Central Bank of Armenia

Working Paper //

MIGRATION IN THE ESTIMATED NEW KEYNESIAN DSGE MODEL FOR ARMENIA AND RUSSIA

Haykaz Igityan

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WP//

CBA Working Paper

Monetary Policy Department

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Abstract

This paper develops DSGE model for Armenia and Russia allowing the Armenian household to supply labor both domestically and abroad. Paper models migration cost arising in a reallocation process of labor, which is absent in a basic New Keynesian framework. Russian migration policy is also taken into account during the construction of the model. The model is estimated by Bayesian technique using 22 Armenian and Russian time series and 22 structural shocks. The estimation shows that the model reproduces Armenian and Russian data. Estimated structural shocks are supported by stylized facts. The model is able to generate real remittances data and Russian migration policy. Historical simulations show that the estimated model predicts quite well the observed movements of the business cycle. The paper also discusses the transmission mechanism of different shocks on migration. The model is used in a forecasting and policy analysis system of the Central Bank of Armenia as a satellite model.

JEL classification: E12, E24, E27, F22, F24.

Keywords: New Keynesian DSGE model, labor migration, remittances, Bayesian estimation.

Author's email address: haykaz.igityan@cba.am

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1. Introduction

Dynamic stochastic general equilibrium (DSGE) models have become a powerful tool for macroeconomic analysis. These models are derived from microeconomic foundations and usually with an ad-hoc simple monetary rule. Such models also include nominal and real frictions to capture properties of data, for example, habit in consumption, investment adjustment costs, capital utilization, stickiness of prices and wages. DSGE models are used for analyzing business cycle fluctuations and effects of monetary policy.

One of the main features of Armenian labor market is the migration of labor force. Migrants send a huge amount of their earnings to Armenia on a regular basis, which is a significant part of their families' income. Russia is considered to be the main destination country for Armenian working migrants. The choice of Russia is associated with close mentality between Armenian and Russian peoples, a visa-free regime, the absence of language barriers etc. Despite the above-mentioned priorities, Russian migration policy plays a significant role in migration dynamics, which is considered in this paper.

Most policy analysis models have an assumption about perfect labor mobility, which limits the role of mobility on macro dynamics and monetary policy. Walsh (2011) discusses a lot of literature associated with this problem and concludes, that some costs are raised in a reallocation process of resources. Then he discusses many methods to solve this problem, one of which is quadratic costs of adjusting the employment in a way suggested by Rotemberg (1983). Literature is limited on focusing how to include migration in DSGE models and how to model the so-called migration costs. Spange and Yates (2008) construct two-country DSGE model and include migration cost in a utility function, which is a quadratic cost of migration change. Engler (2008) develops small open economy DSGE model with labor migration and includes the disutility parameter in a utility function of household, indicating his or her relative preference for working at home rather than working abroad. Hauser (2012) discusses this problem in the same manner but in a two-

country DSGE set-up. Mandelman and Zlate (2008) discuss two-country DSGE model for USA and Mexico, considering the increase of migration costs with arrests at the border. Del Rosario (2010) models migration costs as a constant on migrant's wage in a small open economy DSGE model.

This paper models migration costs as a ratio of Armenian and Russian wages, times the constant. The first part is a constraint over the decision of household members to migrant or stay in Armenia. The coefficient is calibrated in a way that total migration costs are count to be 0.8% of GDP in steady state. There is also Russian migration policy in the model, which affects the migration decision of Armenian households.

In terms of migration, Russia is the external sector for Armenia. So, this paper considers Armenia as a small open economy, the external sector for which is only Russia. The Armenian economy is linked to Russian economy by trade, international risk sharing condition, interest rate parity and migration. The exchange rate of Armenian national currency in the model is the exchange rate of Armenian dram against Russian ruble. Armenian economy is very small compared to the Russian economy, which means that structural shocks of the Armenian economy do not hit the Russian economy or their impacts are very small. On the other hand, there is exogenously determined external sector for Russia.

The developed model is estimated by Bayesian method. Twenty two time series are used, which represent Armenian and Russian economies. The model contains twenty two shocks to provide the stochastic singularity. Estimation gives plausible values of the structural parameters of Armenian and Russian economies. Prices in Armenia turn out to be a little bit sticky than in Russia. The average duration of wage contracts in Armenia is estimated to 4.8 quarters while in Russia it is 2.6 quarters. The posterior mean of the coefficient describing the relative disutility of working as a migrant is lower than one, which rejects the prior belief about the home biasedness of the Armenian households. The Frisch elasticity of labor supply of the working migrants in Russia is estimated to relatively low

value, which reflects the fact that migration is more volatile along the extensive than the intensive margin. On the other side, employment in Armenia is more volatile along the intensive margin.

The estimated model is able to reproduce the dynamics of the observed variables. Moreover, without inputting the remittances data, the model generates the time series of real remittances, which is very close to HP filtered observed data. The estimated political factor of the model is linked to Russian migration legislation. Using a Kalman filter, unobservable shocks of the Armenian and Russian economies are estimated, which shows that the main stylized features of the Armenian and Russian economies are well captured by the model.

The paper then discusses the exercise to test the model's ability to forecast the main variables of the model. As the exercise shows, the model predictions of the migration, real wages in Armenian and Russia are very close to the observed business cycle movements up to twelve quarters. The paper analyzes also the historical contribution of shocks to model variables based on the estimation of posterior means. As it turned out, the decline of migration in 2009 is mostly contributed by negative productivity shock in Armenia, positive risk premium shock in Russia and the decline of work permissions in Russia. On the other hand, the recent decrease of migration is supported by negative consumption preference or the Smets-Wouters internal risk premium shock in Armenia, negative wage mark-up shock in Russia, positive risk premium shock in Russia and the sanctions against Armenian working migrants in Russia. Next, the paper discusses the transmission mechanisms of shocks and investigate how model variables react to different shocks. It is found out that one of the main channels through which shocks impact on migration is the real exchange rate of Armenian dram against Russian ruble.

The model developed in this paper is used by the forecasting team of Central Bank of Armenia as a satellite model. It gets as an input the three-year forecast of the main macroeconomic variables from the Core model (QPM) and the Model of external sector

and gives the story of Armenian and Russian labor market developments. Moreover, the real remittances gap's medium term forecast goes back to QPM.

This paper relates to several studies. Heterogeneity of labor in the utility function of Armenian household is introduced following Blundell et al (2001). Relative disutility parameter of the migrant is included in the model in a way discussed in Engler (2008) and Hauser (2012). Russian migration policy is captured in the budget constraint of Armenian households in a way suggested by Mandelman and Zlate (2008). Following Calvo (1983), prices of domestically produced goods are sticky both in Armenian and Russia. The low pass-through of the exchange rate to import prices is introduced following Monacelli (2003). Staggered wages are modeled in a way suggested by Gali (2008). Small open economy issues are discussed following Gali and Monacelli (2005). Completeness of financial assets is assumed by following Schmitt-Grohe and Uribe (2003). As the model developed in this paper is estimated by Bayesian method, it also relates to studies in this field such as An and Schorfheide (2006), Canova (2007), Smets and Wouters (2007).

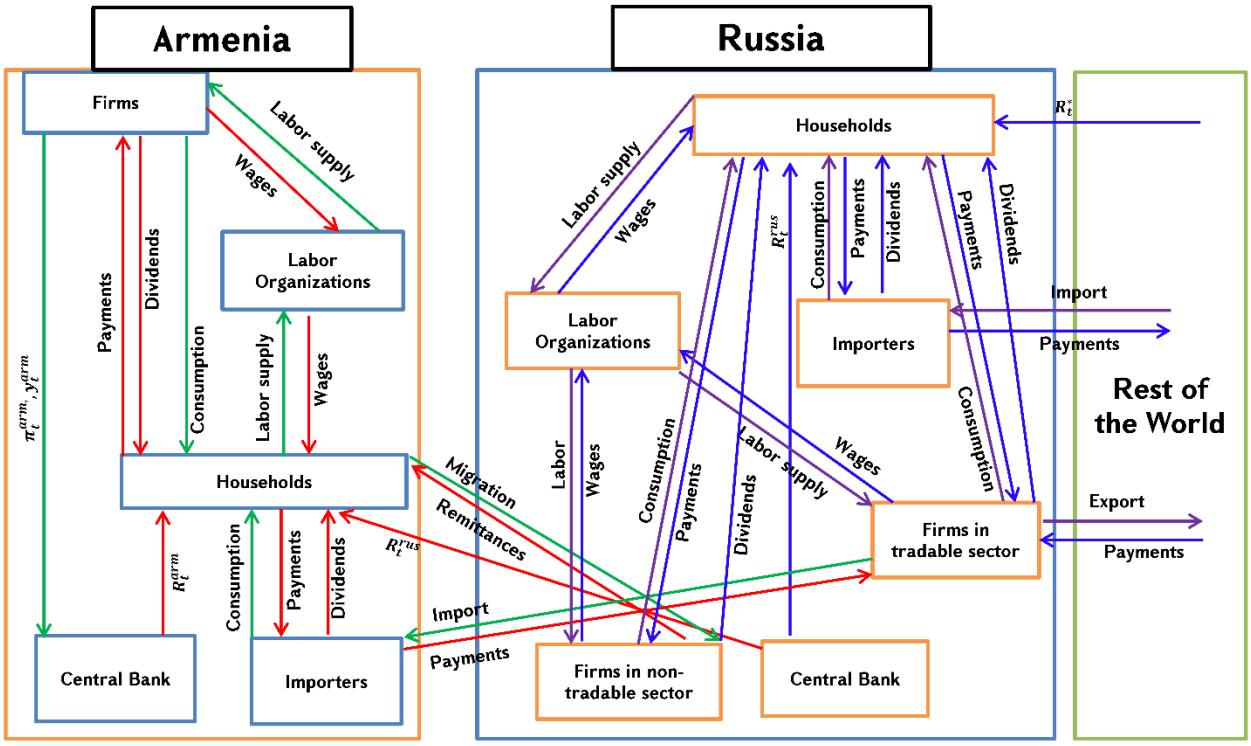
There have been already developed DSGE models for Armenian economy: Stepanyan and Tevosyan (2008), Mkrtchyan et al (2009) and Barseghyan (2013).

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 describes data used in the estimation process. Section 4 presents the results of the estimation. Section 5 discusses the main properties and abilities of the model. Section 6 explains how the model is used in forecasting and policy analysis system of the Central Bank of Armenia. Section 7 concludes.

2. The Model

This paper discusses Armenian labor force migration issues. The destination country for 90% of cases is Russian Federation. So, the external world for Armenia in terms of migration is Russia, which is very big for Armenia. Each period Armenian household decides over working in the home or migrate to Russia. Migrants work in construction and service (non-tradable) sectors. A big part of their earnings they send back to home as remittances. There are two types of firms in Armenia: producers of home goods and importers, which import goods from Russia. There are discussed labor organizations in the model, which is a modeling technique to show incompleteness and wage rigidities in a labor market. Central Bank operates under inflation targeting regime. The Russian economy is divided into tradable and non-tradable sectors. There is an exogenously determined external sector for Russia. Foreign shocks impact Armenian economy through Russia. The schematic representation of the model is captured in picture 1.

