

The Transfer Problem

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Abstract

This paper studies the transfer problem in a model featuring comparative advantage, monopolistic competition, trade costs, and firm heterogeneity. The results are very different from those of the previous literature. First, a transfer creates a secondary burden in situations in which it would not do so if monopolistic competition and trade costs were not in the model. Second, a transfer gives rise to changes in inequality. Third, a transfer is not neutral to world welfare: it reduces it when heterogeneity is unbiased but may reduce or increase it when heterogeneity is factor biased.

J.E.L. Classification: F1, F4.

1 Introduction

"The more things change the more they are the same. After the First World War economists discussed the effects of a unilateral transfer - such as reparations - on the terms of trade. And in the 1950s, as the end of the Marshall Plan comes into sight, economists must once again consider an identical analytic problem - the possible effects of a cessation of unrequited imports on the terms of trade." Samuelson (1952, p. 278). And again, in the XXI century we are confronted with analytically similar problems such as the effects of rebalancing of Asian trade surpluses, of debt adjustments in the Euro-zone, of intra-European transfers or foreign aid. The contemporary observer will

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find similarities also in the debate about alternative ways of repaying foreign debt, for instance: "The easiest method would be to allow the exchange value of the German mark to fall by the amount required to give the necessary bounty to exports and then to resist any agitation to raise money-wages. But it is precisely this method which the Dawes scheme's device of 'transfer protection' expressly forbids." Keynes (1929, p. 6). In contrast, the defenders of the Dawes scheme insisted on the merit of deflation, not devaluation, as the appropriate method to generate enough savings to pay the debt. But "If [...] deflation is enforced, how will this help? Only if, by curtailing the activity of business, it throws men out of work, so that, when a sufficient number of millions are out of work, they will then accept the requisite reduction in their money-wages. Whether this is politically and humanly feasible is another matter." Keynes (1929, p. 7). It is amazing how closely related these words are to the current vicissitudes of the Euro-zone.

I must confess that writing on the Transfer Problem after Keynes, Ohlin, and Samuelson gives me the shivers. But my objective here is very modest. I simply reexamine the Transfer Problem in the light of models featuring monopolistic competition and heterogeneous firms. As I hope to demonstrate, this paper is worth reading because it discovers new effects of the transfers on the economy. In particular, it finds significant consequences of transfers on productivity, income distribution, factor allocation, country welfare and world welfare that do not arise in traditional models of international trade.

A transfer has two possible effects on the economy concerned: The primary effect, or primary burden, consists in the resources to be transferred; the secondary effect, or secondary burden, consists in the general equilibrium repercussions of the transfer on the endogenous variables and, especially, on welfare.¹ The existence of a primary burden is, of course, incontrovertible while the existence of the secondary burden has been the object of contention in the entire literature on the Transfer Problem. In most of this literature, the debate on the secondary burden has evolved around the terms of trade effects of the transfer. The focus on the terms of trade was natural since the conceptual framework was the one we now call the "neoclassical model" of trade. In this model, the only possible way a transfer creates a secondary

¹Keynes, referring to the Dawes Committee, uses the term 'Budgetary Problem' to refer to the primary effect and 'Transfer Problem' to refer to the secondary effect. Samuelson (1952) uses the terms 'primary burden' and 'secondary burden'. Incidentally, Keynes' criticism of the 'transfer protection' device does not imply that he criticized the Dawes scheme overall. The scheme (better known as Dawes Plan) was praised by contemporary political commentators and got Dawes to share the Nobel Peace Prize in 1925 for having contributed with his plan to reducing the tension between Germany and France after the First World War.

burden is via an adverse change in the terms of trade. Samuelson makes this point very clearly in his two famous articles (1952, 1954), which provide of a comprehensive analysis on the conditions under which a transfer impacts the terms of trade. Today we may reexamine the secondary burden armed with models featuring monopolistic competition and heterogeneous firms. The importance of such re-examination had not escaped Samuelson's crisp analysis when, in summing up the results of his study, he wrote:

Only if one brings in Chamberlinian phenomena of monopolistic competition do substantive effects arise, ...
Samuelson (1954, p. 288)

This line of research was not pursued by Samuelson and remained almost entirely unexplored even after the appearance and vast utilisation of monopolistic competition models in international trade theory. Re-examining the Transfer Problem in the light of monopolistic competition and heterogeneous firms models reveals new economic mechanisms and gives rise to new results. These new results are better appreciated after a brief review of the literature.

2 Brief review of the literature

The first debate on the transfer problem was hosted by The Economic Journal in a series of papers and comments published in the spring and summer 1929. The debate was sparked off by John Maynard Keynes (1929) and found remarkable commentators in Berthil Ohlin (1929) and Jacques Rueff (1929). The literature subsequently flourished with Pigou (1932), Metzler (1942, 1951), Samuelson (1952, 1954), Johnson (1956), Mundell (1960) to mention but a few of the great names that took part in that early debate. As already stated, most of these papers adopted neoclassical models of trade, except Metzler (1942) who, under the influence of the recent publication of the General Theory, experimented with the study of the transfer problem in a Keynesian macro model. Later works have studied the transfer problem taking into account elements such as the presence of non-traded goods (McDougall, 1965), multiple countries (Dixit, 1983; Yano, 1983), many goods (significant, 1978), and non-identical preference (Jones, 1970, 1975). A common result of all these papers since the beginning is that in the case of identical and homothetic preferences and unity elasticity of substitution between goods a transfer has no effect whatsoever on the pre-transfer equilibrium. This result played the role of a conceptual benchmark for many of these papers. I show that this result is no longer valid in models featuring monopolistic competition, trade costs, and heterogeneous firms.

