves a cost $s$ per borrower, but it reveals
the quality of the current project. As usual (Manove et al. (2001)), screening is assumed to be non observable and non contractible, and it cannot be sold to borrowers as a service.

We present the one- and two-period equilibria below. The comparison between the two cases will show that having a longer-term horizon increases the use of screening in bank lending.

We first consider one-period lending. Given the common-knowledge proportions of the borrower types and their success probabilities, banks offer screening or collateral-based contracts to their potential borrowers. Borrowers choose their banks, apply for loans and receive funding if their application is successful. At the end of the period, payoffs for borrowers and banks are realized.

We then look at the interaction between banks and borrowers over two periods. We assume that borrowers have a new project in the second period, that also requires an investment of one unit of capital, and that capital needs to be borrowed. The type of a given borrower, and hence the probability of having a good or a bad project, remains the same across periods; we have the same borrowers in the population, with the same success probabilities for good and bad projects. The two banks will offer first-period screening and/or collateral contracts. This will result in some information acquisition about their first-period borrowers. In the second period, each bank will offer loan contracts to its own first-period borrowers, and perhaps also to borrowers with which they did not have a lending relationship in the first period.

As in Manove et al. (2001) we define equilibrium outcomes as those in which banks make nonnegative profits, borrowers have a nonnegative payoff, banks are maximizing their profits and borrowers are maximizing their payoffs, and there are no viable contracts, i.e. contracts offered by a bank that provide it with nonnegative profits and provide at least one borrower with a strictly larger payoff.

III Bank competition

We first look at the case of a competitive banking system: banks bid for potential borrowers, applicants can freely choose their lender, and competition will drive down bank profits. Lenders can use screening or collateral to identify creditworthy borrowers. We begin by examining the case where screening reveals just the quality of the current project and later on look at the case of liquidation costs is also supported empirically. Benmelech and Bergman (2009) find that collateral that easier to redeploy is associated with lower credit spreads and higher loan-to-value ratios. Eisfeldt and Rampini (2009) find that ease of repossession is associated with higher debt capacity. Liberti and Mian (2010) confirm that banks are sensitive to the specificity of assets pledged as collateral and thus to their liquidation costs.

We later consider the case where screening reveals borrower type: it gives deeper information about a potential borrower’s business skills, and that information can be reused when examining further loan applications from the same borrower.
where it reveals a borrower’s type.

A The one-period equilibrium

We consider the case of two banks that compete for borrowers that are offered single one-period loans. Since the average borrower is not creditworthy, banks cannot lend without selecting their borrowers by either requiring collateral or screening them.

The timing of events is as follows:

1. Each bank offers contracts \((R, C)\) sequentially, where \(R\) is the interest rate, \(C \geq 0\) is the collateral requirement.
2. Each entrepreneur applies to one bank for one loan contract, maximizing their payoff: if offered equal payoffs, borrowers allocate themselves in equal proportions to the two banks.
3. Each bank decides whether or not to screen the applicants that apply for a given contract. Screening entails a cost \(s\) per borrower.
4. Banks approve or reject applications.
5. If the approved project is successful, it yields \(X\), from which the borrower repays \(R\) to the bank.

We assume that banks do not give loan applicants verifiable evidence of a positive screening outcome until a loan contract is signed, so borrowers are unable to take the screening result and take it to another, competing bank. If such free riding were possible, there would be inefficiently low screening.

**Proposition III.1** When both banks use collateral, there is a unique separating equilibrium in which

1. The collateral requirement is given by \(C = p_B \frac{p_G X - R}{p_G (1 - p_B) - (1 - \alpha) p_B (1 - p_G)}\), while the interest rate charged by banks is \(R_c = \frac{(1 - p_B) \tilde{R} - (1 - \alpha) p_B (1 - p_G) X}{p_G (1 - p_B) - (1 - \alpha) p_B (1 - p_G)}\).
2. Banks make zero profits.
3. Borrowers with good projects get a loan and their final expected payoff is \(P(C) = p_G X - \tilde{R} - \frac{\alpha (1 - p_G) p_B}{p_G (1 - p_B) - (1 - \alpha) p_B (1 - p_G)} (p_G X - \tilde{R})\).
4. Borrowers with bad projects do not get a loan.

**Proof** See Appendix.

When collateral is used to identify creditworthy projects there is no pooling equilibrium. The intuition behind this result is that a borrower’s rate of substitution between interest and collateral...
is different across the two project types. A new contract can change the interest-collateral mix in a way that appeals only to good project borrowers, and thus reduce the effective credit risk for the lender, so as to make it profitable. At the same time, for separation collateral needs be sufficiently high to yield bad-project borrowers a negative payoff, \( p_B(X - R) - (1 - p_B)C \leq 0 \). As indicated by the empirical literature (Booth and Booth (2006), Cerqueiro, Ongena, and Roszbach (2014)), there is an inverse relationship between interest rates and collateral requirements. Finally, note that in the case above screening borrowers would not add any information for the bank and would not be used at the same time when collateral separates the projects.

When banks instead use screening to identify good projects, competition will again drive the interest rate charged by banks to the level where their profit net of screening costs is zero.

**Proposition III.2** When both banks screen their potential borrowers:

- The interest rate charged by banks will be \( R_s = \frac{1}{p_G} \left( \bar{R} + \frac{s}{\lambda + (1 - \lambda)p_L} \right) \).
- Banks make zero profits.
- Borrowers with good projects get a loan and their final expected payoff is \( P(S) = p_G X - \bar{R} - \frac{s}{\lambda + (1 - \lambda)p_L} \).
- Borrowers with bad projects are screened, but they do not get a loan.

**Proof** See Appendix.

A higher level of the per-borrower screening costs \( s \) will increase the break-even interest rate \( R_s \) charged by banks and will reduce the good-project borrowers’ payoff. Another important factor that influences the break-even interest rate is the average quality of the borrower pool: a lower average quality (a lower proportion of high-quality borrowers in the formula: \( \lambda + (1 - \lambda)p_L \)) increases the rate. The reason is that the bank screens all borrowers that apply for a loan, but only lends to creditworthy borrowers. This creates an important contrast between screening borrowers and requiring collateral. The per-borrower cost of screening depends on the average quality of the borrower pool, while the cost of collateral does not: the amount of collateral required depends on the gap between the quality of good and bad projects (between \( p_G \) and \( p_B \)) and the probability that projects that get a loan default (\( 1 - p_G \)), leading to liquidation costs. As a result, a higher frequency of high-quality borrowers in the overall pool will increase the preference for screening relative to collateral. This mechanism is also present in Manove et al. (2001).

Banks will have to compare the liquidation costs of collateral and the screening costs to choose the borrower selection method. Given that banks compete with each other, the choice will be determined by maximizing the surplus obtained by borrowers with good projects. The condition for screening contracts being preferred to collateral is given below.
Proposition III.3 If screening costs are relatively low compared to the liquidation costs of collateral, $s < (\lambda + (1 - \lambda)p_L)\frac{\alpha(1-p_G)p_B}{p_G(1-p_B)-(1-\alpha)(1-p_G)p_B}(p_GX - \bar{R})$, then screening is preferred to collateral in the one-period game.

Proof See Appendix.

As one may expect, the cutoff point $s^* = (\lambda + (1 - \lambda)p_L)\frac{\alpha(1-p_G)p_B}{p_G(1-p_B)-(1-\alpha)(1-p_G)p_B}(p_GX - \bar{R})$ is increasing in the average quality of the borrower pool $(\lambda + (1 - \lambda)p_L)$ and the liquidation cost of collateral $(\alpha)$. We use this cutoff point as a benchmark in the following sections as a way to examine whether the possibility of longer lending relationships has a significant influence on the preference for screening or requiring collateral.

B Two periods

We now analyze the interaction between banks and borrowers over two periods\textsuperscript{10}. In each period borrowers need one unit of capital. The proportion of the two types in the overall population and their probabilities of getting a good project are the same in both periods. Also, as stated before, the type of a borrower does not change from the first period to the second. In each of the two periods, borrowers get funding from one of the banks, but they can switch banks between periods. We provide banks’ choice of instruments in the initial and subsequent selection of loan applicants.