Picture 1. Model Environment



2.1. Armenia Economy

2.1.1. Households

Consumption basket of the Armenian representative household is expressed by the following index:

$$C_t^{arm} = \left[(1-\gamma_1)^{\frac{1}{\eta_1}} C_{H,t}^{arm\frac{\eta_1-1}{\eta_1}} + \gamma_1^{\frac{1}{\eta_1}} C_{F,t}^{arm\frac{\eta_1-1}{\eta_1}}\right]^{\frac{\eta_1}{\eta_1-1}} \quad (1)$$

where $C_{H,t}^{arm}$ is the consumption of home produced goods, $C_{F,t}^{arm}$ is the consumption of imported goods from Russia, γ_1 is the share of imported goods in Armenian household consumption or the degree of openness and η_1 is the elasticity of substitution between these two goods. Aggregate consumption indexes of home produced $(C_{H,t}^{arm})$ and imported $(C_{F,t}^{arm})$ goods are represented by Dixit-Stiglitz constant elasticity of substitution functions:

$$C_{H,t}^{arm} = \left[\int_0^1 C(j)_{H,t}^{arm\frac{\varepsilon_1-1}{\varepsilon_1}} dj\right]^{\frac{\varepsilon_1}{\varepsilon_1-1}} \quad (2)$$

$$C_{F,t}^{arm} = \left[\int_0^1 C(j)_{F,t}^{arm\frac{\varepsilon_1-1}{\varepsilon_1}} dj\right]^{\frac{\varepsilon_1}{\varepsilon_1-1}} \quad (3)$$

where ε_1 is the elasticity of substitution between varieties of goods and $j \in [0,1]$ denotes the continuum of goods. Household tries to minimize expenditures of each type of domestic $\int_0^1 P(j)_{H,t}^{arm} C(j)_{H,t}^{arm}\, dj$ and foreign $\int_0^1 P(j)_{F,t}^{arm} C(j)_{F,t}^{arm}\, dj$ goods, where $P(j)_{H,t}^{arm}$ and $P(j)_{F,t}^{arm}$ are respectively prices for j-th home produced and imported goods. The optimal allocation of expenditures within each category of goods brings to the demand functions of the form:

$$C(j)_{H,t}^{arm} = \left(\frac{P(j)_{H,t}^{arm}}{P_{H,t}^{arm}}\right)^{-\varepsilon_1} C_{H,t}^{arm} \quad (4)$$

$$C(j)_{F,t}^{arm} = \left(\frac{P(j)_{F,t}^{arm}}{P_{F,t}^{arm}}\right)^{-\varepsilon_1} C_{F,t}^{arm} \quad (5)$$

Composite price indexes are defined as follows:

$$P_{H,t}^{arm} = \left[\int_0^1 P(j)_{H,t}^{arm\,1-\varepsilon_1} dj\right]^{\frac{1}{1-\varepsilon_1}} \quad (6)$$

$$P_{F,t}^{arm} = \left[\int_0^1 P(j)_{F,t}^{arm\,1-\varepsilon_1}\, dj\right]^{\frac{1}{1-\varepsilon_1}} \tag{7}$$

Finally, households try to minimize expenditures of each type of consumption goods. The cost minimization problem is represented by:

$$\min\{P_t^{arm} C_t^{arm} - P_{H,t}^{arm} C_{H,t}^{arm} - P_{F,t}^{arm} C_{F,t}^{arm}\} \tag{8}$$

where P_t^{arm} is the consumer price index in Armenia. Solution of the above optimization problem gives demand functions for these two types of goods:

$$C_{H,t}^{arm} = (1-\gamma_1)\left(\frac{P_{H,t}^{arm}}{P_t^{arm}}\right)^{-\eta_1} C_t^{arm} \tag{9}$$

$$C_{F,t}^{arm} = \gamma_1\left(\frac{P_{F,t}^{arm}}{P_t^{arm}}\right)^{-\eta_1} C_t^{arm} \tag{10}$$

After permutation (9) and (10) in (8), consumer price index gets the form:

$$P_t^{arm} = \left[(1-\gamma_1)P_{H,t}^{arm\,(1-\eta_1)} + \gamma_1 P_{F,t}^{arm\,(1-\eta_1)}\right]^{\frac{1}{1-\eta_1}} \tag{11}$$

A typical household in the home country every period decides over working in Armenia (N_t^{arm}) or becomes a migrant (N_t^m) and works in Russian non-tradable sector. Thus, the utility function gets the following form:

$$U_{t+j} = E_t \sum_{j=0}^{\infty} \beta^j \left(\log\left(C_{t+j}^{arm}\right) - \chi_{arm}\frac{N_{t+j}^{arm\,(1+\varphi_{arm})}}{1+\varphi_{arm}} - \chi_m \frac{N_{t+j}^{m\,(1+\varphi_m)}}{1+\varphi_m}\right) \tag{12}$$

where E_t denotes the expectation operator conditional on information available at time t, β is the discount rate, φ_{arm} is the inverse of the Frisch elasticity of labor supply for employed in Armenia and φ_m is the same coefficient for migrant, $\frac{\chi_m}{\chi_{arm}}$ represents the relative preference working as a migrant over staying in the home country. χ_{arm} is calibrated to 1 and the issue comes to the estimating the coefficient χ_m. Actually, χ_m could be bigger or less than one. In a first case, the household member has a relative preference to stay and work in Armenia. In the second one, he or she prefers to migrate and work in Russia.

According to a lot of surveys among Armenian households, which includes also the report of UN mission in Armenia (2014), two most significant factors, affecting working migration, are economic and political. The first includes an economic situation in both countries and the second one represents legislation system of destination country against working migrants. Two these factors are taken into account during a construction of this model. The basic New Keynesian framework that is the heart of most DSGE policy models has assumptions about the ability of labor to move among alternative uses extremely. In a migration process, costs arise because resources are not fully mobile. The maximization of utility function is subject to the sequence of intertemporal budget constraints of the form:

$$C_t^{arm} + \frac{B_t^{arm}}{P_t^{arm}} + \frac{E_t^{arm} B_t^{rus}}{P_t^{arm}} + COST_t POLITICS_t N_t^m = \frac{W_t^{arm}}{P_t^{arm}} N_t^{arm} +$$

$$+ (1-\tau)\frac{E_t^{arm} W_t^{rus}}{P_t^{arm}} N_t^m + \frac{R_{t-1}^{arm} B_{t-1}^{arm}}{P_t^{arm}} + \frac{E_t^{arm} R_{t-1}^{rus} B_{t-1}^{rus} \varepsilon_{prem,t-1}^{arm}}{P_t^{arm}} + Div_t \qquad (13)$$

The left-hand side of the budget constraint presents expenditures, written in real terms and the right one represents incomes of households. Every period household decides about purchasing home (B_t^{arm}) and Russian (B_t^{rus}) bonds. He or she also gets incomes from bonds of the previous period. R_{t-1}^{arm} and R_{t-1}^{rus} are respectively gross interest rates on Armenian and Russian bonds from the previous period, $\varepsilon_{prem,t}^{arm}$ is a country-specific risk premium. E_t^{arm} is the nominal exchange rate of Armenian dram to Russian ruble (increase means depreciation of national currency), W_t^{arm} is the nominal wage in home country, W_t^{rus} is the nominal wage in Russia expressed in ruble, τ is a share of income that migrants spend in destination country, Div_t are dividends of firms, which are owned by households. The forth piece of expenditures side includes costs associated with migration and policy factors. Costs are expressed by:

$$COST_t = \sigma \frac{W_t^{arm}}{E_t^{arm} W_t^{rus}} \qquad (14)$$

The ratio of wages is a constraint over a decision of household to migrate. When wage in the home country goes up, household members prefer to stay and work in Armenia

which is expressed by an increase of migration costs. In an opposite case the motivation of migration increases, which is also true with exchange rate depreciation. Coefficient σ is calibrated in a way that migration costs of the economy in a steady state are to be equal to 0.8 percent of GDP. Political factor is modeled as an autoregressive process of first order.

The household's utility maximization problem gives the following first order conditions:

$$\frac{1}{C_t^{arm}} - \lambda_t = 0 \tag{15}$$

$$-\frac{\partial U_{t+j}}{\partial N_t^{arm}} \Bigg/ \frac{\partial U_{t+j}}{\partial C_t^{arm}} = MRS_t^{arm} \tag{16}$$

$$\frac{\lambda_{t+1} R_t^{arm}}{P_{t+1}^{arm}} - \frac{\lambda_t}{P_t^{arm}} = 0 \tag{17}$$

$$\frac{\lambda_{t+1} E_{t+1}^{arm} R_t^{rus} \varepsilon_{prem,t}^{arm}}{P_{t+1}^{arm}} - \frac{\lambda_t E_t^{arm}}{P_t^{arm}} = 0 \tag{18}$$

$$\frac{\lambda_t (1-\tau) E_t^{arm} W_t^{rus}}{P_t^{arm}} - \chi_m N_t^{m\varphi_m} - \lambda_t COST_t POLITICS_t = 0 \tag{19}$$

where λ_t is a Lagrange multiplier. From (16) it is calculated the intratemporal optimality condition which shows the equalization of marginal utility of consumption to the marginal value of labor:

$$MRS_t^{arm} = C_t^{arm} \chi_{arm} N_t^{arm\varphi_{arm}} \tag{20}$$

Equation (21) is the consumption Euler equation which represents the trade-off to the economy of moving consumption across time. It is derived by substituting (15) to (17) and from the condition that $\lambda_{t+1} = \frac{\beta}{C_{t+1}^{arm}}$.

$$\beta R_t^{arm} E_t \left(\frac{C_t^{arm}}{C_{t+1}^{arm}} \frac{P_t^{arm}}{P_{t+1}^{arm}} \right) = 1 \tag{21}$$

Combination of (17) and (18) gives the uncovered interest rate parity, which implies that under the assumption of complete financial markets the expected nominal return in national currency from domestic and foreign bonds must be the same:

$$R_t^{arm} = E_t\left(\frac{E_{t+1}^{arm}}{E_t^{arm}}\right)R_t^{rus}\varepsilon_{prem,t}^{arm} \tag{22}$$

where E_t is the expectation operator.

Using (15), (16) and (19), making some rearrangements, using some definitions and log-linearizing around deterministic steady state, migration decision equation gets the following form (small letters represent percentage deviations of variables from their steady state levels and coefficients with 'SS' subscript denote values of those variables in steady state):

$$n_t^m = -\frac{1}{\chi_m\varphi_m}\left(1+\frac{COST^{SS}}{C^{SS}}\right)cost_t - \frac{\varphi_{arm}}{\varphi_m}n_t^{arm} - \frac{COST^{SS}POLITICS^{SS}}{\chi_m C^{SS}}politics_t \tag{23}$$

Migration is a decreasing function of costs associated with migration and employment in Armenia and increasing with favorable migration legislation policy of Russian Federation. Real remittances (tr_t), which are the second component in income part of household budget constraint (see 13), in a log-linearized form are written as:

$$tr_t = n_t^m + w_{r,t}^{rus} + q_t^{arm} \tag{24}$$

Real money transfers of working migrants from Russia depend on migration, real Russian wages and real exchange rate of Armenian dram (the increase of q_t^{arm} means real depreciation of Armenian dram to Russian ruble).