While the literature on the transfer problem is immense, only a few papers adopt models of monopolistic competition. The only papers I have found are Brakman and Marrewijk (1995) and Corsetti, Martin and Pesenti (2013). Brakman and Marrewijk (1995) use a model of monopolistic competition to study the effect of tied aid to developing countries. Their model differs from mine especially in that they use a model without comparative advantage, with homogenous firms, non-identical preferences between countries, home biased expenditure and no-trade costs. Corsetti, Martin and Pesenti (2013) use a model of monopolistic competition and heterogeneous firms to distinguish between the extensive and intensive margin of trade as channels through which a transfer may affect welfare. Their model differs from mine in that it features no comparative advantage, no selection into entry, unbiased heterogeneity, and a partition between exporting and non-exporting firms. Only very recently, new studies for the first time provided empirical evidence on the effect of transfers. Devereux and Smith (2007) provide an empirical study on the effects of Franco-Prussian reparation payments. Lane and Milesi-Ferretti (2004) provide evidence that countries with net external liabilities tend to have depreciated real exchange rates.²

The results of my study contrast sharply with those of the previous literature on the transfer problem. A transfer brings about adjustments in the terms of trade, in the degree of specialization, in the skill premium, and in welfare even in situations where the previous literature found that a transfer is neutral on the equilibrium. All these effects are due, for the most part, to monopolistic competition, comparative advantage and trade costs. When heterogeneity is unbiased there are no additional effects other than those due to monopolistic competition, comparative advantage and trade costs. Instead when heterogeneity is biased there are also effects on average productivity that depend on the relative factor abundance of the donor. Furthermore, while in the previous literature a transfer is neutral on world welfare, in my model it is not. Monopolistic competition and trade costs ensure that a transfer surely reduces world welfare while biased heterogeneity allows a transfer to increase world welfare.

The remainder of the paper is as follows: Sect. 3 lays out the model, Sect. 4 discusses the general equilibrium effects of transfer while Sect. 5 discusses the welfare effects, Sect. 6 concludes and the appendix Sect. 7

²In terms of the model structure a few other papers relate to the present article. In particular Bernard, Redding and Schott (2007), Costinot and Vogel (2010), Burstein and Vogel (2015) and Crozet and Trionfetti (2013) since they all use heterogeneous firms and comparative advantage. Aside from the similarities and differences in the model structure, the most important difference is, of course, in the research objective.

provides mathematical details.

3 The model

The world economy is composed of two countries, indexed by $c = A, B$; in which live two factors, indexed by $j = H, L$; which produce two differentiated goods, indexed by $i = Y, Z$. Each country is endowed with a positive share ν_j^c of the world's endowments denoted \bar{H} and \bar{L} .

Technology. Production requires fixed and variable inputs in each period. The variable input technology takes the CES form here represented by the marginal cost which, for a firm in industry i of country c , is

$$mc_i^c = \{(\lambda_i)^\sigma (w_L^c)^{1-\sigma} [a(\xi)]^{\sigma-1} + (1 - \lambda_i)^\sigma (w_H^c)^{1-\sigma} [b(\xi)]^{\sigma-1}\}^{\frac{1}{1-\sigma}} \quad (1)$$

where $\lambda_i \in (0, 1)$ is a constant technology parameter of industry i , the variables w_H^c and w_L^c denote, respectively, the price of H and L in country c , and $\sigma \neq 1$ measures the elasticity of substitution between factors. The variable ξ is a random variable with cumulative distribution $G(\xi)$ and with support (ξ_0, ∞) where $\xi_0 > 0$. The continuous, positive, increasing, and differentiable functions $a(\xi)$ and $b(\xi)$ contribute to determining factor productivity. Let $\beta(\xi) \equiv b(\xi)/a(\xi)$ be monotonic. I will say that heterogeneity is *H-biased* if $\beta'(\xi) > 0$; is *L-biased* if $\beta'(\xi) < 0$; is *unbiased* if $\beta'(\xi) = 0$.

Firms seeking to enter the market face fixed entry costs. Paying the fixed entry costs gives the right to draw ξ . Upon drawing ξ the firm is able to compute its profit and decides to stay in the market if such profit is non-negative or decides to exit otherwise.³ If the firm decides to stay it remains attached to its value of ξ until death do them part. At any point in time any firm has a probability of death equal to $\bar{\delta}$. Let ξ_i^{*c} be the least value of ξ in industry i of country c such that profit is zero. A firm that decides to produce faces fixed production costs. We may assume that fixed costs are homogenous or heterogenous across firms. These alternative assumptions give qualitatively the same results. I assume homogeneous fixed costs since this assumption allows to be focused on heterogeneity in the production process (which is the heart of the matter). Incidentally, this is the assumption most commonly adopted in the literature (Melitz, 2003; Bernard, Redding and Schott, 2007;

³Given that $G(t)$ and δ are constant over time, it is irrelevant for the equilibrium value of the endogenous variables whether the firm decides to stay on the basis of expected profit or the basis of current profit.

and many others). Specifically, I assume that the fixed input technology is represented by the cost function $\widetilde{mc}_i^c \equiv \left\{ \frac{1}{1-G(\xi_i^{*c})} \int_{\xi_i^{*c}}^{\infty} (mc_i^c)^{1-\sigma} dG \right\}^{\frac{1}{1-\sigma}}$, which is the average marginal cost in the industry and may conveniently be written as

$$\widetilde{mc}_i^c = \left[(\lambda_i)^\sigma (w_L^c)^{1-\sigma} (\widetilde{a}_i^c)^{\sigma-1} + (1-\lambda_i)^\sigma (w_H^c)^{1-\sigma} (\widetilde{b}_i^c)^{\sigma-1} \right]^{\frac{1}{1-\sigma}}. \quad (2)$$

where $\widetilde{a}_i^c \equiv \left(\frac{1}{1-G(\xi_i^{*c})} \int_{\xi_i^{*c}}^{\infty} (a(\xi))^{\sigma-1} dG \right)^{\frac{1}{\sigma-1}}$ and analogously for \widetilde{b}_i^c . Thus, the fixed production cost is $F_i \widetilde{mc}_i^c$ where F_i is a positive constant and the fixed entry cost is $F_{ie} \widetilde{mc}_i^c$ where F_{ie} is a positive constant. This specification represents the fixed input as a homogenous, non-traded, composite good produced in a perfectly competitive market by assembling in a CES all the varieties of the domestic industry output (similarly to Ethier, 1980). But it may also be interpreted as in Yeaple (2005), who assumes that the fixed cost is represented by output that must be produced by the firm and that ultimately cannot be sold; with the difference that in the present model this output requires a unit cost function \widetilde{mc}_i^c .