In the first period, banks will have to either screen potential borrowers or require collateral, since the average borrower is not creditworthy. At the beginning of the second period, each bank has a group of borrowers that received a loan from it in the first period. Those borrowers had a good project in the first period, so they are of relatively high quality\textsuperscript{11}. However, not all borrowers that had a good project in the first period are guaranteed to also have a good project in the second period; some of them are low-type borrowers that happened to be lucky. As a result, further selection (via collateral requirements or direct screening) by the first-period bank may be warranted before second-period lending. We call the bank the inside (or incumbent) bank with respect to those borrowers. At the same time, each bank may want to attract borrowers it did not lend to in the first period, some of which have received a loan from the other bank.

\textsuperscript{10}Repeated interaction is the key feature of relationship banking (Boot (2000)). Bharath et al. (2007) find that “the probability of a relationship lender providing a future loan is 42%, while for a non-relationship lender, this probability is 3%. Loan repayments followed by new loans from the same lender are more likely to be a feature of credit lines, overdrafts or loan commitments, which play a major role in corporate lending (Shockley and Thakor (1997)).

\textsuperscript{11}For brevity, we present the case where this pool is still not creditworthy without any borrower selection mechanism, i.e., \(\frac{\lambda}{\lambda + (1 - \lambda)p_L}p_G \frac{(1 - \lambda)p_L}{\lambda + (1 - \lambda)p_L} \left( p_Lp_G + (1 - p_L)p_B \right) X - \bar{R} < 0 \). It is straightforward to verify that our main result regarding banks’ first-period choice also holds in the complementary case, where the average borrower is creditworthy and can receive a loan without screening or collateral requirements.
We call the bank the outside bank with respect to those borrowers. The outside bank is at a competitive disadvantage: it faces a pool that borrowers that did not receive a loan from it in the first period. That pool includes not only the other bank’s first-period borrowers - that are relatively high-quality -, but also the borrowers that did not get a loan from either bank in the first period and are therefore of poor quality. Since it has superior knowledge of its own borrowers, the incumbent will be able to ward off the poaching attempts of its competitor; it will offer a contract that provides a surplus just above the outside option to second-period borrowers with good projects and make positive profits. This hold-up problem for successful first-period borrowers implies that the incumbent bank can make positive profits on those borrowers.

The timing of events in the second period is as follows:

1. Each bank offers contracts \( (R_2, C_2) \), where \( R_2 \) is the second-period interest rate, and \( C_2 \geq 0 \) is the collateral requirement. The incumbent bank is the first to make an offer to its first-period borrowers. The outside bank can then make a counteroffer, and borrowers choose the preferred contract.

2. Each entrepreneur applies to one bank for one loan contract, maximizing their payoff: if offered equal payoffs, borrowers allocate themselves in equal proportions to the two banks.

3. Each bank decides whether or not to screen the applicants that apply for a given contract.

4. Banks approve or reject applications.

5. If the approved project is succeeds, it yields \( X \), from which the borrower repays \( R \).

We solve the game by backward induction. In the first period, the two banks will offer screening or collateral-based contracts and in either case borrowers with good projects will choose the contract that provides them with the higher two-period surplus. At the beginning of the second period, banks know that the inside borrowers had a good project in the first period, so they are of above average quality. We prove the following in the appendix.

**Lemma III.4** If \( s \leq s_2 \), where \( s_2 = \frac{\alpha p_B (1-p_G)(1-p_B)}{p_G (1-p_B)(1-p_G)} \frac{\lambda + (1-\lambda) p_L^2}{\lambda + (1-\lambda) p_B^2} \), then the inside bank uses screening in period 2. Otherwise, both banks use collateral in both periods.

Note that, as we show in the proof of the following proposition in the appendix, whenever screening is used by the outside bank in period 2 \( (s \leq p_L \frac{\alpha p_B (1-p_G)(1-p_B)}{p_G (1-p_B)(1-p_G)} \frac{\lambda + (1-\lambda) p_L^2}{\lambda + (1-\lambda) p_B^2} < s^* \), so we are below the one-period cutoff level of screening costs), screening is also used by both banks in the one-period game. We therefore restrict our attention to the case when the outside bank uses collateral (around the one-period cutoff \( s^* \)), to allow collateral to be used in the one-period game.

As we have mentioned above, collateral-based selection is not sensitive to the average quality of the borrower pool. In contrast, the viability of screening depends on the average quality of
the borrower pool which is screened. The inside bank is faced with a relative high-quality pool: borrowers that received a loan in the first period had a good project in that period and therefore have a high probability of having good projects in the second period \(p(G|G) = \frac{\lambda + (1-\lambda)p_G^2}{\lambda + (1-\lambda)p_L^2}\). If screening costs are relatively low \((s \leq s_2)\), given the higher quality of the borrower pool, the inside bank will prefer to screen borrowers, since that maximizes the joint surplus. The contract offered to the first-period borrowers will provide them with a surplus which is higher or equal to the surplus they would get by accepting the contract offered by the outside bank.

The outside bank will be faced with a relatively low-quality pool, since it is only borrowers with a bad project in the first period that are willing to switch. If screening costs are around \(s^\ast\), the benchmark one-period cutoff, the outside bank will offer the one-period collateral contract to the borrowers it is trying to poach.

As a result, when screening costs are low \((s \leq s_2)\), the inside bank will screen, while the outside bank will require collateral. The inside bank will offer a screening contract that provides borrowers a surplus which is just above the one-period collateral contract surplus \((P(C))\). To do that, banks will offer (slightly below) the following interest rate: \(R_{\text{inside}} = \frac{R}{p_G} + \frac{1}{p_G}(p_GX - R)\). At this rate the inside and outside contract payoffs are equalized:

\[
p_G(X - R_{\text{inside}}) = P(C) = \frac{(p_G - p_B)(p_GX - R)}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)}\]

and incumbent banks will make positive profits by lending to their first-period borrowers.

For very high values of screening costs \((s > s_2)\), the inside bank has to resort to requiring collateral, and will therefore not be able to profit from its competitive advantage of facing a higher-quality borrower pool. Both the inside and the outside bank make zero expected profits. The first period is then reminiscent of the one-period game, when, at those high values of screening, collateral would again be chosen.

The interesting case is therefore that of “average” screening costs, where the inside bank screens borrowers and the outside bank requires collateral. In what follows, we restrict our attention to values \(p_L \frac{\alpha p_G(1 - p_C)(pGX - R)}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)} < s < \frac{\alpha p_B(1 - p_C)(pGX - R)}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)}\), an interval which also includes the one-period cutoff \(s^\ast\). Given this outcome in the second period, we analyze the banks’ choice in the first period.

The collateral-based first-period contract will have to reflect the expected second-period bank profits. Borrowers with good projects will anticipate the banks’ future ability to extract informational rents and will require a discount in the first period. We denote the second-period profit per first-period borrower by \(\pi_{2G1} > 0\), where \(G1\) stands for having had a good project in the first period. The banks’ new zero-profit condition can be written as

\[12\]
\[ p_G R_{1,C} + (1 - p_G) C_1 + \pi_{2G1} = 0, \]

where \( R_{1,C} \) and \( C_1 \) are the interest rate and collateral required by banks in the first period. The amount of collateral will still have to be large enough to separate good- and bad-project borrowers:
\[ C_1 \geq \frac{p_B(X - R_{1,C})}{1 - p_B}. \]
These conditions give us the first-period interest rate and collateral.

**Proposition III.5** The interest rate and collateral in the first period are given by:

\[
R_{1,C} = \frac{(1 - p_B) \bar{R} - (1 - p_B) \pi_{2G1} - (1 - \alpha)p_B(1 - p_G)X}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)},
\]
\[
C_1 = p_B \frac{p_G X - \bar{R} + \pi_{2G1}}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)}.
\]

Compared to the one-period case, the interest rate will be lower to compensate for the second-period rents extracted by incumbent banks: \( R_{1,C} < R_C \). At the same time, however, the amount of collateral required by banks will have to increase to pre-empt bad-project borrowers attracted by the low interest rate: \( C_1 > C \). If screening is used in the first period, interest rate is given by

\[
R_{1,S} = \frac{\bar{R}}{p_G} + \frac{s}{p_G} \left( 1 + \frac{1}{\lambda + (1 - \lambda)p_L} \right) - \frac{\lambda(1 - \lambda)p_L^2}{\lambda + (1 - \lambda)p_L} \frac{1}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)} < R_S.
\]

and is lower than the screening interest rate in the one-period game.