2.1.2. CPI inflation, law of one price gap, real exchange rate and terms of trade

Log-linearization of aggregate price index (11) around steady state yields:

$$p_t^{arm} = (1-\gamma_1)p_{H,t}^{arm} + \gamma_1 p_{F,t}^{arm} \tag{25}$$

Substituting p_{t-1}^{arm} from both sides of the above equation, one gets the inflation equation as a function of home produced and imported good inflations:

$$\pi_t^{arm} = (1-\gamma_1)\pi_{H,t}^{arm} + \gamma_1\pi_{F,t}^{arm} \tag{26}$$

Important relationship is the terms of trade of the domestic economy, which measures the competitiveness of the economy and is expressed as a home produced good price relative to import price in domestic currency:

$$S_t^{arm} = \frac{P_{H,t}^{arm}}{P_{F,t}^{arm}} \tag{27}$$

In a log-linearized form, it is written as:

$$s_t^{arm} = p_{H,t}^{arm} - p_{F,t}^{arm} \tag{28}$$

Law of one price (LOP) gap means that the law of one price fails to hold. It is given by the ratio of the foreign price index in terms of domestic currency to the domestic currency price of import. Since in this paper it is discussed that the external sector for Armenia is only Russia, LOP gap is expressed by the following expression:

$$\Psi_t^{arm} = \frac{E_t^{arm} P_t^{rus}}{P_{F,t}^{arm}} \tag{29}$$

The real exchange rate is given as the ratio of price index of Russia in terms of domestic currency to the domestic price index:

$$\mathrm{Q}_t^{arm} = \frac{E_t^{arm} P_t^{rus}}{P_t^{arm}} \tag{30}$$

Log-linearization of the above LOP gap equation around steady state gives:

$$\psi_t^{arm} = e_t^{arm} + p_t^{rus} - p_{F,t}^{arm} \tag{31}$$

Making some rearrangements and after log-linearization real exchange rate equation would be expressed by the following:

$$q_t^{arm} = \psi_t^{arm} - (1-\gamma_1) s_t^{arm} \tag{32}$$

Equation (32) shows that LOP gap is positively related to the real exchange rate and the relation of terms of trade is negative.

2.1.3. International risk sharing

Under the assumption of complete financial markets, first order condition is expressed as an Euler equation must hold also for any other country. In this paper it implies that it would be true for Russia the following (derivation is shown in part 2.2.1):

$$\beta R_t^{rus} E_t \left(\frac{C_t^{rus}}{C_{t+1}^{rus}} \frac{P_t^{rus}}{P_{t+1}^{rus}} \right) = 1 \tag{33}$$

Combining expression (33) with (21) and using the definition of real exchange rate, one gets:

$$C_t^{arm} = \omega C_t^{rus} Q_t^{arm} \tag{34}$$

for all t, where ω is a constant depending on initial conditions. Using Euler equation with (34) and log-linearizing it around steady state, the consumption equation gets the form:

$$c_t^{arm} = c_{t+1}^{rus} + q_{t+1}^{arm} - (r_t^{arm} - \pi_{t+1}^{arm}) \tag{35}$$

Equation (35) shows that consumption is an increasing with higher expectation of foreign consumption and real exchange rate depreciation expectations and is decreasing with real interest rate:

2.1.4. Labor market

Households supply differentiated labor ($N(i)_t^{arm}$) to labor organizations which they combine to homogeneous labor and supply to firms according to the following Dixit-Stiglitz constant elasticity of substitution function:

$$N_t^{arm} = \left[\int_0^1 N(i)_t^{arm\,1-\frac{1}{\epsilon_w^{arm}}} di \right]^{\frac{\epsilon_w^{arm}}{\epsilon_w^{arm}-1}} \tag{36}$$

where ϵ_w^{arm} is the elasticity of substitution between varieties of supplied labor. Labor organization represents household and it is introduced in this model in a way suggested by Gali (2008) to express incompleteness in the labor market and wage stickiness. Labor organization tries to minimize expenditures associated with purchasing each type of labor:

$$\min\{W_t^{arm} N_t^{arm} - W(i)_t^{arm} N(i)_t^{arm}\} \tag{37}$$

Because labor is differentiated, the wage for the i-th type of labor is $W(i)_t^{arm}$ and households or labor organizations representing them have the power to set wages, which is given for firms. From labor organizations optimization problem demand for i-th type of labor is represented by the following equation:

$$N(i)_t^{arm} = \left(\frac{W(i)_t^{arm}}{W_t^{arm}} \right)^{-\epsilon_w^{arm}} N_t^{arm} \tag{38}$$

Permutation of demand equation to the (37) gives the aggregate wage index:

$$W_t^{arm} = \left[\int_0^1 W(i)_t^{1-\epsilon_w^{arm}} di\right]^{\frac{1}{1-\epsilon_w^{arm}}} \tag{39}$$

According to Calvo (1983), it is assumed that only $(1-\theta_w^{arm})$ fraction of households every period could negotiate about wages. So θ_w^{arm} is a wage stickiness coefficient. Households set $W_t^{*\,arm}$ optimal wage in a way that they cannot change it in the future. In a symmetric equilibrium the optimization condition associated with that problem is expressed by the following:

$$\sum_{j=0}^{\infty}(\beta\theta_w^{arm})^j\, E_t\left[\Lambda_{t,t+j}^{arm}\left(\frac{W_t^{*\,arm}}{P_{t+j}^{arm}} - \mathrm{M}_w^{arm} MRS_{t+j}^{arm}\right)N_{t+j}^{arm}\right] = 0 \tag{40}$$

where $\beta^j\Lambda_{t,t+j}^{arm} = \beta^j\left(\frac{C_{t+j}}{C_t}\right)^{-1}\frac{P_t}{P_{t+j}}$ is the stochastic discount factor and $\mathrm{M}_w^{arm} = \frac{\epsilon_w^{arm}}{\epsilon_w^{arm}-1}$ is a markup over wages in competitive market. Log-linearization of the above condition brings to the following optimal wage setting rule:

$$w_t^{*arm} = \mu_w^{arm} + (1-\beta\theta_w^{arm})\sum_{j=0}^{\infty}(\beta\theta_w^{arm})^j\, E_t\left[mrs_{t+j}^{arm} + p_{t+j}^{arm}\right] \tag{41}$$

where $\mu_w^{arm} = \log(\mathrm{M}_w^{arm})$. Under the assumptions that every period only θ_w^{arm} part of households can negotiate about their wages, aggregate wage index is defined:

$$W_t^{arm} = \left(\theta_w^{arm}(W_{t-1}^{arm})^{1-\epsilon_w^{arm}} + [1-\theta_w^{arm}](W_t^{*\,arm})^{1-\epsilon_w^{arm}}\right)^{\frac{1}{1-\epsilon_w^{arm}}} \tag{42}$$

Defining wage inflation as a $\pi_{W,t}^{arm}$, replacing w_t^{*arm} in (41) by (42) and using some manipulation techniques, the New Keynesian Phillips curve for wage inflation gets the form:

$$\pi_{W,t}^{arm} = \beta\pi_{W,t+1}^{arm} + \lambda_w^{arm}[mrs_t^{arm} - (w_t^{arm} - p_t^{arm})] \tag{43}$$

where $\lambda_w^{arm} = \frac{(1-\theta_w^{arm})(1-\beta\theta_w^{arm})}{\theta_w^{arm}(1+\epsilon_w^{arm}\varphi_{arm})}$. This equation shows that when real wage is below the desired level, households increase it by pressing the wage inflation to go up. On the other hand, taking the identity that $\pi_{W,t}^{arm} = w_t^{arm} - w_{t-1}^{arm}$ real wage will be defined by the following equation:

$$w_{r,t}^{arm} = w_{r,t-1}^{arm} + \pi_{W,t}^{arm} - \pi_t^{arm} \tag{44}$$

2.1.5. Production

Firms produce homogeneous good. Aggregate output in the economy is represented by production function of the form:

$$Y_t^{arm} = A_t^{arm}(N_t^{arm})^{(1-\alpha_1)} \tag{45}$$

where $(1-\alpha_1)$ is a share of labor in total recourses used in production process. This paper abstracts from the capital and considers it as a constant. A_t^{arm} is a technology which is common for all firms and each of them has access to it. The last follows the first order autoregressive process. Firms minimize their expenditures associated with production of goods, which is given by the following:

$$min\left\{\frac{W_t^{arm}N_t^{arm}}{P_{H,t}^{arm}}\right\} \tag{46}$$

Solution of the firm optimization problem gives the function of marginal cost:

$$MC_t^{arm} = \frac{W_t^{arm}(Y_t^{arm})^{\frac{\alpha_1}{1-\alpha_1}}}{(1-\alpha_1)P_{H,t}^{arm}(A_t^{arm})^{\frac{1}{1-\alpha_1}}} \tag{47}$$

Log-linearization around deterministic steady state brings to:

$$mc_t^{arm} = w_{r,t}^{arm} - \gamma_1 s_t^{arm} + \alpha_1 n_t^{arm} - a_t^{arm} \tag{48}$$

The above equation shows that marginal cost is increasing with real wage and employment growth and decreasing with terms of trade and total productivity in the economy.

2.1.6. Price setting

There are a lot of firms operating in monopolistic competitive market. They have a monopolistic power and set prices following Calvo (1983). In each period a firm with probability $(1-\theta_H^{arm})$ receives a signal to adjust prices and with probability θ_H^{arm} does not

receive the signal for price optimization. An optimal price which chooses the j-th firm is denoted by $P(j)_t^{*\,arm}$. A typical firm maximizes its profit function:

$$\sum_{j=0}^{\infty}(\beta\theta_H^{arm})^j\, E_t\left[\Lambda_{t,t+j}^{arm}\left(\frac{P(j)_{H,t}^{*\,arm}}{P_{H,t+j}^{arm}} - MC_{t+j}^{arm}\right)Y(j)_{H,t+j}^{arm}\right] \tag{49}$$

subject to the demand curve for its production:

$$Y(j)_{H,t+j}^{arm} = \left(\frac{P(j)_{H,t}^{*\,arm}}{P_{H,t+j}^{arm}}\right)^{-\varepsilon_1} Y_{H,t+j}^{arm} \tag{50}$$

The optimization condition of the above discussed problem is expressed as:

$$E_t\sum_{j=0}^{\infty}(\beta\theta_H^{arm})^j\Lambda_{t,t+j}^{arm}\frac{P(j)_{H,t}^{*\,arm}}{P_{H,t+j}^{arm}}Y(j)_{H,t+j}^{arm}$$

$$= E_t\sum_{j=0}^{\infty}(\beta\theta_H^{arm})^j\,\Lambda_{t,t+j}^{arm}(1+\mu_1)MC_{t+j}^{arm}Y(j)_{H,t+j}^{arm} \tag{51}$$

where $\mu_1 = \frac{1}{\varepsilon_1 - 1}$ is a mark-up over prices. In a symmetric equilibrium for any j-th firm, $P(j)_{H,t}^{*\,arm} = P_{H,t}^{*\,arm}$ and $Y(j)_{H,t+j}^{arm} = Y_{H,t+j}^{*\,arm}$ it becomes:

$$E_t\sum_{j=0}^{\infty}(\beta\theta_H^{arm})^j\Lambda_{t,t+j}^{arm}\frac{P_{H,t}^{*\,arm}}{P_{H,t}^{arm}\pi_{H,t,t+j}^{arm}}Y_{H,t+j}^{*\,arm} = E_t\sum_{j=0}^{\infty}(\beta\theta_H^{arm})^j\,\Lambda_{t,t+j}^{arm}(1+\mu_1)MC_{t+j}^{arm}Y_{H,t+j}^{*\,arm} \tag{52}$$

where $\pi_{H,t,t+j}^{arm} = \pi_{H,t+1}^{arm}\times\pi_{H,t+2}^{arm}\times\pi_{H,t+3}^{arm}\times\cdots\times\pi_{H,t+j}^{arm}$. Also optimal price ratio is denoted by $p_{H,t}^{*arm} = \frac{P_{H,t}^{*\,arm}}{P_{H,t}^{arm}}$ and after log-linearization and some algebraic manipulations it gets the form:

$$p_{H,t}^{*\,arm} = (1-\beta\theta_H^{arm})E_t\sum_{j=0}^{\infty}(\beta\theta_H^{arm})^j\left(mc_{t+j}^{arm} + \pi_{H,t,t+j}^{arm}\right) \tag{53}$$