Demand. The representative consumer has preferences represented by a Cobb-Douglas index, with shares $\gamma_i \in (0, 1)$, $\gamma_Y + \gamma_Z = 1$, defined over CES aggregates whose elasticity of substitution between varieties is $\varsigma > 1$. Gross domestic product is $I^c = w_L^c \nu_L^c \bar{L} + w_H^c \nu_H^c \bar{H}$. Factors of production are taxed for the sole purpose of raising the resources to be transferred. I assume per-capita taxation so as to rule out any direct distributional consequence of the transfer. Thus, national disposable income is $\Delta^A = I^A - T$, and $\Delta^B = I^B + T$, where by convention T is the transfer from A to B . When $T > 0$ country A is the donor and B the recipient, and vice-versa when $T < 0$.

The demand emanating from domestic residents, s_{id}^A , and from foreign residents, s_{ix}^A , for the output of a firm in industry i of country A is:

$$s_{id}^A = \left(\frac{p_{id}^A}{P_i^A} \right)^{1-\varsigma} \gamma_i \Delta^A; \quad s_{ix}^A = \left(\frac{p_{ix}^A}{P_i^B} \right)^{1-\varsigma} \gamma_i \Delta^B, \quad (3)$$

where s stands for sales, d for domestic, and x for foreign; p_{id}^A and p_{ix}^A are the price faced by consumers and P_i^c is the price index. Analogous functions obtain for s_{id}^B and s_{ix}^B . Total firm sales are represented by $s_i^c(\xi) = s_{id}^c(\xi) + s_{ix}^c(\xi)$.

Profit maximization and zero profit. With monopolistic competition and under the large-group assumption, the profit-maximizing prices for the domestic and the foreign market are:

$$p_{id}^c(\xi) = \frac{\varsigma}{\varsigma - 1} mc_i^c(\xi); \quad p_{ix}^c(\xi) = \frac{\varsigma}{\varsigma - 1} \frac{1}{\tau_i} mc_i^c(\xi). \quad (4)$$

The second expression in (4) takes into account iceberg transport costs by which for one unit of good shipped only a fraction $\tau_i \in [0, 1]$ arrives at its destination. The notation $mc_i^c(\xi)$ reminds us that firms with different ξ have different marginal costs; they therefore apply different prices and will have different sales as a result. Indeed, for any two firms with draws ξ' and ξ'' the relative sales are

$$\frac{s_i^c(\xi')}{s_i^c(\xi'')} = \left[\frac{mc_i^c(\xi')}{mc_i^c(\xi'')} \right]^{1-\varsigma}. \quad (5)$$

After drawing ξ a firm decides to stay in the market if $\pi_i^c(\xi) \geq 0$ and decides to quit otherwise. Thus, recalling that the firm's profit may be written as $\pi_i^c(\xi) = s_i^c(\xi) / \varsigma - F_i \widetilde{mc}_i^c$, the zero profit condition is⁴

$$s_i^c(\xi_i^{*c}) = \varsigma F_i \widetilde{mc}_i^c. \quad (6)$$

Aggregation. Applying equations (5) and (6) to $s_i^c(\xi) / s_i^c(\xi_i^{*c})$ gives the sales of any firm as function of the cut off value ξ_i^{*c} ; then by aggregation we obtain average sales in any particular industry and country:

$$\bar{s}_i^c = \left[\frac{\widetilde{mc}_i^c}{mc_i^{*c}} \right]^{1-\varsigma} \varsigma F_i \widetilde{mc}_i^c. \quad (7)$$

And the average profit in industry i of country c is:

$$\bar{\pi}_i^c = \left[\frac{\bar{s}_{id}^c}{\varsigma} - F_i \widetilde{mc}_i^c \right]. \quad (8)$$

Using (4) we may compute the average domestic price, the average export price and the price indices:

$$\widetilde{p}_{id}^c = \frac{\varsigma}{\varsigma - 1} \widetilde{mc}_i^c, \quad \widetilde{p}_{ix}^c = \frac{\varsigma}{\varsigma - 1} \frac{1}{\tau_i} \widetilde{mc}_i^c, \quad (9)$$

$$P_i^A = \left[M_i^A (\widetilde{p}_{id}^A)^{1-\varsigma} + M_i^B (\widetilde{p}_{ix}^A)^{1-\varsigma} \right]^{\frac{1}{1-\varsigma}}, \quad (10)$$

$$P_i^B = \left[M_i^B (\widetilde{p}_{id}^B)^{1-\varsigma} + M_i^A (\widetilde{p}_{ix}^B)^{1-\varsigma} \right]^{\frac{1}{1-\varsigma}}, \quad (11)$$

where M_i^c is the mass of firms.

⁴Since mc_i^c is monotonic in ξ there is one and only one ξ_i^{*c} .