**Proof** See Appendix.

The proposition provides the first step to our key finding: because collateral is now higher in the first period, the liquidation costs associated with it will be higher as well. If banks have used screening in period one, there will again be positive second-period profits for the banks coming from their informational advantage relative to their initial borrowers. In the first period, banks will have to offer good-project borrowers a discount to compensate them for the rents extracted in the second period, equal to the expected second period profits from first-period good project borrowers:
\[
R_{1,S} = \frac{\bar{R}}{p_G} + \frac{s}{p_G} \left( 1 + \frac{1}{\lambda + (1 - \lambda)p_L} \right) - \pi_{2G1} < R_S.
\]
As a result, the break-even interest rate that they charge first-period borrowers will be below the one-period rate.

**Proposition III.6** In the two-period model, the cutoff level of screening costs, \( s^{**} \), below which screening is preferred to requiring collateral in initial lending is higher than the one-period cutoff level \( s^* \).
The proposition establishes our key result: for values of screening costs just above \( s^* \), namely \( s^* \leq s \leq s^{**} \) borrowers are required to post collateral under one-period lending relationships, but they are screened under repeated lending.

The proposition indicates that the possibility of repeated lending moves banks away from collateral requirements and makes them more likely to screen borrowers. In practice, this may be good news for borrowers with insufficient collateral: they would be denied credit in short-term relationships, but they may be granted funding if longer-term relationships are likely. The lower incidence of collateral in the case of relationship lending has been consistently documented empirically in many countries (Li, Lu and Srinivasan (2013), Degryse and Cayseele (2000), Elsas and Krahnen (1998)). As suggested by Petersen and Rajan (1994, 1995), long-term relationships may improve access to credit. We identify a mechanism that can generate the advantage of repeated interaction between banks and borrowers.

The intuition behind our result is straightforward. Repeated lending over the length of a bank relationship can create a holdup situation and informational rents to incumbent lenders. To compensate for that, banks will have to offer more favorable conditions in initial lending. In the case of borrower screening, the initial discount will just result in lower interest rates, without an effect on the screening costs. In the case of collateral, the lower initial interest rate will raise the collateral requirement and therefore the expected liquidation costs. The higher cost of collateral will shift the balance in favor of borrower screening.

The existence of the holdup problem has been documented empirically (see for instance Bharath et al. (2007), Ioannidou and Ongena (2010)). Note that in our case the outside bank can get the same signal quality as the inside bank: it can eventually distinguish between good and bad projects. It also has the same selection techniques available - direct screening and collateral requirements - as the inside bank. The higher degree of adverse selection it faces pushes the outside bank to require collateral rather than screen; nevertheless, even in that case its borrower selection is more expensive than that its competitor. As a result, the inside bank can retain its high-quality borrowers and makes a positive profit. Borrowers anticipate the future rent extraction by the bank and require an initial discount to compensate for it; the discount in turn makes screening more likely in initial lending.

The total two-period payoff for borrowers with good projects in the first period will be

\[
P(S_1, S_2) = p_G(X - R_{1,S}) + \frac{\lambda+(1-\lambda)p_2^G}{\lambda+(1-\lambda)p_L} \times P(C),
\]

where the last sum denotes expected second-period payoff for borrowers with good projects in period 1. After plugging in the expression for the first-period interest rate one can see that this discount translates into an increase for total borrower payoff: \( p_G X - \bar{R} - \frac{s}{\lambda+(1-\lambda)p_L} + \pi_{2G1} + \frac{\lambda+(1-\lambda)p_2^L}{\lambda+(1-\lambda)p_L} \times P(C) \). Here the first two terms denote consumer payoff from the one-period game, while the additional surplus, \( \pi_{2G1} \), is equal to the discount. In case of collateral the total two-period payoff for borrowers with good projects in the
first period will be \( P(C_1, S_2) = p_G(X - R_{1,C}) - (1 - p_G)C_1 + \frac{\lambda + (1-\lambda)p_L^2}{\lambda + (1-\lambda)p_L} \times P(C) \), which then amounts to \( P(C) + \frac{(p_G - p_B)\pi_{2G_1}}{\alpha(p_G - p_B)(1-p_L)-1} + \frac{\lambda + (1-\lambda)p_L^2}{\lambda + (1-\lambda)p_L} \times P(C) \), so that the additional surplus is less than the discount, due to the liquidation costs.

**Proposition III.7** Whenever \( s^{**} \leq s \leq s_2 \), banks use collateral in initial lending but then shift to screening for their first-period good project borrowers. First-period bad-project borrowers will pledge collateral, either with the inside or the outside bank. First-period good-project borrowers will face higher interest rates compared to the first period, but will not be required to post collateral.

**Proof** In the Appendix we show that \( s^{**} < s_2 \), after which the result is immediate.

The less pronounced preference for collateral in the first period is matched by a lower incidence of collateral in subsequent periods. After the initial lending, banks have a pool of known borrowers whose quality is relatively high. The higher quality may mean that banks are able to shift from collateral to screening, removing the need for collateral. The reason is that the per-borrower cost of screening depends on average borrower quality; an improved average quality in the subsequent lending makes screening relatively cheaper and more attractive. Again, this may be good news for borrowers with few assets available to pledge as collateral. This result is supported in the findings of Berger, Frame and Ioannidou (2011), where at initial lending the incidence of collateral is less likely for unobservably riskier borrowers. However, with longer relationship unobservably riskier borrowers are more likely to pledge collateral.

### C Screening and information about borrower types

In the previous section we assumed that screening borrowers only provides information about their current projects. However, it can be argued that screening can provide deeper information about the overall, persistent quality of a borrower. That information can be useful in future lending. In contrast, collateral only provides information about the current project that requires financing. If the success probability of the project is high enough, then borrowers will be willing to pledge collateral, and this will allow banks to separate good and bad projects. However, this may at the same time reduce the bank’s incentive to acquire further information about the borrower.

We model this intuition about the role of screening by assuming that screening reveals not just the quality of the current project, but also the borrower’s type. In the one-period case, there is no practical difference between high- and low-type borrowers with good projects. In the two-period case, in contrast, the two groups are no longer identical. If a borrower is found to be of high type in the first period - and that can be done only through screening - then it is obvious that they will have a good project in the second period. The same cannot be said of low-type borrowers.
We look at the two-period interaction between banks and borrowers under this additional assumption. To make comparisons easier, we once again focus on values of screening costs close to the one-period cutoff $s^\ast$.

If banks decide to use collateral to select borrowers in the first period, then the two-period equilibrium is similar to the one described in the previous section. Borrowers receive a contract $(R_{1,C}, C)$ in the first period, where the interest rate $R_{1,C}$ is below the one-period break-even level. In the second period, banks make positive profits on their initial borrowers.

If banks decide to screen in the first period, they can then distinguish between high- and low-type borrowers. At the beginning of the second period, they will have this information available with respect to their first-period customers.

High-type borrowers will again have good projects in the second period, and banks can lend to them without any further selection. To prevent them from switching to the outside bank, the interest rate charged by the inside bank $R_2$ has to provide borrowers the one-period surplus from collateral contracts $P(C)$. (This is because, as before, if screening costs are around $s^\ast$ the outside bank will use collateral in contracts dedicated to the initial customers of its competitor bank). The inside bank will make positive profits on those borrowers.

In contrast, the group of low-type initial borrowers is not creditworthy on average and requires collateral or direct screening to identify second-period good projects. The low-type borrowers are of worse average quality than the larger pool of borrowers facing one-period lenders. If screening costs are around the cutoff level $s^\ast$, then incumbent banks will prefer to require collateral rather than screen those borrowers’ projects. Since the competing bank will make collateral-based offers to the same borrowers, the incumbent bank will have no competitive advantage with respect to its initial low-type borrowers and will make zero profits on them.