Using some techniques, the above equation would have the following form:

$$p_{H,t}^{*\,arm} = (1-\beta\theta_H^{arm})mc_t^{arm} + \beta\theta_H^{arm}E_t\left(p_{H,t+1}^{*\,arm} + \pi_{H,t+1}^{arm}\right) \tag{54}$$

Aggregate price index of the home produced goods (equation 6) is written by:

$$P_{H,t}^{arm} = \left[\int_0^1 P(j)_{H,t}^{arm\,-\frac{1}{\mu_1}}\,dj\right]^{-\mu_1} = \left[\theta_H^{arm}\left(P_{H,t-1}^{arm}\right)^{-\frac{1}{\mu_1}} + (1-\theta_H^{arm})\left(P_{H,t}^{*\,arm}\right)^{-\frac{1}{\mu_1}}\right]^{-\mu_1} \tag{55}$$

Log-linearization of (55) around a steady state and after some algebraic manipulations optimal price ratio gets the form:

$$p_{H,t}^{*\,arm} = \frac{\theta_H^{arm}}{1-\theta_H^{arm}}\pi_{H,t}^{arm} \tag{56}$$

This equation is true for every t. Substituting (56) to (54), it gets the form, which is called New Keynesian Phillips curve:

$$\pi_{H,t}^{arm} = \beta\pi_{H,t+1}^{arm} + \lambda_H^{arm} mc_t^{arm} \tag{57}$$

where $\lambda_H^{arm} = \frac{(1-\beta\theta_H^{arm})(1-\theta_H^{arm})}{\theta_H^{arm}}$, and inflation is a function of marginal cost and expected inflation.

Importers are operating in the monopolistic competitive market too. They set prices following Calvo (1983). With probability $(1-\theta_F^{arm})$ firms set optimal price and with a probability of θ_F^{arm} they keep prices unchanged. The j-th importer solves the following profit maximization problem:

$$\sum_{j=0}^{\infty}(\beta\theta_F^{arm})^j\, E_t\left[\Lambda_{t,t+j}^{arm}\left(\frac{P(j)_{F,t}^{*\,arm}}{P_{F,t+j}^{arm}} - \frac{E_t^{arm}P_t^{rus}}{P_{F,t}^{arm}}\right)Y(j)_{F,t+j}^{arm}\right] \tag{58}$$

subject to the demand curve for its good:

$$Y(j)_{F,t+j}^{arm} = \left(\frac{P(j)_{F,t}^{*\,arm}}{P_{F,t+j}^{arm}}\right)^{-\varepsilon_1} Y_{F,t+j}^{arm} \tag{59}$$

Steps of the optimization problem and the derivation of New Keynesian Phillips curve are the same as in a case with home produced goods. The exception is that real marginal cost for importers is LOP gap $\frac{E_t^{arm}P_t^{rus}}{P_{F,t}^{arm}} = \Psi_t^{arm}$. The New Keynesian Phillips curve of the imported goods from Russia has the following form:

$$\pi_{F,t}^{arm} = \beta\pi_{F,t+1}^{arm} + \lambda_F^{arm}\psi_t^{arm} \tag{60}$$

where inflation of imported goods is a positive function of inflation expectations and law of one price gap.

2.1.7. Equilibrium

Value added in the economy is divided into the consumption of home-produced goods and migration costs. It is represented by the following equation:

$$Y_t^{arm} = C_{H,t}^{arm} + cost_t N_t^m \tag{61}$$

Log-linearization of the above expression around steady state and after simple manipulations, the market clearing condition gets the form:

$$y_t^{arm} = (1-\gamma_1)C^{ss\,arm}(c_t^{arm} - \eta_1\gamma_1 s_t^{arm}) + COST^{ss} N_M^{ss}\left(cost_t + n_{M,t}\right) \tag{62}$$

2.1.8. Monetary policy

Central bank operates with a Taylor (1993) type interest rate rule. Interest rate responses to movements of inflation expectation from its target and the expected deviation of output from its non-inflation driving level. It is given by the following:

$$R_t^{arm} = (R_{t-1}^{arm})^{\rho_r^{arm}} \left\{ \left(\frac{\pi_{t+1}^{arm}}{\pi^{ss,arm}} \right)^{\mu_\pi^{arm}} \left(\frac{Y_{t+1}^{arm}}{Y^{ss,arm}} \right)^{\mu_y^{arm}} \right\}^{(1-\rho_r^{arm})} \tag{63}$$

where ρ_r^{arm} is an interest rate persistence coefficient, μ_π^{arm} and μ_y^{arm} are respectively coefficients of responses of interest rate to deviations of inflation expectation and output gap from their targets. The log-linearized version of monetary policy rule is expressed:

$$r_t^{arm} = \rho_r^{arm} r_{t-1}^{arm} + (1-\rho_r^{arm})\left(\mu_\pi^{arm}\pi_{t+1}^{arm} + \mu_y^{arm} y_{t+1}^{arm}\right) \tag{64}$$

2.2. Russian Economy

2.2.1. Households

Consumption basket of the representative Russian household consists of tradable and non-tradable goods and is represented by the following constant elasticity of substitution function:

$$C_t^{rus} = \left[(1-\gamma_2)^{\frac{1}{\eta_2}} C_{T,t}^{rus\,\frac{\eta_2-1}{\eta_2}} + \gamma_2^{\frac{1}{\eta_2}} C_{NT,t}^{rus\,\frac{\eta_2-1}{\eta_2}} \right]^{\frac{\eta_2}{\eta_2-1}} \tag{65}$$

where $C_{T,t}^{rus}$ is a consumption of tradable goods, $C_{NT,t}^{rus}$ are non-tradable goods, γ_2 is a share of non-tradable goods in household's consumption, η_2 is the elasticity of substitution between these two types of goods. Aggregate consumption index of non-tradable good is expressed by Dixit-Stiglitz function:

$$C_{NT,t}^{rus} = \left[\int_0^1 C(j)_{NT,t}^{rus\,\frac{\varepsilon_2-1}{\varepsilon_2}}\, dj\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}} \tag{66}$$

where ε_2 is the elasticity of substitution between varieties of goods and $j \in [0,1]$ denotes the continuum of goods. Household tries to minimize expenditures of each type of non-tradable goods $\int_0^1 P(j)_{NT,t}^{rus} C(j)_{NT,t}^{rus}\, dj$, where $P(j)_{NT,t}^{rus}$ is the price of j-th type of non-tradable good. From expenditures optimization problem one gets the demand function of each type of good:

$$C(j)_{NT,t}^{rus} = \left(\frac{P(j)_{NT,t}^{rus}}{P_{NT,t}^{rus}}\right)^{-\varepsilon_2} C_{NT,t}^{rus} \tag{67}$$

Aggregate price index of non-tradable goods is derived by substituting (67) into expenditures minimization function, which gets the form:

$$P_{NT,t}^{rus} = \left[\int_0^1 P(j)_{NT,t}^{rus\,1-\varepsilon_2}\, dj\right]^{\frac{1}{1-\varepsilon_2}} \tag{68}$$

Household tries to minimize expenditures of two types of goods by solving the minimization problem:

$$\min\{P_t^{rus} C_t^{rus} - P_{T,t}^{rus} C_{T,t}^{rus} - P_{NT,t}^{rus} C_{NT,t}^{rus}\} \tag{69}$$

where P_t^{rus} is a consumer price index, $P_{T,t}^{rus}$ and $P_{NT,t}^{rus}$ are respectively prices of tradable and non-tradable goods. A solution of the above problem gives demand functions for tradable and non-tradable goods.

$$C_{T,t}^{rus} = (1-\gamma_2)\left(\frac{P_{T,t}^{rus}}{P_t^{rus}}\right)^{-\eta_2} C_t^{rus} \tag{70}$$

$$C_{NT,t}^{rus} = \gamma_2\left(\frac{P_{NT,t}^{rus}}{P_t^{rus}}\right)^{-\eta_2} C_t^{rus} \tag{71}$$

After permutation of (70) and (71) into (69), one gets the consumer price index of the form:

$$P_t^{rus} = \left[(1-\gamma_2)P_{T,t}^{rus\,(1-\eta_2)} + \gamma_2 P_{NT,t}^{rus\,(1-\eta_2)}\right]^{\frac{1}{1-\eta_2}} \tag{72}$$

Consumption of tradable goods consists of home produced and imported goods, which is represented by the following:

$$C_{T,t}^{rus} = \left[(1-\gamma_3)^{\frac{1}{\eta_3}} C_{H,t}^{rus\,\frac{\eta_3-1}{\eta_3}} + \gamma_3^{\frac{1}{\eta_3}} C_{F,t}^{rus\,\frac{\eta_3-1}{\eta_3}}\right]^{\frac{\eta_3}{\eta_3-1}} \tag{73}$$

where $C_{H,t}^{rus}$ and $C_{F,t}^{rus}$ are respectively consumptions of home produced and imported goods, γ_3 is a share of imported goods in overall consumption of tradable goods, η_3 is the elasticity of substitution between imported and home produced tradable goods. On the other hand, consumption indexes of home produced $\left(C_{H,t}^{rus}\right)$ and imported $\left(C_{F,t}^{rus}\right)$ goods are given by the following constant elasticity of substitution functions:

$$C_{H,t}^{rus} = \left[\int_0^1 C(j)_{H,t}^{rus\,\frac{\varepsilon_2-1}{\varepsilon_2}}\, dj\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}} \tag{74}$$

$$C_{F,t}^{arm} = \left[\int_0^1 C(j)_{F,t}^{rus\,\frac{\varepsilon_2-1}{\varepsilon_2}}\, dj\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}} \tag{75}$$

The optimal allocation of resources within each category of goods gives demand functions of the form:

$$C(j)_{H,t}^{rus} = \left(\frac{P(j)_{H,t}^{rus}}{P_{H,t}^{rus}}\right)^{-\varepsilon_2} C_{H,t}^{rus} \tag{76}$$

$$C(j)_{F,t}^{rus} = \left(\frac{P(j)_{F,t}^{rus}}{P_{F,t}^{rus}}\right)^{-\varepsilon_2} C_{F,t}^{rus} \tag{77}$$

Aggregate price indexes of home produced and imported tradable goods are represented by the followings:

$$P_{H,t}^{rus} = \left[\int_0^1 P(j)_{H,t}^{rus\,1-\varepsilon_2}\, dj\right]^{\frac{1}{1-\varepsilon_2}} \tag{78}$$

$$P_{F,t}^{rus} = \left[\int_0^1 P(j)_{F,t}^{rus\,1-\varepsilon_2}\, dj\right]^{\frac{1}{1-\varepsilon_2}} \tag{79}$$

Household minimizes expenditures of the above mentioned goods:

$$\min\{P_{T,t}^{rus} C_{T,t}^{rus} - P_{H,t}^{rus} C_{H,t}^{rus} - P_{F,t}^{rus} C_{F,t}^{rus}\} \tag{80}$$