General Equilibrium. In addition to profit-maximizing prices and to the zero profit conditions discussed above, there are four additional sets of equilibrium conditions. First, stationarity of the equilibrium requires the mass of potential entrants, M_{ei}^c , to be such that at any instant the mass of successful entrants, $[1 - G(\xi_i^*)] M_{ei}^c$ equals the mass of incumbent firms who die, $\bar{\delta} M_i^c$:

$$[1 - G(\xi_i^*)] M_{ei}^c = \bar{\delta} M_i^c. \quad (12)$$

Second, free entry ensures that the expected benefit from entry equals the entry cost:

$$[1 - G(\xi_i^{*c})] \bar{\pi}_i^c / \bar{\delta} = F_{ei} \widetilde{m} c_i^c. \quad (13)$$

The left-hand-side is the present value - prior to entry - of the expected profit stream until death; the right-hand-side is the entry cost. Third, we need to ensure goods market equilibrium. Replacing (10)-(11) into (3) gives average demands as functions of average prices, $s_{id}^c(\widetilde{p}_{id}^c)$ and $s_{ix}^c(\widetilde{p}_{ix}^c)$, which allows writing the goods market equilibrium equations as

$$\bar{s}_i^c = s_{id}^c(\widetilde{p}_{id}^c) + s_{ix}^c(\widetilde{p}_{ix}^c). \quad (14)$$

Fourth, equilibrium in factor market requires that factor demand inclusive of all fixed factors inputs, denoted L_i^c and H_i^c , be equal to factor supply

$$L_Y^c + L_Z^c = \nu_L^c \bar{L}. \quad (15)$$

$$H_Y^c + H_Z^c = \nu_H^c \bar{H}. \quad (16)$$

After replacing equations (9) and (10)-(8) into (13)-(16) and remembering that each of these is required to hold for any i and any c we count 11 independent equilibrium conditions and 12 endogenous variables. The equilibrium conditions are the four free-entry conditions (13), any three out of the four goods market equilibrium conditions (14), and the four factor market equilibrium (15)-(16). The endogenous are $\{\xi_i^{*c}\}$, $\{w_L^c, w_H^c\}$ and $\{M_i^c\}$. The equilibrium value of all other endogenous variables can be computed from these. The choice of a numéraire makes the model determined.

4 General Equilibrium Effects

The key elements in the model above are differences in factor proportions, monopolistic competition, and heterogeneous firms. I will study the role played by each of this elements in giving rise to the effects of a transfer. I will begin, however, by discussing the effect of a transfer in the neoclassical version of the factor proportions model of trade since this will be the benchmark from which our model departs.

4.1 The neoclassical benchmark

The neoclassical version of the factor proportions model features homogeneous goods, homogenous firms and perfect competition. With regard to the transfer problem the neoclassical model gives rise to the following result:

Proposition 1 : "*Samuelson neutrality proposition*". *In the neoclassical model of trade with identical Cobb-Douglas preferences a transfer has no impact on any endogenous variable. Therefore, there is no terms of trade effect and there is no secondary burden.*

Proof. See appendix Sect. 7.1. ■

The logic of this result is that the transfer leaves demand for goods unchanged. Thus, relative prices remain unchanged and so do all other endogenous variables. Samuelson obtains this result first in a model without trade costs (Samuelson, 1952) and then in the model with trade costs (Samuelson, 1954).⁵ In both cases he obtains the result in a model with fixed output but argues with crystal clear logic that the same result should obtain when output may vary. The reason is that in neoclassical models no change in output may occur without change in relative prices. Thus, if a transfer leaves relative prices unchanged it also leaves output unchanged even when output may vary. Indeed, this is confirmed formally in Mundell's famous paper on the pure theory of international trade (1960, Sect. IV); however he only proves it for the case of free trade. To complete the picture I prove in Sect. 7.1 that Proposition 1 is valid also in the the case of variable output and costly trade. My fundamental contribution so far (if I am allowed to be facetious) is to dub the result in Proposition 1 the "Samuelson neutrality proposition". This proposition is an excellent starting point for our study because it ensures that any effect we find when using our model will be due exclusively to its features; namely, monopolistic competition and biased heterogeneity.

4.2 The role of monopolistic competition

In this section I study the role of monopolistic competition. I do not need to abandon the assumption of firm heterogeneity but shall do so since I prefer to cover firm heterogeneity separately in the next sub-section. To eliminate firm heterogeneity all I need to do is to assume that $G(\xi)$ is degenerate.

By its very nature a transfer reduces expenditure in the donor country and increases it in the recipient country thereby operating an international

⁵Samuelson (1954), Table IV (p. 286), cell where the row "Basic convention of unitary income elasticity and identical tastes" intersects the column "Real transport costs"; case of elasticity of substitution between goods equal to 1 (Cobb-Douglas).

reallocation of expenditure. Such reallocation may give rise to excesses of demand and supply which are the only channels through which a transfer can affect the initial equilibrium. Indeed, the analysis of the transfer problem may be posed as the study of the changes in endogenous variables required to eliminate the excess demand and supply created by the transfer. As we have seen above, in the neoclassical benchmark the transfer leaves demands unchanged. Instead, when monopolistic competition and trade costs are introduced into the model a transfer gives rise to excess demand and excess supply. This is why I begin my analysis by studying the effect of a transfer on the excess demand and supply for any variety. This can be done by differentiating equation (14) with respect to T only so as to obtain the following expressions for the percentage changes in demand, denoted \widehat{D}_i^c :

$$\widehat{D}_i^A = -\frac{(1-\theta^2) M_i^B (\widetilde{p}_{id}^B)^{1-\varsigma} dT}{\theta M_i^A (\widetilde{p}_{id}^A)^{1-\varsigma} + (\theta^2 \iota^B + \iota^A) M_i^B (\widetilde{p}_{id}^B)^{1-\varsigma}} \leq 0 \Leftrightarrow dT \geq 0, \quad (17)$$

$$\widehat{D}_i^B = \frac{(1-\theta^2) M_i^A (\widetilde{p}_{id}^A)^{1-\varsigma} dT}{\theta M_i^B (\widetilde{p}_{id}^B)^{1-\varsigma} + (\theta^2 \iota^A + \iota^B) M_i^A (\widetilde{p}_{id}^A)^{1-\varsigma}} \geq 0 \Leftrightarrow dT \geq 0, \quad (18)$$

where ι^c is country c 's share in world disposable income. Thus,

Proposition 2 *Excess demand.* *In monopolistic competition with trade costs a transfer creates an excess demand (excess supply) for any variety of both goods produced by the recipient (donor).*