The second-period profit of the inside bank per first-period good borrower will be

$$
\pi'_{2G1} = \frac{\lambda}{\lambda + (1-\lambda)p_L} (p_G R_{inside} - \bar{R});
$$

around $s^\ast$, $\pi'_{2G1}$ is higher than its equivalent when screening only reveals the quality of a borrower’s current project ($\pi_{2G1} = \frac{\lambda+(1-\lambda)p'_L}{\lambda+(1-\lambda)p_L} (p_G R_{inside} - \bar{R}) - \frac{s}{\lambda+(1-\lambda)p_L}$).

Going to the first period, banks will have to offer initial interest rates $R'_{1,S}$ that compensate good-project borrowers for the information rents the bank will extract in the second period. The banks will have to make zero profits over the two periods on each good-project borrower:

$$
\pi_1 + \pi'_{2G1} = p_G R'_{1,S} - \bar{R} - \frac{s}{\lambda + (1-\lambda)p_L} + \pi'_{2G1} = 0.
$$
The good-project borrower surplus over the two periods will be given by

\[ P(S_1, S_2) = p_G X - \bar{R} - \frac{s}{\lambda + (1 - \lambda)p_L} + \pi_2G_1 + \frac{\lambda + (1 - \lambda)p_L^2}{\lambda + (1 - \lambda)p_L} \times P(C). \]

Borrowers with good projects in the first period will prefer screening to collateral-based initial contracts if the expected two-period surplus is higher: \( P(S_1, S_2) > P(C_1, S_2) \). The competing banks will offer them the preferred contract. Once again, for values of screening costs around the one-period cutoff \( s^* \) initial screening will be preferred to collateral.

**Proposition III.8** If screening costs are equal to the one-period cutoff level \( s^* \), then screening will be preferred to collateral requirements in initial lending.

**Proof** See Appendix.

The intuition is once again that offering better initial conditions, including a lower interest rate, to borrowers in the first period will entail increased liquidation costs under collateral contracts. As shown in the appendix, the initial discount is deeper if screening reveals borrower types, and this increases the relative disadvantage of collateral. An additional factor pushing the balance in favor of initial screening is the potential second-period screening costs made necessary by initial collateral contracts. If screening reveals borrower type, the duplication of screening costs is avoided.

The cutoff point \( s_T^{**} \) above which collateral is required in the first period is higher than in the case where screening only indicates the quality of the current project, which is in turn higher than the one-period cutoff: \( s_T^{**} > s^{**} > s^* \). When collateral is used in the first period, once again for screening costs \( s \in (s_T^{**}, s_2) \) screening will be used in the second period by the inside banks for their first-period good-project borrowers, so again banks switch from requiring collateral to screening over time. However, for screening costs below \( s_T^{**} \), inside banks will screen in the first period, and then in the second period either lend without any further selection (to high-type borrowers), or require collateral (in the case of low-type borrowers). In the later stages of the bank relationship what will be observed is a pool of high-quality borrowers that are financed directly and a pool of low-quality borrowers that is required to post collateral.

Summing up, if screening reveals additional information about borrowers, beyond the quality of their current project, that will increase the preference for screening in the initial borrower selection and reduce overall selection costs. The result is stronger the higher the reusability of the information acquired through screening (Chan, Greenbaum and Thakor (1986)), that is, the higher the probability that borrower types remain stable over time.
IV The monopolistic bank

We also examine the case of a bank monopoly - there is just one bank that can supply funds to potential borrowers. The bank is faced with the same population of borrowers, with the same borrower types as in the previous sections. Given that the average borrower is not creditworthy, the bank will need to screen or use collateral in order to lend.

As above, suppose first that borrowers and banks interact over just one period. If the bank uses collateral to separate good- and bad-project borrowers, then the amount of collateral required should be large enough so that bad-project borrowers are not willing to choose the collateral contract. At the same time, good-project borrowers should still find it worthwhile to pledge collateral. The conditions for collateral-based separation are:

\[ p_G(X - R) - (1 - p_G)C \geq 0, \quad p_B(X - R) - (1 - p_B)C < 0, \]

which means that, for a given interest rate \( R \), the required amount of collateral is at least \( \frac{p_G(X - R)}{1 - p_B} \).

The bank’s expected profit will be:

\[ \Pi_C = (\lambda + (1 - \lambda)p_L)(p_GR + (1 - p_G)(1 - \alpha)C - \bar{R}). \]

This profit is maximized when the interest rate \( R \) is just below \( X \) (\( R = X - \epsilon \)) and the collateral is just above zero (\( C = \frac{p_G\epsilon}{1 - p_B} \)). Borrowers with bad projects do not apply for loans, and borrowers with good projects receive a loan, but the monopolistic bank appropriates almost the entirety of the surplus created by their project. The total bank profit will therefore be:

\[ \Pi_C = \lambda(1 - \lambda)p_L(p_GR - \bar{R}) + \alpha \left( p_G(X - \epsilon) + (1 - p_G)(1 - \alpha)C - \bar{R} \right). \]

If the bank screens its potential borrowers, its profits will be:

\[ \Pi_S = \lambda(p_GR - \bar{R}) + (1 - \lambda)p_L(p_GR - \bar{R}) - s. \]

The fact that the bank can charge a relatively high interest rate allows it to require a minimal amount of collateral in order to select creditworthy borrowers. This allows the bank to minimize expected liquidation costs. Unless screening costs are negligible, collateral is very lumpy (the minimal amount of collateral that can be pledged is large), or the private benefits bad-project borrowers derive from being in business are really high, collateral will be preferred to screening.

If borrowers apply for funding in two successive two periods (and screening does not reveal borrower types), the quality of the borrower pool will improve over time and screening will become
cheaper in later periods. However, the amount of collateral required to separate types and hence the liquidation cost will still be minimal.

If screening reveals borrower types, and the bank chooses initial screening, there is no need to screen high-type borrowers again in the second period. The bank can use either screening or collateral to separate good- and bad-project low-quality borrowers. Thus as we move from a single-period to a multiple-period horizon the cost of screening decreases, while the cost of using collateral does not. That being said, a monopoly bank can charge very high interest rates and a “minimal” amount of collateral is enough to separate good and bad projects. This may imply that collateral is relatively cheaper even over longer time horizons. As a result, bank monopolies are more favorable to collateral in both the short and the long term than competitive banking systems.

V Welfare

We now consider the question whether overall welfare is higher under successive arm’s length interactions compared to repeated interaction over two periods. The idea is to compare the situation in which a borrower is unlikely to meet the same borrower for the next loan to the situation where she can apply to the same bank for successive loans. We focus on the competitive case where screening does not reveal information about borrower type.

**Proposition V.1** For any value of screening costs $s$, total welfare in the one-period case is lower or equal to the per-period welfare in the two-period case. The inequality is strict for any value of screening costs between $s^*$ and $s_2$.

**Proof** See Appendix.

This result indicates that the possibility of repeated interaction between banks and borrowers allows a better management of selection costs and therefore increases overall welfare.

For any value of screening costs above $s^*$, borrowers are required to post collateral in single-period interaction. In the repeated case, for value of screening costs between $s^*$ and $s^{**}$ borrowers are screened in the first period, and in the second period inside borrowers are screened and outside borrowers are required to post collateral. Since screening costs are above $s^*$, welfare will be lower in the first period compared to one-period interaction (in both cases good projects receive funding, but expected screening costs are higher than the expected liquidation costs for collateral in the one-period case). However, welfare is higher in the second period, when relatively high-quality inside borrowers are screened, and only outside borrowers are required to post collateral. Overall, per-period welfare is higher in the repeated case, although the difference declines as screening costs increase and we approach $s^{**}$. 
For values of screening costs between \( s^{**} \) and \( s_2 \), borrowers are required to post collateral in the first period, and inside borrowers are screened and outside borrowers required to post collateral in the second period. The first-period collateral requirement \( C_1 \) is higher than in the single-period case, and as a result expected liquidation costs are higher and welfare is lower. However, as screening costs increase towards \( s_2 \), the first-period interest rate discount decreases, and therefore collateral requirements and expected liquidation costs decrease. In the second period, welfare is higher than in the single-period case, but the difference is decreasing as the screening costs increase. Overall, per-period welfare is again higher in the case of repeated interaction.