Demand functions for home produced tradable and imported goods are derived from optimization problem, which have the following forms:

$$C_{H,t}^{rus} = (1-\gamma_3)(1-\gamma_2)\left(\frac{P_{H,t}^{rus}}{P_{T,t}^{rus}}\right)^{-\eta_3}\left(\frac{P_{T,t}^{rus}}{P_t^{rus}}\right)^{-\eta_2} C_t^{rus} \tag{81}$$

$$C_{F,t}^{rus} = \gamma_3(1-\gamma_2)\left(\frac{P_{F,t}^{rus}}{P_{T,t}^{rus}}\right)^{-\eta_3}\left(\frac{P_{T,t}^{rus}}{P_t^{rus}}\right)^{-\eta_2} C_t^{rus} \tag{82}$$

So, price index of tradable goods gets the form:

$$P_{T,t}^{rus} = \left[(1-\gamma_3)P_{H,t}^{rus(1-\eta_3)} + \gamma_3 P_{F,t}^{rus(1-\eta_3)}\right]^{\frac{1}{1-\eta_3}} \tag{83}$$

A typical Russian household maximizes the following utility function:

$$U_{t+j} = E_t \sum_{j=0}^{\infty} \beta^j \left(\log\left(C_{t+j}^{rus}\right) - \chi_{rus} \frac{N_{t+j}^{rus\,1+\varphi_{rus}}}{1+\varphi_{rus}}\right) \tag{84}$$

where E_t denotes the expectation operator conditional on information available at time t, β is the discount rate, φ_{rus} is the inverse of the Frisch elasticity of labor supply, χ_{rus} is a preference parameter. N_t^{rus} is a supplied labor and it is expressed as a function of employed in tradable ($N_{T,t}^{rus}$) and non-tradable ($N_{NT,t}^{rus}$) sectors:

$$N_t^{rus} = \left[(1-\gamma_4)^{\frac{1}{\eta_4}} N_{T,t}^{rus\,\frac{\eta_4-1}{\eta_4}} + \gamma_4^{\frac{1}{\eta_4}} N_{NT,t}^{rus\,\frac{\eta_4-1}{\eta_4}}\right]^{\frac{\eta_4}{\eta_4-1}} \tag{85}$$

where γ_4 is a share of employed in the non-tradable sector and η_4 is the elasticity of substitution between two types of labors. In this paper, it is assumed that labor is mobile between two sectors and wages are equalized. To provide this condition, the elasticity of substitution between two types of labor must go to infinity ($\eta_4 \to \infty$).

The maximization of the utility function is subject to the sequence of budget constraints of the form:

$$C_t^{rus} + \frac{B_t^{rus}}{P_t^{rus}} + \frac{E_t^{rus} B_t^*}{P_t^{rus}} = \frac{W_t^{rus}}{P_t^{rus}} N_t^{rus} + \frac{R_{t-1}^{rus} B_{t-1}^{rus}}{P_t^{rus}} + \frac{E_t^{rus} R_{t-1}^* B_{t-1}^* \varepsilon_{prem,t-1}^{rus}}{P_t^{rus}} + Div_t \quad (86)$$

Every period household buys B_t^{rus} internal and B_t^* external bonds and gets $R_{t-1}^{rus} B_{t-1}^{rus}$ and $E_t^{rus} R_{t-1}^* B_{t-1}^*$ payments from previously purchased bonds, where R_t^{rus} and R_t^* are respectively gross interest rates of internal and external bonds. E_t^{rus} is the nominal effective exchange rate of ruble, W_t^{rus} is a nominal wage in economy and $\varepsilon_{prem,t}^{rus}$ is a country-specific risk premium. Households are the owners of firms and get dividends (Div_t).

First order conditions of the household utility maximization problem are the followings:

$$\frac{1}{C_t^{rus}} - \lambda_t = 0 \quad (87)$$

$$-\frac{\partial U_{t+j}}{\partial N_t^{rus}} \Bigg/ \frac{\partial U_{t+j}}{\partial C_t^{rus}} = MRS_t^{rus} \quad (88)$$

$$\frac{\lambda_{t+1} R_t^{rus}}{P_{t+1}^{rus}} - \frac{\lambda_t}{P_t^{rus}} = 0 \quad (89)$$

$$\frac{\lambda_{t+1} E_{t+1}^{rus} R_t^* \varepsilon_{prem,t}^{rus}}{P_{t+1}^{rus}} - \frac{\lambda_t E_t^{rus}}{P_t^{rus}} = 0 \quad (90)$$

(88) gives the equation of marginal rate of substitution between consumption and leisure:

$$MRS_t^{rus} = C_t^{rus} \chi_{rus} N_t^{rus \varphi_{rus}} \quad (91)$$

Substituting λ_t by $\frac{1}{C_t^{rus}}$ in equation (89) and from the condition that $\lambda_{t+1} = \frac{\beta}{C_{t+1}^{arm}}$, the intertemporal consumption equation would be:

$$\beta R_t^{rus} E_t \left(\frac{C_t^{rus}}{C_{t+1}^{rus}} \frac{P_t^{rus}}{P_{t+1}^{rus}} \right) = 1 \quad (92)$$

Uncovered interest rate parity equation is derived by combining (89) and (90) conditions and gets the form:

$$R_t^{rus} = \frac{E_{t+1}^{rus}}{E_t^{rus}} R_t^* \varepsilon_{prem,t}^{rus} \quad (93)$$

It shows that differences in interest rates are a result of expected change of nominal exchange rate and country specific risk premium.

2.2.2. Law of one price gap, real exchange rate and terms of trade

Log-linearization of (72) and (83) around steady state brings to the set of following equations:

$$p_t^{rus} = (1-\gamma_2)p_{T,t}^{rus} + \gamma_2 p_{NT,t}^{rus} \tag{94}$$

$$p_{T,t}^{rus} = (1-\gamma_3)p_{H,t}^{rus} + \gamma_3 p_{F,t}^{rus} \tag{95}$$

Substituting p_{t-1}^{rus} from (94) and $p_{T,t-1}^{rus}$ from (95), one would get inflation of consumer price index as a function of tradable and non-tradable goods inflations and tradable good inflation as a function of home produced and imported goods inflations respectively:

$$\pi_t^{rus} = (1-\gamma_2)\pi_{T,t}^{rus} + \gamma_2 \pi_{NT,t}^{rus} \tag{96}$$

$$\pi_{T,t}^{rus} = (1-\gamma_3)\pi_{H,t}^{rus} + \gamma_3 \pi_{F,t}^{rus} \tag{97}$$

Similar to the Armenian block of the model, terms of trade and law of one price (LOP) gap are expressed by the equations:

$$S_t^{rus} = \frac{P_{H,t}^{rus}}{P_{F,t}^{rus}} \tag{98}$$

$$\Psi_t^{rus} = \frac{E_t^{rus} P_t^*}{P_{F,t}^{rus}} \tag{99}$$

where P_t^* is the price index of the rest of the world. Internal exchange rate is defined by the ratio of prices of non-tradable good to home produced tradable good:

$$S_{I,t}^{rus} = \frac{P_{NT,t}^{rus}}{P_{H,t}^{rus}} \tag{100}$$

Real exchange rate in a log-linearized form gets the following form:

$$q_t^{rus} = \psi_t^{rus} - (1 - \gamma_3 + \gamma_2\gamma_3)s_t^{rus} - \gamma_2 s_{I,t}^{rus} \tag{101}$$

This means that changes of real exchange rate are a result of movements of LOP gap, terms of trade and internal exchange rate.

2.2.3. International risk sharing

Under the assumption of complete financial markets, first order condition is expressed as an Euler equation must hold also for any other country:

$$\beta R_t^* E_t \left(\frac{C_t^*}{C_{t+1}^*} \frac{P_t^*}{P_{t+1}^*} \right) = 1 \tag{102}$$

Combining expression (102) with (92) and using the definition of real exchange rate, one gets:

$$C_t^{rus} = \vartheta C_t^* \mathrm{Q}_t^{rus} \tag{103}$$

for all t, where ϑ is a constant depending on initial conditions. The external world is very big and Russia could not influence on it. So, it is represented as a closed economy, when production goes only to consumption ($C_t^* = Y_t^*$). Using Euler equation with (103) and log-linearizing it around steady state, the consumption equation gets the form:

$$c_t^{rus} = y_{t+1}^* + q_{t+1}^{rus} - (r_t^{rus} - \pi_{t+1}^{rus}) \tag{104}$$

2.2.4. Labor market

It is assumed that labor is mobile between tradable and non-tradable sectors of the economy in a way that wages are equalized in both sectors. As shown in Appendix 2, mean wages in Russian tradable and non-tradable sectors are going very close to each other. In the second picture of the same appendix, there are expressed growth rates, which also do not differ a lot from each other. This stylized fact supports the assumption that wages are equalized in both sectors. So, this paper discusses one mean wage in the Russian economy.

As in Armenian block of the model, there are labor organizations which combine differentiated labor to homogeneous and supply to firms both of tradable and non-tradable sectors. Steps of derivation of the New Keynesian Phillips curve for wages are the same as in 2.1.4., and it has the following form:

$$\pi_{W,t}^{rus} = \beta \pi_{W,t+1}^{rus} + \lambda_w^{rus} [mrs_t^{rus} - (w_t^{rus} - p_t^{rus})] \tag{105}$$

This equation shows that nominal wage is the increasing function of wage expectations and the difference between reserve and real wages in the economy. Real wage is defined by the identity:

$$w_{r,t}^{rus} = w_{r,t-1}^{rus} + \pi_{W,t}^{rus} - \pi_t^{rus} \tag{106}$$

2.2.5. Production

Firms in Russia operate in tradable and non-tradable sectors. They produce homogeneous tradable ($Y_{T,t}^{rus}$) and non-tradable ($Y_{NT,t}^{rus}$) goods using the following Cobb-Douglas production functions:

$$Y_{T,t}^{rus} = A_{T,t}^{rus}\left(N_{T,t}^{rus}\right)^{(1-\alpha_2)} \tag{107}$$

$$Y_{NT,t}^{rus} = A_{NT,t}^{rus}\left(N_{N,t}^{rus}\right)^{(1-\alpha_3)} \tag{108}$$

where $A_{T,t}^{rus}$ and $A_{NT,t}^{rus}$ are respectively productivity levels in tradable and non-tradable sectors. $(1-\alpha_2)$ and $(1-\alpha_3)$ are shares of labor in total resources. In tradable sector usage of labor is low with comparison to non-tradable sector ($\alpha_2 > \alpha_3$): $N_{T,t}^{rus}$ and $N_{N,t}^{rus}$ represent labor in tradable and non-tradable sectors respectively. The last one is given by the constant elasticity of substitution function:

$$N_{N,t}^{rus} = \left[(1-\gamma_5)^{\frac{1}{\eta_5}} N_{NT,t}^{rus\,\frac{\eta_5-1}{\eta_5}} + \gamma_5^{\frac{1}{\eta_5}} N_t^{m\,\frac{\eta_5-1}{\eta_5}}\right]^{\frac{\eta_5}{\eta_5-1}} \tag{109}$$

where $N_{NT,t}^{rus}$ are Russian household members working in the non-tradable sector, N_t^m are Armenian migrants, γ_5 is a share of migrants in total employment of Russian non-tradable sector, η_5 is the elasticity of substitution between them. Armenian migrants do not play a role in wage determination and take it as given, but they have an impact on marginal cost by their weight, which is very small ($\gamma_5 = 0.004$). Firms in both sectors solve cost minimization problems, which are:

$$min\left\{\frac{W_t^{rus} N_{T,t}^{rus}}{P_{H,t}^{rus}}\right\} \tag{110}$$

$$min\left\{\frac{W_t^{rus}N_{N,t}^{rus}}{P_{NT,t}^{rus}}\right\} \tag{111}$$

Solution of the above optimization problems gives marginal cost equations:

$$MC_{T,t}^{rus} = \frac{W_t^{rus}\left(N_{T,t}^{rus}\right)^{\alpha_2}}{(1-\alpha_2)P_{H,t}^{rus}A_{T,t}^{rus}} \tag{112}$$

$$MC_{NT,t}^{rus} = \frac{W_t^{rus}\left(N_{N,t}^{rus}\right)^{\alpha_3}}{(1-\alpha_3)P_{NT,t}^{rus}A_{NT,t}^{rus}} \tag{113}$$

(112) and (113) equations show that marginal cost in each sector is an increasing function of wages and input labor, and decreasing with the price of that sector's good and technology level.