Proof. Expressions (17)-(18). ■

To understand Proposition 2 consider, for instance, a transfer from A to B . The proposition states that the transfer will create an excess demand for any variety produced in B and an excess supply for any variety produced in A . The reason is that - in any equilibrium with trade costs - foreign expenditure on any domestic variety is smaller than domestic expenditure on that same variety because the price paid by foreign residents is higher than the price paid by domestic residents; the price difference being due to trade costs. A transfer gives rise to an increase in total expenditure in B and a decline in A of equal magnitude, but the share of the transfer spent on any A 's variety by residents of B is smaller than the share spent on any A 's variety by residents of A . Thus the transfer creates an excess supply for all A 's varieties and an excess demand for all B 's varieties. In other words, for any symmetric expenditure shock between countries the effect of the domestic shock on domestic varieties dominates the effect of the foreign shock on domestic varieties except in the case of free trade ($\theta = 1$) where

the net effect is zero. We now understand a crucial difference between the neoclassical benchmark and our model with respect to the effects of transfers: in the former the international reallocation of expenditure does not give rise to excess demand and supply while in the latter it does. Lastly, we note that Proposition 2 rests only on monopolistic competition and trade costs; it needs neither comparative advantage nor firm heterogeneity but remains valid when we take these features on board.

We may now study the consequences of excess demand and supply. These excesses are absorbed by changes in the endogenous variables. Let us examine these changes one by one.

First as a consequence of the excess supply and demand the price of all varieties will decline in the donor country and will increase in the recipient country. Thus,

Proposition 3 *Terms of Trade.* *In monopolistic competition with trade cost a transfer gives rise to a deterioration (improvement) of the terms of trade of the donor (recipient).*

Proof. Direct consequence of Proposition 2. ■

Unlike in the neoclassical benchmark, a transfer affects the terms of trade when monopolistic competition and trade costs are in the model. Furthermore, there are additional consequences due to the fact that the excess demand or supply is in general different for varieties of different goods. This can be seen by using (17)-(18) to take ratios $\widehat{D}_Y^c/\widehat{D}_Z^c$ and observing that

$$\text{If } T > 0, \text{ then } \frac{M_Y^A (\widetilde{p}_{Zd}^B)^{1-\varsigma}}{M_Z^A (\widetilde{p}_{Yd}^B)^{1-\varsigma}} \geq \frac{M_Y^B (\widetilde{p}_{Zd}^A)^{1-\varsigma}}{M_Z^B (\widetilde{p}_{Yd}^A)^{1-\varsigma}} \Rightarrow \begin{cases} \frac{\widehat{D}_Y^A}{\widehat{D}_Z^A} \leq 1 \\ \frac{\widehat{D}_Y^B}{\widehat{D}_Z^B} \geq 1 \end{cases} \quad (19)$$

$$\text{If } T < 0, \text{ then } \frac{M_Y^B (\widetilde{p}_{Zd}^A)^{1-\varsigma}}{M_Z^B (\widetilde{p}_{Yd}^A)^{1-\varsigma}} \geq \frac{M_Y^A (\widetilde{p}_{Zd}^B)^{1-\varsigma}}{M_Z^A (\widetilde{p}_{Yd}^B)^{1-\varsigma}} \Rightarrow \begin{cases} \frac{\widehat{D}_Y^A}{\widehat{D}_Z^A} \geq 1 \\ \frac{\widehat{D}_Y^B}{\widehat{D}_Z^B} \leq 1 \end{cases} \quad (20)$$

Thus, for instance, the inequalities and implications in (19) tell us that a transfer from A to B (i.e. $T > 0$) when A has the comparative advantage in Y (as a consequence of which the second inequality would hold with $>$) causes a smaller excess supply for Y than for Z in A (i.e., $\widehat{D}_Y^A < 0$ and $\widehat{D}_Z^A < 0$ but $\widehat{D}_Y^A/\widehat{D}_Z^A < 1$) and a bigger excess demand for Y than for Z in B (i.e., $\widehat{D}_Y^B > 0$ and $\widehat{D}_Z^B > 0$ but $\widehat{D}_Y^B/\widehat{D}_Z^B > 1$). Analogously, a transfer from B to A ($T < 0$) when B has the comparative advantage in Y causes $\widehat{D}_Y^A > 0$ and $\widehat{D}_Z^A > 0$ but $\widehat{D}_Y^A/\widehat{D}_Z^A > 1$ and $\widehat{D}_Y^B < 0$ and $\widehat{D}_Z^B < 0$ but $\widehat{D}_Y^B/\widehat{D}_Z^B < 1$. All the other cases may be read analogously. This leads to the following proposition.

Proposition 4 *Price of goods, price of factors, and specialization.*

In monopolistic competition, trade costs, and comparative advantage a transfer causes in both countries an increase of the relative demand for the good in which the donor country has the comparative advantage. This has three consequences:

1. *An increase in both countries in the relative price of the good in which the donor country has the comparative advantage.*
2. *An increase in both countries in the relative price of the factor intensively used in the good in which the donor country has the comparative advantage.*
3. *An increase in both countries in the relative mass of varieties of the good in which the donor country has the comparative advantage.*

Proof. Direct consequence of (19)-(20). ■

The second consequence is directly implied by the first through the Stolper-Samuelson relationship between goods and factor price. Thus, a transfer not only changes disposable income but also gives rise to changes in the relative income of different factors. For instance, a transfer from an *H-abundant* country increases the skill premium in all countries thereby creating more inequality.