Finally, for values of screening costs just below \( s^* \), banks will choose screening in the case of one-period interaction. In the case of repeated interaction, banks will screen potential borrowers in the first period, screen inside borrowers in the second period, and require collateral from outside borrowers in the second period. First-period welfare will be the same as in the one-period case, while second-period welfare will be higher.

We can also look at borrower welfare (surplus) and bank profits separately. Banks make positive profits in the second period, and negative profits in the first period, and just break even over the two periods. This pattern is consistent with the well-known holdup problem. However, the welfare of good-project borrowers is the same or marginally higher in the second period, and is unambiguously higher in the initial period compared to single-period lending. The reason is the additional surplus created by lower selection costs, which accrues to borrowers under bank competition.

Summing up, repeated interaction between banks and borrowers can enhance overall welfare. This is more likely in later periods, when better knowledge of borrower groups allows banks to optimize selection costs. Banks may have to offer initial discounts to borrowers; those discounts compensate for the rents they are able to extract in later periods. Borrower surplus is higher in initial periods as a result of the discount, and may be equal or marginally higher in the second period compared to single-period interaction.

\section*{VI Discussion and conclusions}

Both collateral and screening can be used by banks to select their borrowers. At the same time, both have their costs. Appropriating and liquidating collateral can destroy value, and screening requires the bank to expend resources on collecting and analyzing information about borrowers.

Our paper looks at the equilibrium choice between screening and collateral. It finds that screening becomes cheaper and therefore the favored solution in later stages of a bank relationship.
Moreover, screening is more likely to be used instead of collateral in the initial stage of repeated relationship. The results are stronger in the presence of a competitive banking system, and when screening allows banks to acquire information about borrower types that can be used again in further lending. Finally, welfare is higher in the case of repeated lending as a result of lower borrower selection costs.

We provide a theoretical explanation for the decline in collateral requirements in the later stages of lending relationships, which has been well documented in empirical studies, including those that have focused on lines of credit (Berger and Udell (1995), Chakraborty and Hu (2006)). Also in line with existing empirical evidence (Ioannidou and Ongena (2010)), we show that interest rates do not necessarily decline over time, as banks switch from requiring collateral to screening inside borrowers.

Our paper is also related to work on unsecured lending contracts, where (partial) debt collection after default is possible at a cost. Using U.S. data Severino et al. (2014) find that an increase in the level of bankruptcy protection increases the amount of unsecured debt with the existing clients, but does not lead to higher delinquency - a result that is consistent with more informed lending by the bank when it is costlier for the lender to appropriate assets after default.

Our finding is not conditional on market power allowing banks to accumulate extraordinary rents (Petersen and Rajan (1995)); indeed, we allow banks to compete and the surplus generated by the bank relationship to be captured by potential borrowers. What counts is the possibility to use the information from initial loans when optimizing selection mechanisms for subsequent funding. Moreover, our findings indicate that an environment with competing relationship lenders can lower collateral requirements and increase borrower surplus in initial lending (while borrower surplus in subsequent periods is marginally higher compared to one-period transactional lending). These results may go some way in explaining why bank lending through revolving lines of credit has survived given the growth of bond markets and lending by non-bank institutional investors (Nini (2013)).

Although relatively minor in our model, the holdup problem may be reduced through information sharing (when outside banks know which borrowers received loans from competing banks in previous periods, their adverse selection problem is reduced), by using credit lines where the bank can refuse to offer further loans, but the conditions of those future loans are predetermined (as in von Thadden (1995)), or when there are new borrowers coming into the market after the first period, and they are of average quality (as in Dell’Ariccia and Marquez (2005)).

Our findings can be extended to include other factors that change the trade-off in long-term relationships. Future shocks to the market value of available collateral can increase the preference for screening either for the bank or for the borrower. While their currently available collateral is satisfactory, borrowers may worry that future shocks may reduce the value of their assets in place and make them unable to post sufficient collateral. In this case borrowers may be excluded from credit markets even if they have high-quality projects. It has indeed been documented that
shocks to the value of assets in place that can be used as collateral affect access to financing (Gan, (2007)).

Furthermore, the reduction in collateral requirements in long lending relationships will have welfare implications if one extends the model to take account of social costs of secured lending. Collateral may have systemic costs when collateralized asset values are correlated across the economy. Bernanke and Gertler (1989), Kiyotaki and Moore (1997) show that fluctuations in the value of collateral can affect credit availability and aggregate output. Moreover, our finding that the potential for longer-term relationships reduces the use of collateral in the initial stages of bank relationships, points to a possible increase in initial access to credit. The lack of collateral is indeed still a major hindrance to small firms' access to credit (Chakraborty and Hu (2006)).

In the light of our findings, supporting stable lending relationships may be a useful policy. Supporting local, small banks (Berger et al. (2005), Agrawal and Hauswald (2010), Beck, Degryse and van Horen (2014)), as well as the integration of international banks in the local network (De Haas and van Horen (2013)), and ensuring that banks are well capitalized (Kapan and Minoiu (2013)), all of which are favorable to long-lasting bank-borrower contacts, may enhance overall welfare, significantly increase borrower surplus at least in the initial stages, and reduce collateral requirements even in the initial selection of potential borrowers.
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I Appendix

Proof of proposition III.1

Banks offer contracts with an interest rate \( R \) and collateral \( C \). Borrowers enter the market choosing from the set of available loan contracts, or opt out and choose the outside option, denoted by \( \theta_0 \). For each adopted \((R, C)\) contract we define a contract outcome \((R, C, p)\) where \( p \) denotes the average risk of the pool that has adopted the contract. A competitive equilibrium is a collection of \( \Theta \) of contract outcomes of the form \((R, C, p)\) such that i) each member of \( \Theta \) earns non-negative profits to the lenders; ii) each contract in \( \Theta \) provides borrowers a non-negative payoff; iii) each contract in \( \Theta \) maximizes the borrowers’ payoff given the choice of each borrower from the set \( \Theta \cup \theta_0 \); iv) there exists no new contract \( \theta \notin \Theta \) that generates strictly higher payoff for at least one borrower, given other borrowers’ choice from \( \Theta \cup \{\theta_0, \theta\} \). We normalize the outside option to zero.

We first show that there is no pooling equilibrium.

Suppose instead there is a pooling equilibrium and denote it by \((R, C, p)\). The expected payoff for a borrower with a good project that uses collateral is given by

\[
P(C) = p_G(X - R) - (1 - p_G)C.
\]

The expected payoff for a borrower with a bad project that uses collateral is given by

\[
p_B(X - R) - (1 - p_B)C.
\]

Since the pooling contract is selected by both type of borrowers, the expected profit that the lender generates is \( pR + (1 - p)(1 - \alpha)C - \bar{R} \), where \( p \) is the success probability of the average pool of borrowers: \( p = \lambda + (1 - \lambda)p_L \). The borrower surplus maximization condition would require that banks’ expected profit be zero, since otherwise a slight decrease in interest rate (or decrease in collateral) could further increase all borrowers’ surplus.

It can be shown that a new contract \((R', C')\) can enter, attract only good-project borrowers and make a positive profit for banks, i.e.

\[
\begin{align*}
p_G R' + (1 - p_G)C' &< p_G R + (1 - p_G)C \quad (1) \\
p_B R' + (1 - p_B)C' &> p_B R + (1 - p_B)C \quad (2) \\
p_G R' + (1 - p_G)(1 - \alpha)C' - \bar{R} &> 0 \quad (3)
\end{align*}
\]
The first inequality means that for good-project borrowers, expected payments are lower (i.e., the surplus is higher) under the new contract. The second inequality means that for bad-project borrowers, expected payments are lower under the original contract. The third inequality means that, when only borrowers with good projects select the new contract, the new contract generates positive profits.