2.2.6. Price setting

According to Calvo (1983), every period firms with probability $(1-\theta_i^{rus})$ receive a signal to optimize its' prices and set $P_{i,t}^{*rus}$ optimal price $(i = H, NT, F)$ while the remaining θ_i^{rus} part of firms cannot adjust and change prices. New Keynesian literature discusses different factors that do not allow firms to change prices in a short run in response to economic conditions. Steps and assumptions of derivation of New Keynesian Phillips curves are same as in Armenian block of the model (see 2.1.6), the final forms of which are the followings:

$$\pi_{H,t}^{rus} = \beta\pi_{H,t+1}^{rus} + \lambda_H^{rus}mc_{T,t}^{rus} \tag{114}$$

$$\pi_{NT,t}^{rus} = \beta\pi_{NT,t+1}^{rus} + \lambda_{NT}^{rus}mc_{NT,t}^{rus} \tag{115}$$

$$\pi_{F,t}^{rus} = \beta\pi_{F,t+1}^{rus} + \lambda_F^{rus}\psi_t^{rus} \tag{116}$$

where λ_i^{rus} $(i = H, NT, F)$ is a coefficient on marginal cost and represents the combination of structural parameters of the model. The LOP gap is an important factor driving the inflation of imported goods and it is called marginal cost of importers.

2.2.7. Equilibrium

The whole value added in the economy is divided into outputs in tradable and non-tradable sectors:

$$Y_t^{rus} = Y_{T,t}^{rus} + Y_{NT,t}^{rus} \tag{117}$$

The whole production of non-tradable sector is consumed at home, and it is true the following identity:

$$Y_{NT,t}^{rus} = C_{NT,t}^{rus} \tag{118}$$

On the other hand, production of the tradable sector is divided into home and foreign consumptions (export):

$$Y_{T,t}^{rus} = C_{H,t}^{rus} + Y_{EX,t}^{rus} \tag{119}$$

Given the demand function for home produced tradable goods of the Russian household as $C_{H,t}^{rus} = (1-\gamma_3)\left(\frac{P_{H,t}^{rus}}{P_{T,t}^{rus}}\right)^{-\eta_3} C_{T,t}^{rus}$ and following much of the literature, demand function of foreign households for Russian tradable goods is represented by the following:

$$Y_{EX,t}^{rus} = \gamma_3\left(\frac{P_{H,t}^{rus}}{E_t^{rus}P_t^*}\right)^{-\eta_3} Y_t^* \tag{120}$$

where export is a function of external demand and relative prices.

2.2.8. Monetary policy

The model is closed by adding monetary policy rule of a Taylor (1993) type:

$$R_t^{rus} = (R_{t-1}^{rus})^{\rho_r^{rus}}\left\{\left(\frac{\pi_{t+1}^{rus}}{\pi^{ss,rus}}\right)^{\mu_\pi^{rus}}\left(\frac{Y_{t+1}^{rus}}{Y^{ss,rus}}\right)^{\mu_y^{rus}}\right\}^{(1-\rho_r^{rus})} \tag{121}$$

The Central bank is forward looking with an interest rate persistence component and responds to inflation expectations and the future output gap.

2.2.9. External world

Russian economy is small relative to external world and cannot impact on it. The external sector is modelled exogenously. Foreign demand, inflation and interest rate follow first order autoregressive processes:

$$y_t^* = \rho_y^* y_{t-1}^* + \sigma_{y,t}^* \tag{122}$$

$$\pi_t^* = \rho_\pi^* \pi_{t-1}^* + \sigma_{\pi,t}^* \tag{123}$$

$$r_t^* = \rho_r^* r_{t-1}^* + \sigma_{r,t}^* \tag{124}$$

where ρ_y^*, ρ_π^*, $\rho_r^* \in (0,1)$ are persistence coefficients and $\sigma_{y,t}^*$, $\sigma_{\pi,t}^*$, $\sigma_{r,t}^*$ are independent and identically distributed error terms with zero mean.

All the equations of the log-linearized model are written in Appendix 1.

3. Data

For estimation of the log-linearized model 22 structural shocks and 22 time series are used with quarterly frequency (see Appendix 3.1.), 19 of which start from the first quarter of 2003 and end in the second quarter of 2015. Data about Armenian labor market based on Household's integrated living conditions surveys, which concepts are mainly based on principles recommended by International Labor Organization (ILO), start from 2008. So, data of Armenian employed start from the first quarter of 2008 and end in the second quarter of 2015. Information about Russian employment starts from the first quarter of 2006. In this model, there are used employed in tradable (agriculture and industry) and non-tradable (construction, trade and service) sectors. The time series of consumptions represent real private consumptions published by Statistical services for both countries. Armenian interest rate is an interbank interest rate and the Russian interest rate is money market rate. Data of Armenian GDP is used, while for Russian block GDP is divided into the output of tradable and non-tradable sectors. Real wages are calculated by dividing average nominal wages to CPI index. CPI of Russia is divided to tradable (goods) and non-tradable (services) price indexes. For an Armenian case, the CPI is divided into home produced and imported goods price index from Russia. The real exchange rate of Armenian dram is calculated by using CPI indexes of Armenia and Russia and nominal exchange rate. The real effective exchange rate of Russian ruble represents exchange rate of ruble to bilateral currency basket of USD and Euro. Data on migration is approximated to the border crossing. Time series (CPI, GDP and interest rate) of the foreign sector are calculated by weights of Russian bilateral exchange rate basket.

In a first stage, all data except interest rates are seasonally adjusted using the X12 method. In a next step, all series are logged (100*log) besides interest rates. Time series are detrended using univariate Kalman filter, which is expressed by $(3.1 - 3.4)$ equations:

$$X_t = X_trend_t + X_gap_t \tag{3.1}$$

where X_t is observed data, which is divided into trend and gap components.

$$\Delta X_trend_t = X_trend_t - X_trend_{t-1} \tag{3.2}$$

ΔX_trend_t is a percentage change of trend in an one-quarter. Trend change depends on its previous lag, steady state level of growth and disturbance component:

$$\Delta X_trend_t = \alpha_1 \Delta X_trend_{t-1} + \frac{(1-\alpha_1) ss_growth}{4} + \varepsilon_t^{trend} \tag{3.3}$$

where α_1 is a coefficient of persistence, $(1-\alpha_1)$ is a speed of going to its steady state growth level, ss_growth is a yearly steady state growth rate of observed macroeconomic variable and ε_t^{trend} is an independent and identically distributed normal error term with zero mean. Gap component is represented by a first order autoregressive process:

$$X_gap_t = \alpha_2 X_gap_{t-1} + \varepsilon_t^{gap} \tag{3.4}$$

where α_2 is a persistence of previous lag and ε_t^{gap} is interpreted in the same way as trend component shock.

Appendix 3.2. provides data after all transformations, which are used as an input in estimation process of the model.

The twenty two shocks in the model are consumption preference shocks both in Armenia and Russia, a risk premium shock (Armenia and Russia), a productivity shock in Armenia, productivity shocks in Russian tradable and non-tradable sectors, price mark-up shocks of home produced and imported goods in Armenia, price mark-up shocks of home produced tradable, non-tradable and imported goods in Russia, a wage mark-up shock (Armenia and Russia), a monetary policy shock (Armenia and Russia), a migration policy shock, a demand shock in Armenia, a labor supply shock in Russia, foreign demand, inflation and interest rate shocks. Besides monetary policy shocks, all shocks are assumed to follow a first order autoregressive process with i.i.d. normal error terms such that $\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \sigma_t$, where the shock $\varepsilon \in \{\varepsilon_C^{arm}, \varepsilon_{prem}^{arm}, \varepsilon_{prem}^{rus}, a^{arm}, a_T^{rus}, a_{NT}^{rus}, \varepsilon_{\pi_H}^{arm}, \varepsilon_{\pi_F}^{arm}, \varepsilon_{\pi_H}^{rus}, \varepsilon_{\pi_{NT}}^{rus}, \varepsilon_{\pi_F}^{rus}, \varepsilon_{\pi_W}^{arm}, \varepsilon_{\pi_W}^{rus}, \varepsilon_Y^{arm}$ $politics, \varepsilon_Y^{arm}, \varepsilon_N^{rus}, y_t^*, \pi_t^*, r_t^*\}$, $0 < \rho_\varepsilon < 1$ and $\sigma_t \sim N(0, \sigma_\varepsilon)$. Monetary policy shocks $\sigma_{r,t}^{arm}$ and $\sigma_{r,t}^{rus}$ are independent and identically distributed normal error terms.

4. Estimation

The estimation process of DSGE models, motivated with Bayesian ideas, consists of two stages. At the first stage, the approximation modal value of the posterior distribution is determined with numerical optimization methods. At the second stage, the shape of parameters distribution in the neighborhood of the posterior mode is estimated by applying MCMC techniques. Thus a sample from model parameters posterior distribution is generated. Most of the authors of DSGE models divide parameters into two groups: calibrated and estimated. Then the estimation process with Bayesian techniques is applied only to some of the parameters. Such a mixed approach is quite common in literature and could lead to more efficient estimation of the non-calibrated parameters (see Canova 2007). Together with 22 time series, model has 22 shocks to provide a stochastic singularity.