4.3 The role of firm heterogeneity

When firms are heterogeneous and entry is endogenous a transfer may affect productivity but, as we shall see in the next two propositions, the effects on productivity arise only when heterogeneity is biased. To see this it is convenient to use (7) and (8) to write the free entry condition (13) as follows:

$$\Upsilon_i^c \equiv \int_{t_i^*}^{\infty} \left\{ \left[\frac{mc_i^c}{mc_i^{*c}} \right]^{1-\varsigma} - 1 \right\} g(\xi) d\xi = \frac{\tilde{\partial} F_{ei}}{F_i}. \quad (21)$$

Proposition 5 *When heterogeneity is unbiased a transfer has no impact on productivity.*

Proof. If heterogeneity is unbiased, i.e., $a(\xi) = b(\xi)$, then the marginal cost ratio mc_i/mc_i^* , which reduces to $mc_i/mc_i^* = a(\xi)/a(\xi_i^*) = b(\xi)/b(\xi_i^*)$. Therefore, equation (21) determines ξ_i^* independently of the rest of the model and, in particular, independently of T . ■

In other words, when heterogeneity is unbiased there is a dichotomy between the free entry conditions and the other equilibrium conditions of the model. This implies that a transfer, though it impacts goods prices, factor prices, and masses, has no impact on productivity. This is readily understood if we note that a model with unbiased heterogeneity is - on average - identical to a model with homogenous firms.

When heterogeneity is biased, instead, factor prices do not cancel out from the marginal cost ratio mc_i/mc_i^* and the free entry condition (21) relates ξ_i^* and factor prices in the following way (see Sect. 7.2 for the mathematical passages):

$$\frac{d\widehat{\xi_i^{*c}}}{d\widehat{\omega^c}} \begin{cases} \leq 0 \\ > 0 \end{cases} \text{ as } \beta'(\xi) \begin{cases} \geq 0 \\ < 0 \end{cases} \forall \sigma \neq 1 \quad (22)$$

where $\omega^c \equiv w_H^c/w_L^c$ and $\widehat{}$ represent percentage changes. Expression (22) shows the channel through which a transfer affects productivity: a transfer affects the relative factor price, ω^c , - recall Proposition 4 - and the change in ω^c affects productivity. We may state the effect of a transfer on productivity in the following way:

Proposition 6 *In monopolistic competition, trade costs, comparative advantage, and biased heterogeneity, a transfer causes a decline (increase) in average productivity of both industries in both countries if the donor is abundant (scarce) in the factor towards which heterogeneity is biased.*

Proof. Expression (22) ■

It is interesting to understand the economic logic of expression (22). Consider, for instance a transfer from A to B when A is H -abundant, which gives rise to an increase in the skill premium in both countries (i.e., an increase ω^c). Since firms are heterogeneous in skill intensity they are affected differently even when they face the same increase in the skill premium. Specifically, highly skill intensive firms will lose competitiveness with respect to the least skill intensive firms since the former use more intensively the factor whose relative price has increased. Now, if heterogeneity is skill-biased ($\beta'(t) > 0$) then the least skill intensive firms will also be the least productive firms. Their relative position improves and some previously unprofitable firms will become profitable and decide to stay in the market (i.e. ξ_i^{*c} declines).

4.4 The role of comparative advantage

The importance of comparative advantage is best assessed by removing it from the model while keeping monopolistic competition and firm heterogeneity, be it biased or not.

Proposition 7 *In the absence of comparative advantage a transfer is neutral on specialization, productivity, and the skill premium.*

Proof. Direct consequence of (19)-(20). ■

The reason for Proposition 7 is simply that in the absence of a comparative advantage $M_Y^A/M_Z^A = M_Y^B/M_Z^B$ and $\tilde{p}_{Yd}^A/\tilde{p}_{Zd}^A = \tilde{p}_{Yd}^B/\tilde{p}_{Zd}^B$ therefore the excess demand is identical for all varieties of all goods in the same country as we see by inspection of (19)-(20). Naturally, the effect of a transfer on the terms of trade stated in Proposition 3 remains.

It is useful to devote a few words to summarise what we have learnt in Sect 4. A transfer brings about an excess demand for any variety produced by the recipient and an excess supply for any variety produced by the donor. The excess demand and supply are of different magnitudes for varieties of different goods except in the absence of a comparative advantage. They are absorbed by changes in goods prices, factor prices and masses of varieties; furthermore, if and only if heterogeneity is biased, they are absorbed by changes in productivity. These effects require monopolistic competition and trade costs. In the absence of one or the other, the results vanish. Thus, only if one brings in monopolistic competition *and* trade costs do substantive effects arise.⁶ These general equilibrium effects give rise to the welfare and distributive effects that we study in the next section.

5 Secondary Burden

Does a transfer give rise to a secondary burden? Are there any effects on world welfare? Are these effects big or small? Answering this question analytically presents two limitations. The first is that one would be bound to use comparative statics techniques and therefore to obtain only local results. The second is that the system contains eleven equations and eleven endogenous and comparative statics proved to be too intricate to be intelligible. I therefore prefer to show the results obtained through systematic numerical solutions of the model. To better isolate the effect of a transfer I make the model symmetric by setting $\lambda_Y = (1 - \lambda_Z)$ and $\bar{H} = \bar{L}$; and by changing the factor proportion in a symmetric way by choosing $\nu_j^A = (1 - \nu_j^B)$. For the simulations, I set $a(t) = 1$ and $b(t) = t$ and choose the Pareto probability distribution with shape parameter κ . I take parameter values from

⁶Is what Samuelson (1954) actually meant by the sentence quoted above? He wrote it in his 1954 paper dedicated to the transfer problem and trade costs but he did it in Sect. XII, which is dedicated to various model extensions and where he does not mention trade costs.

the literature when available and choose arbitrarily the values to assign to world endowments and to fixed costs since results are practically insensitive to these values.⁷ I do not worry about calibration since the objective is not to replicate any real world situation, but rather to gain a sense of how different the results are when we move from the neoclassical model to models containing monopolistic competition and heterogeneous firms.