Denote \( C' = C + \varepsilon \) and \( R' = R - \delta \). Then the first two inequalities reduce to

\[
\frac{1 - p_G \varepsilon}{p_G} < \delta < \frac{1 - p_B \varepsilon}{p_B}
\]

For every \( \varepsilon \) we can find \( \delta \) that satisfies the above. The third inequality can be rewritten as

\[
p_G R + (1 - p_G)(1 - \alpha)C + ((1 - p_G)(1 - \alpha) \varepsilon - p_G) - \bar{R} > 0.
\]

If \( \varepsilon \) and \( \delta \) are small the last term is small (and can be arbitrarily close to zero). By virtue of the (assumed) equilibrium, bank profits are zero for the pooling equilibrium: \( pR + (1 - p)(1 - \alpha)C - \bar{R} = 0 \). But because \( p_G - p > 0 \) the first term is strictly positive, thus the third inequality holds and \((R', C')\) undercuts the supposed equilibrium.

We now show that there is a (unique) separating equilibrium.

First, in any separating equilibrium bad-project borrowers must not receive credit since they are not creditworthy projects by assumption.

\[
p_B(X - R) - (1 - p_B)C \leq 0,
\]

Thus, the bad-project borrowers will not find it worthwhile to pledge collateral if the amount required is high enough: for a given interest rate \( R \) the required amount of collateral is at least \( C = \frac{p_B(X - R)}{1 - p_B} \).

Second, the banks’ per-contract profit on loans to good-project borrowers pledging collateral must be zero:

\[
p_G R + (1 - p_G)(1 - \alpha)C - \bar{R} = 0.
\]

This stems from the definition of a separating equilibrium and the fact maximizing borrowers’ payoff requires that competing banks lower the interest rate \( R \) up to where expected profits are zero. As a result, the \((R, C)\) pair in the equilibrium separating contract is unique. To see this, notice that good project borrowers’ payoff increases in the interest rate:

\[
\frac{\partial P(C)}{\partial R} = -p_G + \frac{p_B(1 - p_G)}{1 - p_B} > 0
\]
Solving, the amount of collateral pledged by banks is given by

$$C = p_B \frac{p_G X - \bar{R}}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)}$$

and the interest rate is

$$R_C = \frac{(1 - p_B)\bar{R} - (1 - \alpha)p_B(1 - p_G)X}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)}.$$

The payoff for good-project borrowers using collateral will be

$$P(C) = (p_G X - \bar{R}) \frac{p_G(1 - p_B) - (1 - p_G)p_B}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B} = p_G X - \bar{R} - \frac{\alpha(1 - p_G)p_B}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B} (p_G X - \bar{R}).$$

**Proof of proposition III.2**

Suppose the bank screens potential borrowers and offers an interest rate \( R \) to borrowers with good projects. The expected payoff for good-project borrowers that are screened and offered an interest rate \( R \) is given by \( p_G(X - R) \). Borrowers with bad projects that are screened will not receive a loan, since they are not creditworthy.

The bank’s per-borrower profits from screening a pool of borrowers out of which a proportion \( p \) have good projects is \( p(p_G R - \bar{R}) - s \); when screening the average pool of borrowers \( p = \lambda + (1 - \lambda)p_L \). The market will be shared equally between the banks and competition will draw bank profits to zero: \( (\lambda + (1 - \lambda)p_L)(p_G R - \bar{R}) - s = 0 \). The competitive interest rate offered to borrowers will be the break-even rate: \( R = \frac{1}{p_G} \left( \bar{R} + \frac{s}{\lambda + (1 - \lambda)p_L} \right) \). The expected payoff for a good-project borrower will therefore be \( P(S) = p_G(X - R) = p_G X - \bar{R} - \frac{s}{\lambda + (1 - \lambda)p_L} \).

**Proof of proposition III.3**

The payoff for borrowers with good projects is \( p_G X - \bar{R} - \frac{\alpha(1 - p_G)p_B}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B} (p_G X - \bar{R}) \) if banks require collateral, and \( p_G(X - R) = p_G X - \bar{R} - \frac{s}{\lambda + (1 - \lambda)p_L} \) if banks choose to screen. In equilibrium, banks will choose the contract that maximizes borrower surplus.

Borrowers have a higher payoff under screening than under collateral if the per contract screening cost \( s \leq (\lambda + (1 - \lambda)p_L)\frac{\alpha(1 - p_G)p_B}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B} (p_G X - \bar{R}) \). Therefore the cutoff level of screening costs above which collateral is preferred is \( s^* = (\lambda + (1 - \lambda)p_L)\frac{\alpha(1 - p_G)p_B}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B} (p_G X - \bar{R}) \).
Proof of lemma III.4 and proposition III.5

At the beginning of the second period, the inside bank has a pool of borrowers which had a good project in the first period, and have a high probability of having a good project in the second period \( p(G|G) = \frac{\lambda + (1 - \lambda)p_L^2}{\lambda + (1 - \lambda)p_L} \). The inside bank can choose to screen those borrowers, or to require collateral. Given the high quality of the borrower pool, screening is relatively cheap for the inside bank. The outside bank can screen or require collateral in its attempt to poach borrowers. If it tries to attract all borrowers that did not have a loan with it in the first period, the outside bank is faced with a below-average borrower pool; the probability of having good-project borrowers is \( \frac{\lambda + (1 - \lambda)p_L(3 - 2p_L)}{\lambda + (1 - \lambda)(3 - 2p_L)} \). The collateral-based contract is not sensitive to the average borrower quality, while the screening contract is. For very low values of screening costs, both banks can offer screening contracts; however, since it can identify a better borrower pool, the inside bank can offer better conditions to its first-period good-project borrowers. The outside bank will offer a contract aimed at the remaining borrowers, where the probability of finding a good project is \( p_L \).

For very high values of screening costs, both banks will offer collateral-based contracts; in that case, the inside bank has no competitive advantage and both banks make zero profits. In the intermediary case, around the one-period cutoff \( s^* \), the inside bank will choose screening, while the outside bank will require collateral. We start by looking at this case.

In the equilibrium with sequential bidding, in period 2 the incumbent banks will offer an interest rate that provides first-period good-project borrowers a surplus which is just above what they can get from the competing bank. Thus only first-period bad-project borrowers go to the outside bank. The contract they get from the outside bank will be the usual one-period collateral-based contract presented in the previous section. The share of borrowers with good projects in this population is given by \( p(G|B) = \frac{(1 - \lambda)p_L(1 - p_L)}{(1 - \lambda)(1 - p_L)} = p_L \), i.e., the probability of having a good project given that the bad project was observed in period 1.

Note that the outside bank prefers screening over collateral whenever

\[
s \leq p_L \frac{\alpha(1 - p_G) p_B}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B}(p_G X - \bar{R}),
\]

which differs from the one-period cutoff (which is higher: \( s^* = (\lambda + (1 - \lambda)p_L) \frac{\alpha(1 - p_G) p_B}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B}(p_G X - \bar{R}) \)). This is because first-period good-project borrowers are held up, while the switching borrowers with bad project have probability \( p(G|B) = \frac{(1 - \lambda)p_L(1 - p_L)}{(1 - \lambda)(1 - p_L)} = p_L \) of having a good project in period 2. When the outside bank offers a collateral-based contract, the borrowers’ payoff will be equal to the one-period collateral case, \( P(C) = \frac{(p_G - p_B)(p_G X - \bar{R})}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)} \).

In this case, the interest rate that provides that borrower surplus with screening is \( R_{inside} = \)
The minimal amount of collateral will be \( C \) period one as well. For assumption, banks anticipate this so that they face the same trade-off as in the one-period game. Since by profits in period two for first-period good project borrowers. In the first period, borrowers’ and both the inside and the outside bank use collateral in period 2. This secures zero expected profits in the second period, banks will be able to offer a discount in the first period. At the beginning of the second period, each bank has again a pool of borrowers that had a good project in the first stage and are therefore of high average quality. The second-period competition will be similar to that under first-period collateral, as outlined above. Given the positive expected profits in the second period, banks will be able to offer a discount in the first period. The lower interest rate will however lead to higher collateral requirements to deter bad-project borrowers. The minimal amount of collateral will be \( C_1 = \frac{p_B X - R_{1,c}}{(1-p_B)(1-\alpha)p_B(1-p_G)} > C \).