4.1. Calibration

As in other similar studies, some parameters are calibrated to match important stylized facts in the data. The values of fixed parameters are summarized in Table 1 (Appendix 4). The discount factor β for both countries is set at 0.99 to imply a real interest rate of 4%. The share of imported goods from Russia in Armenian CPI γ_1 is calibrated to 0.1 to match the fact that in the observed period imported goods from Russia compose 25% of Armenian import, which is 10% of consumption. The share of private consumption in GDP C^{SS} is assumed to be 0.85. Relative average wage in Armenia is two times small to Russian average wage $W_{SS}^{ARM}/(W_{SS}^{RUS} * E_{SS}^{ARM})$, which is observed in the 2003-2015 period. Share of non-tradable goods in Russian CPI γ_2 is approximated to be the share of services, which is 35% in consumer basket. Share of imported goods in the tradable basket of Russian households γ_3 is calculated by the ratio of Russian import to remaining 65% of consumption, which is 70%. The share of employment in Russian non-tradable sector γ_4 is

calibrated to 0.7, which is calculated by dividing the employment in construction, service and trade sectors to total employment. The non-tradable sector's share in Russian output γ_7 is calibrated to 0.6 to match the stylized facts on Russian national accounts. The share of Russian export from tradable sector γ_6 is calculated by dividing the Russian export to tradable sector output and it is fixed to be 0.65. According to some estimates, the seasonal migrants in Armenia are about 200000 people. So, the ratio of migrants in Armenian labor force N_M^{SS} is fixed to 0.15. On the other hand, this share in Russian non-tradable employment γ_6 is calibrated to be 0.4%. According to the estimations and surveys of Statistical Department of the Central Bank of Armenia, migrants spend on consumption in destination country about 30% of their incomes, so the τ is calibrated to 0.3. The coefficient of migration costs σ is set to 0.12 to match the overall migration costs to be 0.8% of GDP. The value of political factor $POLITICS^{SS}$ is calibrated to 3. The size of the last do not impact on model dynamics but only on the magnitude of migration policy shock. Following much of the literature on business cycles (see Alvarez-Parra et al (2011), Hove et al (2012) etc.), the share of capital in tradable sector α_2 is calibrated to 0.5 and the share of the last α_3 is set to 0.3 in the non-tradable sector. The share of capital in Armenian production function α_1 is calibrated to 0.35. Literature discusses the value of elasticity of substitution between tradable and non-tradable goods from 0.2 to 5. In a case of Armenia, this parameter η_1 is fixed to 1.1.

4.2. Prior distributions

The remaining parameters are estimated. Beta distribution is used for the parameters that take sensible values between zero and one, the gamma distribution for parameters restricted to be positive and the inverse gamma distribution for the shock variances (see Appendix 4: Table 2, Table 3, Table 4).

The relative disutility of working as a migrant is calibrated to 1.3 with 0.35 standard deviation, mentioning the home biasedness of Armenian household.

The prior mean of price stickiness coefficients of domestically produced goods in Armenia θ_H^{arm} and Russia θ_H^{rus}, θ_{NT}^{rus} is calibrated to 0.75 to match a yearly duration of prices. To mention the more monopolistic power of importers in comparison with domestic producers, prior means of price stickiness coefficients both in Armenia and Russia are set to 0.65 assuming the price duration of 3 quarters.

The prior mean of the Calvo parameters on wages ($\theta_W^{arm}, \theta_W^{rus}$) is set to 0.75 in order to match a yearly average wage renegotiation frequency, as in Dickens et al (2007). Following much of the literature, for example, Gali et al (2012), Grabek et al (2013) etc., prior means of the inverses of the elasticity of labor supply with wages for three types of workers (φ_{arm}, φ_m and φ_{rus}) are calibrated to 2.0 with 0.4 standard deviations. Given the well-known problem with identification of the elasticity of substitution across different varieties of labor, the prior means of these coefficients are set to the mean of suggested values by Smets and Wouters (2009) and Gali (2011). ϵ_w^{arm} and ϵ_w^{rus} are calibrated to 4.0, which implies a steady state mark-up of 33%.

The problem is also raised with identification of the values of parameters, which describe elasticity of substitution between different groups of goods. These coefficients are elasticity of substitution between tradable and non-tradable goods and elasticity of substitution between domestically produced and imported goods. Prior means of two parameters (η_2 and η_3) are set to 1.4 with 0.25 standard deviations.

The prior means of Taylor rule responses to inflation (μ_π^{arm} and μ_π^{rus}), output (μ_y^{arm} and μ_y^{rus}) and the interest rate smoothing parameters (ρ_r^{arm} and ρ_r^{rus}) equal to 1.5, 0.5 and 0.6 respectively. These values are commonly used in the literature.

As usually done in the literature, for example, Smets and Wouters (2007), all parameters of the autoregressive processes are calibrated at the same size (see Table 3). It is assumed that persistence coefficients of AR(1) processes have a beta distribution with mean value 0.72 and standard deviation of 0.1.

Prior distributions of standard errors of the shocks are calibrated using the volatility in the observed data. Overall, volatilities in the data are the same in Armenia and Russia, expect interest rates and prices. The prior calibration of standard errors of foreign shocks is smaller than the ones for Armenian and Russian structural shocks (see Table 4).

4.3. Parameter estimates

The fourth, fifth and sixth columns of Table 2 (structural parameters), Table 3 (persistence coefficients of AR(1) processes) and Table 4 (standard errors of the shock) show the posterior means together with 90% confidence intervals. The posterior parameter space is explored using the Metropolis-Hastings algorithm. The reported parameters are obtained from running 5 parallel chains with 1000000 draws, acceptance rates of which are 31.08%, 30.87%, 30.98%, 31.04% and 31.03% respectively. Figure 4.2 shows the convergence diagnostics of the likelihood function of the model. The blue line shows the 80% interval range based on the pooled draws from all sequences while the red line shows the mean interval range based on the draws of the individual sequences. The second and third column with the appended (m2) and (m3) show an estimate of the same statistics for the second and third central moments. The convergence is achieved when two lines are stabilized horizontally and should be close to each other. Judging by the comparisons between prior and posterior distributions (see Figure 4.1 in Appendix 4), data seems to be quite informative for most of the parameters. Parameters, which prior distributions are not updated, are elasticities of substitution between varieties of labor in Armenian and Russia, reaction of interest rate to inflation expectations in a Taylor rule of Armenian Central Bank, persistence coefficient of political shock and the persistence coefficient of labor supply of Russian households.

The posterior mean of relative disutility of working as a migrant is estimated to be 0.7, which rejects the home biasedness hypothesis of the Armenian households. They get more disutility from working in Russia with a comparison of the ones working in Armenia.

The posterior mean of Calvo price stickiness coefficient of Armenian home produced goods is 0.86, which corresponds to price duration of 7 quarters. The same coefficient for imported goods from Russia is estimated to 0.54. The judgement about the monopolistic power of importers, about which is spoken in prior distribution part, is proved. The posterior mean of price stickiness coefficient of domestically produced tradable goods is 0.69, which is lower because of the huge oil share in Russian tradable sector. The estimated price duration of imported goods in Russia is 3 quarters. The posterior mean of Calvo coefficient for non-tradable goods is 0.82.

The posterior means of wage stickiness is 0.79 and 0.62 for Armenia and Russia respectively. This shows that Armenian households negotiate about wages in average one time in 4.7 quarters. The duration of Russian households wage contracts is lower and estimated to 2.6 quarters. In average, the frequency of wage negotiations is higher than price duration in Russia. Price stickiness and wage stickiness are very close in Armenia. The posterior means of elasticities of substitution between varieties of labor are not much different from prior means and are estimated to 3.89 and 3.79 for Armenia and Russia respectively. The posterior means of the inverse of labor supply elasticities are 1.28, 2.81 and 1.9 for workers in Armenia, Armenian migrants and Russian households respectively. This means that worker in Armenia will increase a labor supply more than Russian household with an increase of real wages. The Armenian migrant prefers to increase labor supply with wage less because of relatively higher wages in Russia, which means that the substitution effect is much lower for Armenian migrant.

The posterior mean of the elasticity of substitution between tradable and non-tradable goods for Russian household is 1.03, which is different from its prior mean. On the other hand, the elasticity of substitution between domestically produced and imported tradable goods is estimated to be 0.56, which could be explained by some habits of Russian households. For example, when the relative prices of imported and home produced goods increase by 1%, Russian household decrease the demand for import only by 0.56%.

The persistence coefficients of interest rates are estimated to 0.67 and 0.46 for Armenia and Russia respectively. The posterior mean of reaction of interest rate to inflation expectations is 1.45 for Armenia, which is not much different from prior mean. Russian Central Bank reacts to inflation expectations with 1.78 coefficient. On the other hand, both Central Banks react to future output gap with the same size of 0.59.

The fourth, fifth and sixth columns of Table 3 show the posterior means of persistence parameters of autoregressive processes. The point posterior estimates suggest that their inertia in Armenia is approximately the same as in Russia. The only exceptions are the coefficient on the mark-up of imported goods and productivities. These coefficients are estimated to 0.81 and 0.35 respectively in Armenia and Russia for imported goods. The posterior mean of persistence coefficient of productivity shock in Armenia is 0.64, while the same coefficients in Russian tradable and non-tradable sectors are 0.78 and 0.86 respectively. The posterior means of the persistence parameters ρ_C^{arm}, ρ_C^{rus}, $\rho_{\pi_F}^{arm}$, ρ_{prem}^{rus}, $\rho_{a_{NT}}^{rus}$ and ρ_y^* equal to 0.85, 0.88, 0.81, 0.83, 0.86 and 0.85, indicate that shocks to consumption preference in Armenian and Russia, imported goods price mark-up shock in Armenia, risk premium shock in Russia, productivity shock in Russian non-tradable sector and foreign demand shock are substantially more persistence than the other shocks.

The posterior estimates of shock volatilities (see Table 4) confirm the prior assumption about their homogeneity across two countries. Standard deviations of the same structural shocks in Armenia and Russia are estimated approximately at the same size. The only exception is the volatility of the productivity shock in Armenia, which is higher to its Russian counterparts about 1.7 times. The posterior means of standard errors of the foreign shocks are three-four times smaller than the posterior means of standard errors of the shocks in Armenia and Russia, which is a feature of advanced countries. Overall, volatilities of the model shocks are at the reasonable size (the posterior mean of the standard error of the largest shock is 4.3%), which means that all shocks are economically interpretable.

5. Model Properties

This section overviews some of the model properties, which are the ability of the model to generate historical data, estimation of historical structural shocks, generation of Russian migration policy and real remittances data, historical decomposition of the main model variables, historical simulations, impulse response functions and conditional variance decomposition.

5.1. Generation of historical data and estimated structural shocks

Figure 5.1 in Appendix 5 captures the actual and model generated data. The last one is the model's Kalman filtered estimates at the posterior mean. Overall, the model's fit is quite well. The model is able to reproduce dynamics of migration, real wages in Armenia and Russia, interest rates in Armenia and Russia, real exchange rates and inflations data in both countries. The volatilities of the model generated data of employment are higher than the actual data. In Armenia, it is explained by volatilities of average worked hours, which is not included in the data. Labor force surveys in Russia do not cover migrants working in the country, which is a huge amount and could impact on a volatility of GDP.

Estimated structural shocks of the model are shown in figure 5.2. Consumption preference shock is the so-called Smets-Wouters (SW) internal risk premium shock (see Smets-Wouters 2007), which is important in DSGE models because of its capacity to generate business cycles co-movement among output, employment, consumption and migration. Krishnamurthy and Vissing-Jorgensen (2012) use "flight-to-quality" as an example of SW shock. Chari et al. (2009) assume that SW shock may pick up variation in flight-to-quality as well. Fisher (2014) concludes that the SW internal risk premium shock can be interpreted as a shock to the demand for safe and liquid assets. In general, this shock describes financial frictions in the economy.