Secondary Burden by Country. I compute numerically the secondary burden as a percentage of the primary burden. For greater clarity, let W^c be the utilitarian aggregate welfare in country c gross of the transfer:

$$W^c = I^c (P_Y^c)^{-\gamma_Y} (P_Z^c)^{-\gamma_Z} \quad (23)$$

and let $W^c|_{T=0}$ and $W^c|_{T \neq 0}$ be, respectively, the value of W^c calculated at the equilibrium without and with transfer. The primary burden is the welfare value of the transfer, i.e., $T / (P_Y^A)^{-\gamma_Y} (P_Z^A)^{-\gamma_Z}$ when A is the donor and $-T / (P_Y^B)^{-\gamma_Y} (P_Z^B)^{-\gamma_Z}$ when B is the donor. The secondary effect of a transfer is $(W^c|_{T \neq 0} - W^c|_{T=0})$ and is a burden when negative and a benefit when positive. The secondary burden in percentage of the primary is:

$$b_I^{II,A} = \frac{W^A|_{T \neq 0} - W^A|_{T=0}}{\left[T / (P_Y^A)^{-\gamma_Y} (P_Z^A)^{-\gamma_Z} \right]} \quad (24)$$

$$b_I^{II,B} = \frac{W^B|_{T \neq 0} - W^B|_{T=0}}{\left[T / (P_Y^B)^{-\gamma_Y} (P_Z^B)^{-\gamma_Z} \right]} \quad (25)$$

Table 1 reports the values of $b_I^{II,c}$. The results are representative of many systematic simulations. The values of $b_I^{II,c}$ are reported in three different columns representing - from left to right - increasing differences in factor proportions. Contrary to the neoclassical benchmark, the donor country bears a secondary burden and the recipient enjoys a secondary benefit. It is easy to understand the reason for these welfare changes. In the donor country all prices and wages fall but part of expenditure goes to foreign varieties whose increase in price causes a reduction in overall purchasing power of donor country residents. Likewise, *mutatis mutandi*, for welfare in

⁷Bernard et al. (2003) estimate the Pareto shape parameter to 3.7. The empirical literature on the gravity equation finds the elasticity of substitution to range between 2 and 6 (see, for instance, Bernard et al. 2003 or Head and Mayer, 2006). Thus, I run simulations adpting values in this range. I take trade costs to be 25% of value shipped (i.e., $\tau = 0.8$). Other parameters: $\bar{H} = \bar{L} = 1000$, $F = F_e = 2$, $\bar{\delta} = 0.025$.

Table 1: Country Welfare

Secondary Burden in Percentage of Primary Burden			
	→ <i>Increasing Comparative Advantage</i> →		
Country ↓	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = 1$	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = \frac{0.6}{0.4}$	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = \frac{0.7}{0.3}$
<i>A</i> 's sec. burden	6.25	6.29	6.24
<i>B</i> 's sec. benefit	6.41	6.65	6.76

Transfer from *A* to *B* equal to 10% of *A*'s GDP
The neoclassical benchmark predicts no secondary burden.

the recipient country. The percentages turned out not to be very sensitive to changes in factor proportions: all simulations (including those not shown in the table) gave values between five and seven per cent. Simulations with unbiased heterogeneity gave the same absolute values of the secondary burden relative to primary between countries, that is, $b_I^{II,A} = -b_I^{II,B}$.⁸ Instead, as we observe in Table 1, when heterogeneity is biased then $b_I^{II,A} \neq -b_I^{II,B}$. This is due to the productivity effects hitting both countries in the same direction but with different strengths. Lastly, not shown in the table, $b_I^{II,c}$ is almost insensitive to the size of the primary burden while it increases with σ and κ .

Secondary Burden by Factor. All factors bear a secondary burden in the donor country and enjoy a secondary benefit in the recipient country but these secondary effects do not impact factors equiproportionally. Table 2 shows the secondary effect by factor. We observe differences not only in the level but also in the relationship between the secondary effect and factor proportions. We see in Table 2 that as factor proportions diverge, the secondary effect becomes greater for the scarce factor and smaller for the abundant factor.

Skill Premia. Table 3 shows the effects of a transfer from *A* to *B* on inequality as measured by the skill premium. The neoclassical model predicts no change in inequality. The model with monopolistic competition and heterogeneous firms predicts an asymmetric increase in inequality where inequality increases more in the donor country than in the recipient.

World welfare. One other major difference between the neoclassical benchmark and the monopolistic competition models is that in the former

⁸Simulations with unbiased heterogeneity gave values of 6.25, 6.14, and 5.79 respectively for columns two, three and four.

Table 2: Factor Welfare

Secondary Burden in Percentage of Primary Burden				
Factor ↓	→ <i>Increasing Comparative Advantage</i> →			
	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = 1$	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = \frac{0.6}{0.4}$	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = \frac{0.7}{0.3}$	
L^A	5.44	6.20	7.13	
H^A sec. burden	7.06	6.35	5.86	
L^B	5.58	5.10	4.75	
H^B sec. benefit	7.24	8.07	9.10	

Transfer from A to B . Transfer equal to 10% of GDP
The neoclassical benchmark predicts no secondary burden.

Table 3: Inequality

Changes in the skill premium				
Country ↓	→ <i>Increasing Comparative Advantage</i> →			
	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = 1$	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = \frac{0.6}{0.4}$	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = \frac{0.7}{0.3}$	
$\hat{\omega}^A$	0	0.15	0.30	
$\hat{\omega}^B$	0	0.13	0.26	

Transfer from A to B equal to 10% of A 's GDP
The neoclassical benchmark predicts no secondary burden.

Table 4: World Welfare

Percentage Changes			
	\rightarrow <i>Increasing Comparative Advantage</i> \rightarrow		
Direction \downarrow	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = 1$	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = \frac{0.6}{0.4}$	$\frac{\nu_H^A}{\nu_L^A} = \frac{\nu_L^B}{\nu_H^B} = \frac{0.7}{0.3}$
$A \rightarrow B$	-0.0104	-0.0194	-0.0271
$B \rightarrow A$	-0.0088	+0.0017	+0.0125

Transfer equal to 10% of GDP of the donor.
The neoclassical benchmark predicts no changes in world welfare.

the transfer is neutral on world welfare while in the latter it is not. A transfer impacts world welfare through misallocation of resources and productivity changes. The misallocation of resources constitutes a welfare loss. Productivity changes may be positive or negative as we have seen in Proposition 6. The net effect may thus be positive or negative. If heterogeneity is unbiased there are no productivity changes, thus the only effect on world welfare is the negative effect arising from misallocation of resources; thus, a transfer gives rise to a world welfare loss.⁹ When heterogeneity is biased instead, the effect of productivity changes may override the effect of misallocation of resources and give rise to a welfare gain for the world. Table 4 reports the percentage welfare changes for the world economy due to a transfer whose size is ten percent of donor GDP.