We next look at the case where banks have used screening to separate borrowers in the first period. The discount offered in the first period will drive the interest rate below the one-period interest rate with collateral:

\[
R_{1,c} = \frac{(1-p_B)\bar{R} - (1-p_B)\lambda_2G1 - (1-\alpha)p_B(1-p_G)X}{p_G(1-p_B) - (1-\alpha)p_B(1-p_G)} < R_C.
\]

If screening is used, the inside bank will therefore make a second-period profit (on a borrower who had a first-period good project) equal to \( \pi_2 = (\lambda + (1-\lambda)p_B^L) - s(\lambda + (1-\lambda)p_L) \) (i.e., lending profits less screening costs). All first-period good-project borrowers are screened and those that also have a good project in the second period receive a loan. The second-period bank profit per first-period good-project borrower is

\[
\pi_{2G1} = \frac{\lambda + (1-\lambda)p_B^L}{\lambda + (1-\lambda)p_L} (p_G\pi_{inside} - \bar{R}) - s = \frac{\alpha p_B(1-p_G)p_G\pi_{X-R}}{p_G(1-p_B) - (1-\alpha)p_B(1-p_G)} \lambda + (1-\lambda)p_B^L - s.
\]

If \( s > \frac{\alpha p_B(1-p_G)p_G\pi_{X-R}}{p_G(1-p_B) - (1-\alpha)p_B(1-p_G)} \lambda + (1-\lambda)p_B^L \) then clearly \( s > p_L\frac{\alpha p_B(1-p_G)p_G\pi_{X-R}}{p_G(1-p_B) - (1-\alpha)p_B(1-p_G)} \), so that both the inside and the outside bank use collateral in period 2. This secures zero expected profits in period two for first-period good project borrowers. In the first period, borrowers’ and banks anticipate this so that they face the same trade-off as in the one-period game. Since by assumption, \( s > s^* = (\lambda + (1-\lambda)p_L) \frac{\alpha (1-p_G)p_B(1-p_G)\pi_{X-R}}{p_G(1-p_B) - (1-\alpha)(1-p_G)p_B} \), banks choose to use collateral in period one as well.

Suppose now \( s \leq \frac{\alpha p_B(1-p_G)p_G\pi_{X-R}}{p_G(1-p_B) - (1-\alpha)p_B(1-p_G)} \lambda + (1-\lambda)p_B^L \). The collateral required in period 1 will have to be sufficient to separate good- and bad-project borrowers: \( C \geq \frac{p_B X - R_1}{1-p_B} \), where \( R_1 \) is the first-period interest rate. The expected positive second-period profits will be competed away in the first-period collateral-based contract.

\[
p_G R_1 + (1-p_G)(1-\alpha)\frac{p_B(X - R_1)}{1-p_B} - \bar{R} + \pi_{2G1} = 0.
\]

Suppose now \( s \leq \frac{\alpha p_B(1-p_G)p_G\pi_{X-R}}{p_G(1-p_B) - (1-\alpha)p_B(1-p_G)} \lambda + (1-\lambda)p_B^L \). The collateral required in period 1 will have to be sufficient to separate good- and bad-project borrowers: \( C \geq \frac{p_B X - R_1}{1-p_B} \), where \( R_1 \) is the first-period interest rate. The expected positive second-period profits will be competed away in the first-period collateral-based contract.
period. The interest rate in the first-period screening contract will be lower than in the one-period case:

\[ R_{1,S} = \frac{\bar{R}}{p_G} + \frac{s}{p_G} \left(1 + \frac{1}{\lambda + (1 - \lambda)p_L}\right) - \frac{1}{p_G} \frac{\lambda + (1 - \lambda)p_L^2}{\lambda + (1 - \lambda)p_L} \frac{\alpha p_B(1 - p_G)(p_G X - \bar{R})}{p_G(1 - p_B)(1 - p_G) - (1 - \alpha)p_B(1 - p_G)} < R_S. \]

**Proof of proposition III.6**

The expected two-period surplus for a borrower that has a good project in the first period will be

\[ P(C_1, S_2) = p_G(X - R_1) - (1 - p_G)C_1 + \frac{\lambda + (1 - \lambda)p_L^2}{\lambda + (1 - \lambda)p_L} \times P(C). \]

Using the expressions for \( R_1 \) and \( C_1 \) we get

\[ P(C_1, S_2) = \frac{(p_G - p_B)\pi_{2G1}}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)} + \left(1 + \frac{\lambda + (1 - \lambda)p_L^2}{\lambda + (1 - \lambda)p_L}\right) \times P(C). \]

The expected two-period surplus for a borrower that has a good project in the first period will be

\[ P(S_1, S_2) = p_GX - \bar{R} - \frac{s}{\lambda + (1 - \lambda)p_L} + \pi_{2G1} + \frac{\lambda + (1 - \lambda)p_L^2}{\lambda + (1 - \lambda)p_L} \times P(C). \]

Whether screening or collateral is chosen in the first period depends on the two-period surplus for borrowers with good projects in the first period. That surplus is higher under first-period screening \( (P(S_1, S_2) > P(C_1, S_2)) \) if

\[
p_GX - \bar{R} - \frac{s}{\lambda + (1 - \lambda)p_L} + \pi_{2G1} + \frac{\lambda + (1 - \lambda)p_L^2}{\lambda + (1 - \lambda)p_L} \times P(C) >
\]

\[ p_G(X - R_1) - (1 - p_G)C_1 + \frac{\lambda + (1 - \lambda)p_L^2}{\lambda + (1 - \lambda)p_L} \times P(C). \]

Cancelling out similar terms, the condition can be rewritten as

\[ p_GX - \bar{R} - \frac{s}{\lambda + (1 - \lambda)p_L} + \pi_{2G1} > \frac{(p_G - p_B)\pi_{2G1}}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)} + P(C). \]

We now evaluate the condition at \( s^* \), the one-period cutoff level for screening costs. We know that \( p_GX - \bar{R} - \frac{s}{\lambda + (1 - \lambda)p_L} = P(C) \) (the one-period borrower surplus under screening and
Suppose screening was used in the first period. At the beginning of the second period, the inside bank can identify high-type borrowers among its first-period customers. These customers require no further selection. Low-type borrowers are a relatively low-quality pool. If screening costs are around $s^*$, the inside bank will require collateral from them. The outside bank requires collateral are equal at $s^*$.

Proof of proposition III.8

Proof of proposition III.7

Looking at interest rates, we can see that the interest period charged to borrowers that suppose screening was used in the first period. At the beginning of the second period, the inside bank will require collateral from them. The outside bank requires collateral are equal at $s^*$.

We have:

\[
\frac{G(1 - \lambda)p}{B - p} = \frac{G(1 - \lambda)p}{B - p}
\]

Therefore the inequality holds around $s^*$, and screening is used in the first period. We can also solve for the cutoff $s^*$, where the two sides are equal.
collateral from the switching borrowers. The inside bank will make positive profits on high-type borrowers, that will be charged an interest rate $R_{\text{inside}}$ that gives a surplus just above $P(C)$, which they can get by taking the outside bank’s offer. The inside bank’s profit per first-period good-contract borrower is $\pi_{2G1} = \frac{\lambda}{\lambda + (1 - \lambda) p_L} (p_G R_{\text{inside}} - \bar{R})$.

In the first period, the interest rate $R_{1,S}$ charged by the banks will give them zero profits over the two periods: $p_G R'_{1,S} - \bar{R} - \frac{s}{\lambda + (1 - \lambda) p_L} \pi_{2G1} = 0$. Solving, we get $R'_{1,S} = \frac{\bar{R}}{p_G} \left(1 - \frac{\lambda}{\lambda + (1 - \lambda) p_L}\right) + \frac{s}{p_G} - \frac{\lambda}{\lambda + (1 - \lambda) p_L} \frac{1}{p_G} \frac{\alpha(1 - p_G)(1 - p_B)(p_G X - \bar{R})}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B}$. Once again we have that $R'_{1,S} < R_S$ (borrowers are offered a discount compared to the one-period case); moreover, if we are around $s^*$, $R'_{1,S} < R_{1,S}$; the discount is deeper compared to the case where screening only reveals the quality of the current project.

The two-period surplus for a borrower that has a good project in the first period is therefore:

$$P(S_1, S_2)' = p_G (X - R'_{1,S}) + \frac{\lambda + (1 - \lambda) p_L^2}{\lambda + (1 - \lambda) p_L} \times P(C)$$

$$= p_G X - \bar{R} - \frac{s}{\lambda + (1 - \lambda) p_L} \pi_{2G1} + \frac{\lambda + (1 - \lambda) p_L^2}{\lambda + (1 - \lambda) p_L} \times P(C).$$

As mentioned in the text, if collateral is used in the first period, then the equilibrium is similar to that described in the previous section and the two-period borrower surplus will once again be:

$$P(C_1, S_2) = \frac{(p_G - p_B) \pi_{2G1}}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)} + \left(1 + \frac{\lambda + (1 - \lambda) p_L^2}{\lambda + (1 - \lambda) p_L}\right) \times P(C).$$

First-period screening will be preferred when it provides a higher payoff to first-period good-project borrowers: $P(S_1, S_2)' > P(C_1, S_2)$, which is equivalent to

$$p_G X - \bar{R} - \frac{s}{\lambda + (1 - \lambda) p_L} \pi_{2G1} + s - (1 - \lambda) p_L^2 \frac{\alpha(1 - p_G)(1 - p_B)(p_G X - \bar{R})}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B} >$$

$$\frac{(p_G - p_B) \pi_{2G1}}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)} + P(C).$$

The condition holds for screening costs around $s^*$. Moreover, a comparison to inequality I reveals that the difference between the surplus under first-period screening and first-period collateral is larger than in the case where screening only reveals project quality.

Solving, we get the following cutoff level of screening costs:
\[ s^{**}_T = (\lambda + (1 - \lambda)p_L) \frac{\alpha(1 - p_G)(p_GX - \bar{R})}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B} \]

\[ = s^* \times \frac{1 + \frac{\lambda + (1 - \lambda)p_L^2}{\lambda + (1 - \lambda)p_L(1 - p_B) - (1 - \alpha)(1 - p_G)p_B}}{\frac{1}{1 - \frac{\alpha(1 - p_G)}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B}}}, \]

which shows that \( s^{**}_T > s^{**} > s^* \). The cutoff in the two-period case when screening reveals borrower types is higher than in the case where it only reveals the quality of the current project, and that in turn is higher than the one-period cutoff.

**Proof of proposition V.1**

We compare the per-period welfare in the case of repeated relationships to the per-period welfare in the case of single-loan relationships.

In the one-period case, welfare can be written as the sum between borrower surplus and bank profits, which are zero. At \( s^* \), total welfare can be written as

\[
\text{Welfare} = (\lambda + (1 - \lambda)p_L)(p_GX - \bar{R}) - (\lambda + (1 - \lambda)p_L)p_G(p_GX - R). 
\]

Put otherwise, welfare can be seen as the surplus produced by good projects less the expected liquidation costs of collateral (which at \( s^* \) are equal to the expected screening costs).

If screening costs are between \( s^* \) and \( s^{**} \), then borrowers are screened in the first period. In the second period, inside borrowers are screened and outside borrowers are required to post collateral. In the second period, the total borrower surplus will be equal to:

\[
(\lambda + (1 - \lambda)p_L^2)p_G(X - R_{\text{inside}}) + (1 - \lambda)p_L(1 - p_L)(p_G(X - R_G)) 
- (1 - p_G)p_B(p_GX - \bar{R}). 
\]

Second-period bank profits will be equal to:
\[
(\lambda + (1 - \lambda)p_L^2)(p_G R_{\text{inside}} - \bar{R}) - (\lambda + (1 - \lambda)p_L) \times s \\
+ (1 - \lambda)p_L(1 - p_L) \left(p_C R_C + (1 - p_G) \frac{(1 - \alpha)p_B(p_G X - \bar{R})}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B} \right).
\]

Total welfare will be equal to

\[
(\lambda + (1 - \lambda)p_L^2)(p_G X - \bar{R}) - (\lambda + (1 - \lambda)p_L)s \\
+ (1 - \lambda)p_L(1 - p_L) \left(p_G X + (1 - p_G) \frac{\alpha p_B p_G(p_G X - \bar{R})}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B} - \bar{R} \right)
\]

\[
= (\lambda + (1 - \lambda)p_L)(p_G X - \bar{R}) - (\lambda + (1 - \lambda)p_L)s - (1 - \lambda)p_L(1 - p_L)(1 - p_G) \frac{\alpha p_B p_G(p_G X - \bar{R})}{p_G(1 - p_B) - (1 - \alpha)(1 - p_G)p_B},
\]

that is, the surplus generated by the good projects, less screening costs for the inside borrowers and the collateral liquidation costs for the outside borrowers.

In the first period, borrowers are screened and those that have a good project receive loans. Their total surplus is:

\[
(\lambda + (1 - \lambda)p_L)p_G(X - R_{S1}),
\]

while bank profits are \((\lambda + (1 - \lambda)p_L)(p_G R_{S1} - \bar{R}) - s\). Total welfare is the surplus generated by good projects less screening costs:

\[
(\lambda + (1 - \lambda)p_L)(p_G X - \bar{R}) - s.
\]

The one-period welfare is lower than the average per-period welfare in the two-period case if

\[
(\lambda + (1 - \lambda)p_L)(1 - p_G)\alpha C > \frac{1}{2}((1 - \lambda)p_L(1 - p_L)\alpha C + (\lambda + (1 - \lambda)p_L)s + s),
\]

where \(C\) is the one-period collateral requirement.

The level of screening costs at which welfare is equal in the two cases is \(s^w = \alpha(1 - p_G)C^{\frac{\lambda + (1 - \lambda)p_L + \lambda + (1 - \lambda)p_L^2}{1 + \lambda + (1 - \lambda)p_L}}\). It can be shown that \(s^w\) is higher than the cutoff \(s^{**}\) \((s^w > s^{**}\) is equivalent to \((\lambda + (1 - \lambda)p_L^2 - (\lambda + (1 - \lambda)p_L)^2) \left(1 - \frac{\alpha(1 - p_G)p_B}{p_G(1 - p_B) - (1 - \alpha)p_B(1 - p_G)} \right) > 0\), which is true). Therefore, per-period welfare is higher in the case of two-period interaction than in the case of one-period lending.
For screening costs between $s^{**}$ and $s_2$, borrowers will be required to post collateral in the first period, and in the second period inside borrowers will be screened and outside borrowers will receive secured loans.

Second-period borrower surplus, bank profits and welfare will have the same expressions as above.

In the first period, borrowers are required to post collateral. Their total surplus is:

\[
(\lambda + (1 - \lambda)p_L)p_G(X - R_{C1}) - (1 - p_G)\alpha C_1,
\]

while bank profits are $(\lambda + (1 - \lambda)p_L)(p_G R_{C1} + (1 - p_G)C_1 - \bar{R})$. Total welfare is the surplus generated by good projects less liquidation costs:

\[
(\lambda + (1 - \lambda)p_L)(p_G X - \bar{R}) - (1 - p_G)\alpha C_1.
\]

The one-period welfare is lower than the average per-period welfare in the two-period case if

\[
(\lambda + (1 - \lambda)p_L)(1 - p_G)\alpha C > \frac{1}{2}((1 - \lambda)p_L(1 - p_L)(1 - p_G)\alpha C + (\lambda + (1 - \lambda)p_L)(1 - p_G)\alpha C_1 \\
+ (\lambda + (1 - \lambda)p_L)s),
\]

where $C$ is the collateral requirement for one-period lending and $C_1$ is the (higher) initial collateral requirement for two-period lending. ($C_1 = C + p_B \frac{\pi_{2G_1}}{\lambda + (1 - \lambda)p_L} \alpha(1 - p_G)(C - s)$).

Plugging in the expression for $C_1$, we get that the inequality holds as long as $s < \frac{\lambda + (1 - \lambda)p_L^2}{\lambda + (1 - \lambda)p_L} (1 - p_G)\alpha C = s_2$. This means that per-period welfare is higher in the case of repeated interaction for all values of screening costs between $s^{**}$ and $s_2$. 