We observe that a transfer from A to B gives rise to a world welfare loss since the misallocation of resources and the productivity effect both contribute to reducing welfare. Conversely a transfer from B to A may give rise to a welfare gain for the world economy since productivity effects counter the misallocation of resources. Lastly, not shown in the table, the magnitudes of welfare gains are largely sensitive to the size of the primary burden. Typically, doubling the primary burden almost doubles the welfare change.

6 Conclusion

This paper revisited the transfer problem in the light of models featuring monopolistic competition and biased heterogeneity. We found results that are very different from those in the previous literature. First, a transfer

⁹Computations for the case of unbiased heterogeneity gave welfare losses between 0.0068 and 0.0095 per cent of pre-transfer welfare.

creates general equilibrium effects giving rise to a secondary burden even in situations in which the benchmark model does not. Second, a transfer has consequences on inequality and world welfare never studied before.

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7 Appendix

7.1 Transfers in the neoclassical benchmark

To obtain the neoclassical model from ours let Y and Z be homogeneous goods and let firm be homogeneous $\beta(\xi) = 1$ for any ξ . To fix ideas, and without loss of generality, assume that factor intensities and factor proportions are such that country A is the exporter of Y . To avoid notational confusion let \mathbf{p}_{ic} be the price of good i in country c in the neoclassical model. Transport costs and the export pattern imply the following price relationships:

$$\mathbf{p}_{YB} = \frac{1}{\tau} \mathbf{p}_{YA}, \quad \mathbf{p}_{ZA} = \frac{1}{\tau} \mathbf{p}_{ZB}, \quad (26)$$

The general equilibrium system is composed of the following nine equations

$$\frac{(Y_A + Y_B/\tau) \mathbf{p}_{YA}}{(Y_A + \frac{Y_B}{\tau}) \mathbf{p}_{YA} + (\frac{Z_A}{\tau} + Y_B) \mathbf{p}_{ZB}} = \frac{\gamma_Y (I_A - T) - \gamma_Y (I_B + T)}{I_A + I_B} \quad (27)$$

$$\mathbf{p}_{ic} = mc_i^c, \quad i = Y, Z; \quad c = A, B. \quad (28)$$

$$\frac{\partial mc_Y^c}{\partial w_L^c} Y_c + \frac{\partial mc_Z^c}{\partial w_L^c} Z_c = \nu_L^c \bar{L}, \quad c = A, B. \quad (29)$$

$$\frac{\partial mc_Y^c}{\partial w_H^c} Y_c + \frac{\partial mc_Z^c}{\partial w_H^c} Z_c = \nu_H^c \bar{H}, \quad c = A, B. \quad (30)$$

Equation (27) ensures goods market clearing, equations (28) results from profit maximization, equations (29)-(30) ensure equilibrium in factor market. This system and the choice of a numéraire determine the ten endogenous variables: four factor prices $\{w_j^c\}$, two commodity prices $\{\mathbf{p}_{YA}, \mathbf{p}_{ZB}\}$, and four output quantities $\{Y_c, Z_c\}$. The important thing to notice is that the two T representing the transfer cancel each other out, see equation (27). Thus, the transfer has no impact on any endogenous variable. The crucial assumptions for this result are identical preferences and unitary elasticity of substitution between goods. This is the Samuelson neutrality proposition and our benchmark.

7.2 Mathematical passages for inequality 22

Total differentiation of the free entry condition (21) gives

$$\begin{aligned}
0 = & \underbrace{\left\{ (\sigma - 1) (\Lambda_i)^\sigma \omega^{1-\sigma} \int_{t_i^*}^{\infty} \frac{(b_i/b_i^*)^{\sigma-1} - (\alpha_i/a_i^*)^{\sigma-1}}{[\omega^{1-\sigma}/b_i^{*\sigma-1} + \Lambda_i^\sigma/a_i^{*\sigma-1}]^2} dG \right\}}_{\Upsilon_{i,\omega}^c} \hat{\omega} + \\
& \underbrace{\left\{ -(\sigma - 1) \left(\frac{a_i^{*\sigma-1}}{\omega^{\sigma-1}} \varepsilon_{\alpha_i^*} + \Lambda_i^\sigma b_i^{*\sigma-1} \varepsilon_{\beta_i^*} \right) \int_{t_i^*}^{\infty} \frac{\frac{a_i^{\sigma-1}}{\omega^{\sigma-1}} + \Lambda_i^\sigma b_i^{\sigma-1}}{[\frac{a_i^{*\sigma-1}}{\omega^{\sigma-1}} + \Lambda_i^\sigma b_i^{*\sigma-1}]^2} dG \right\}}_{\Upsilon_{i,t_i^*}^c} \hat{\xi}_i^*
\end{aligned}$$

where $\varepsilon_{\alpha_i} \equiv \alpha'_i(\xi) \xi / \alpha_i(\xi)$ and $\varepsilon_{\beta_i} \equiv \beta'_i(\xi) \xi / \beta_i(\xi)$. The signs of $\Upsilon_{i,t_i^*}^c$ and $\Upsilon_{i,\omega}^c$ are

$$\Upsilon_{i,\omega}^c \gtrless 0 \text{ as } \beta'(\xi) \gtrless 0 \quad \forall \sigma \neq 1 \quad (31)$$

$$\Upsilon_{i,t_i^*}^c \gtrless 0 \text{ as } \sigma \gtrless 1. \quad (32)$$

From which (22) follows directly.