From the fourth quarter of 2014 remittances inflowing to Armenia declined significantly, which was a result of Russian crisis. As shown in Figure 5.2, from fourth quarter of 2014 to first quarter of 2015 Russia had a higher risk premium. This came to Armenia, but the relative risk premium of Armenia to Russia did not increase (even it decreased a little bit). The decline of remittances increased Armenian risk premium to the rest of the world and generated higher depreciation expectations against US dollar among households. In that situation households' consumption preference increased. The model generates negative internal risk premium shock in Armenia, which is supported by stylized facts. That period Central Bank of Armenia tightened monetary conditions significantly to break higher inflation expectations. In a period from 2009 to 2010 model estimates positive risk premium shock in Armenia relative to Russia, which is explained by the deeper decline of Armenian economic growth and borrowing from Russia. Negative productivity shock in Armenia in 2010 was linked to bad weather conditions, which brought a huge decline of the agricultural sector. Since 2010 most amount of time productivity has shown positive values because of developing Armenian tradable sector (mostly mining sector and expansion of greenhouses). Model estimates a deep negative price mark-up shock of imported goods from Russia. This is also supported by stylized facts because from the first quarter of 2015 fees on imported goods from Russia were removed. A negative aggregate demand shock from 2009 to 2010 is explained by a decline of investments after crises. In the second quarter of 2015 government did an expansionary policy, which is captured in demand shock figure. Up to 2008 wage mark-up shock in Armenia is estimated to be mostly positive, which is a result of double-digit growth of nominal wages. Model mostly estimates periods of minimal wage increase in Armenia. From the second part of 2013, a positive wage mark-up shock is explained by an increase of nominal wages by some enterprises associated with pension reform.

A significant increase of cost of borrowing in Russia is overviewed by two crises. It is captured in a figure of consumption preference shock for 2008 and 2014. Productivity

shock in tradable sector in Russia after 2010 mostly performs to be positive, because of expansion of oil sector due to an increase in oil prices. Negative productivity shock in the non-tradable sector at the end of 2014 and in the first two quarters of 2015 is explained by a decrease of effective labor. Despite the employment of Russian households is not decreased as a result of 2014 crises, migrants and average working hours are declined, which generate a negative productivity shock in the non-tradable sector. Home produced tradable goods price mark-up shock is much more volatile compared to non-tradable goods price mark-up shock. A huge negative size of imported goods price mark-up shock was generated by Russian government through regulations of imported goods prices. The government did not allow to increase prices of some imported goods because of economic security of Russia, so their prices appeared to be low compared with marginal costs of importers. Prior to 2008, the wealth accumulation process of Russia forced to an increase of households substitution effect. It has generated mostly positive values of labor supply shock up to 2008. After Russian recovery from crises, the relatively higher wealth of households increased the income effect and households preferred to work less. The situation was changed after a recent crisis and in the second quarter of 2015 the value of the shock became positive. Model estimates foreign shocks in a way to generate historical data.

Overall, all the estimated structural shocks of the model are comparable with stylized facts.

5.2. Generation of Russian migration policy and real remittances data

The inverse of model estimated political factor is captured in Figure 5.3 in Appendix 5. The positive movement of it means a favorable situation in Russian migration policy for Armenian migrants. The estimated political factor fits the Russian migration policy quite well. At the end of 2005, Russia tightened the immigration legislation by adding the article "Organization of illegal migration" in the Criminal code, where measures against violating

immigration rules were tightened. The policy authorities simplified the registration process of temporary stay in Russia in 2007. Later in 2009 quotas of the permission of work were declined significantly, which generated negative migration policy shock in that year. Since March 25 of 2011, the deadline for the migration registration at the place of stay of foreign citizens has increased. Since March 2011, the Federal Migration Service of Russia has changed the procedure for registration of time making on the work of foreign citizens arriving in Russia in the visa-free regime. If previously a work permit could be requested on behalf of a foreigner for up to 3 months, now the documents are submitted to the state authority only at the request of the employer, which is decorated for up to one year. The result is the generated positive policy factor in 2011. The Order of First Deputy Mayor of Moscow on October 17, 2011, N957: "About measures to prevent illegal use of the labor of foreign nationals and their residence in the building objects". Heads of construction companies of Moscow were instructed to prevent the facts of illegal use of foreign nationals working and living in the building objects, which brought to higher expenditures of migrants associated with the hiring of rooms and apartments. On June 12, 2013, Russian President Vladimir Putin signed a decree N1239 "On the implementation of the State program to assist the voluntary resettlement to the Russian Federation of compatriots living abroad" containing a new version of Public program, which expanded entry channels into the program and other favorable conditions. Joining the Eurasian Economic Union generated the positive migration policy shock, which has turned out to negative value since 2015 because of a huge amount of backwardations of Armenian migrants.

Figure 5.4 captures real remittances. The red line is real remittances gap calculated by Hodrick-Prescott filter using real remittances data. The green line is filtered by quarterly projection model (QPM), which is a core model for policy analysis and forecasting in Central Bank of Armenia. Remittances data are not entered in the model developed in this paper. This model generates real remittances data (blue line in Figure 5.4). As one can

see, the model performs quite well and estimates the data, which are not input in the model.

5.3. Historical simulations

This exercise is done to test model properties to forecast for different horizons (2, 4, 6, 8 and 12 quarters ahead) and examine the usefulness of the model to capture business cycle movements observed in Armenia and Russia. All simulations are started at the same time, but for different horizons. The model simulations are presented in Figure 5.5 and Figure 5.6 in Appendix 5. Figures show that the model predicts quite well overall movements in migration, real wages in Armenia and Russia even up to 12 quarters. Model predictions of the real exchange rate of Armenian dram against Russian ruble and the ruble's real effective exchange rate, interest rates respectively in Armenia and Russia are also in line with business cycle patterns. This means that real remittances forecast in a 3 years horizon is also quite well because in this model real remittances are the combination of real Russian wages, migration and real effective exchange rate of Armenian dram against Russian ruble. Speaking of employment, it must be mentioned that model estimates effective working hours, which create GDP. Observed data on employment is the number of employed in the economy.

5.4. Historical decomposition

Figures 5.7-5.11 in Appendix 5 show contribution of structural shocks of the model to migration costs, migration, real remittances, Armenian and Russian real wages. Periods are represented on a horizontal axis starting from the first quarter of 2003 and ending in the second quarter of 2015. The vertical axis shows percentage deviation of the variables from steady state. As one can see from Figure 5.8, there are two periods of the pick of migration costs: world economic crises and recent Russian crises. In 2008 the main contributions to higher migration costs had a negative relative risk premium between

Armenia and Russia, which turned out to positive from the first quarter of 2009 because of a huge decline in Armenian GDP, negative consumption preference shock in Armenia and price mark-up shock on Russian tradable goods. Positive risk premium and the lower productivity in Russian non-tradable sector contributed to lower migration costs. In post crises period the relative risk premium volatility between Armenia and Russia is decreased. In a period from the fourth quarter of 2014, the higher migration costs were supported by consumption preference in Armenia, negative mark-up shock of Russian wages and tight monetary policy in Armenia.

Figure 5.7 displays contribution of disturbances to migration. Russian migration policy plays a significant role in migration (epss_m_arm in the figure). A huge decline of migration in 2009 was the result of negative productivity shock in Armenia, positive risk premium shock in Russia and the cut of work permissions in Russia. The recent decline of migration has been mostly supported by negative internal risk premium shock in Armenia, positive risk premium shock in Russia and negative Russian wages. The positive contribution to migration had an increase of borrowing costs in Russia, negative price mark-up shock on imported goods in Russia and positive productivity shock in Armenia.

Figure 5.9 shows the historical decomposition of the model generated remittances data. Almost all the shocks of the model had a visible contribution to the fluctuations of real remittances from steady state. World economic crisis in 2008-2009 and Russian crisis periods were characterized by a huge decline in real remittances inflowing to Armenia. In 2008, it was supported by negative productivity shock, imported goods negative price mark-up shock and Russian contrary migration policy. In a period of Russian crisis, decrease of remittances has been mostly supported by negative internal risk premium shock in Armenia, negative wage mark-up shock in Russia. On the other hand, some Russian shocks impacted on real remittances positively: positive monetary policy shock, negative productivity shock in non-tradable sector and negative price mark-up shock of imported goods.

As shown in Figure 5.10, the main supporters of real wage movements in Armenia were home produced good price mark-up shock, imported good mark-up shock and wage mark-up shocks in Armenia. The rise of Russian risk premium increased Armenian real wage. On the other hand, positive internal risk premium shock decreased real wage.

Figure 5.11 shows contributions of structural shocks on Russian real wage. The deep decline of wage in recent crises was contributed by negative wage mark-up shock, positive price mark-up of non-tradable goods and positive SW shock. 5% positive contribution to real wage made an administratively generated negative price mark-up shock of imported goods by the Russian government.

5.5. Impulse response functions

Figures 5.12-5.31 in Appendix 5 plot impulse responses of variables to the model structural shocks. This tool enables to better understand the functioning of the model and the impact of each shock on migration. Responses are measured as a percentage deviation from steady state. The horizontal axis shows the number of quarters following the shock. Structural shocks of the Armenian economy do not influence on Russia because Armenia is small to Russia. On the other side, shocks of the Russian economy hit Armenia because Russia is one of the main Armenian trading partners and because of it to be the main destination country of Armenian migrants (90% of cases). Shocks from external sector come to Armenian economy through Russia.

A consumption preference shock in Armenia (Figure 5.12) increases the cost of borrowing for a given interest rate, which initially worsens consumption and output. The fall in production lowers firms' demand for labor. This reduces marginal cost and the latter puts pressure on home produced goods prices to fall. As a result, it brings to decline in CPI inflation. The monetary policy reacts to weaker inflation and output by cutting interest rate. The lower interest rate depreciates nominal exchange rate and brings to an increase of marginal cost of importers, which is measured by a law of one price (LOP) gap. Higher

prices of imported goods lead terms of trade of Armenia with Russia to become worse. The last one with the LOP gap depreciates real exchange rate. Weak economic activity reduces marginal rate of substitution in the economy, which declines nominal wages. Nominal wages go down more than deflation and reduce real wages. Migration costs are decreased as a result of low real wages and depreciated real exchange rate. Migration has become less costly and people increase migration. Real remittances sent to Armenia are increased more with a support of depreciated national currency.

Figure 5.13 shows that risk premium shock in Armenia causes a depreciation of nominal exchange rate and increases inflation of imported goods. Higher inflation of imported goods worsens terms of trade, which depreciates real exchange rate. Consumers switch off their consumption from imported to home produced goods. As a result, overall consumption increases and firms increase their hire of labor to support higher demand in the economy. Improved economic activity rises marginal rate of substitution and pushes nominal wages to go up. Nominal wages increase real wages and the last one puts pressure on marginal cost to rise. This leads to a rise of home inflation and together with imported inflation supports to an increase of CPI inflation. Central bank reacts by tightening monetary conditions. The effect of depreciated real exchange rate is performed over higher wages and migration costs are declined. In its turn, the employment's effect is higher than lower migration costs and the migration is cut a little bit. But real remittances sent to Armenia are increased with a support of depreciated real exchange rate.

As shown in figure 5.14, productivity shock in Armenia initially reduces marginal cost of domestic firms, enabling them to decrease prices for their goods. It worsens terms of trade and depreciates real exchange rate, which forces consumers to consume more home produced and less imported goods. Productivity shock decreases the employment in a short term. Low employment reduces the substitution effect between leisure and consumption and impacts on nominal wages to go down. The decline of nominal wages is lower than the deflation in the economy. The result is that the real wage is increased and

its effect dominates the effect of depreciated real exchange rate. Migration costs are respond by becoming high. The low employment increases migration. The latter brings to higher remittances.

Figure 5.15 shows impulse responses to an aggregate demand shock in Armenia. Demand shock increases output on impact. The consumption falls as a result of the substitution effect. Firms hire more workers to service higher demand in the economy, which puts pressure on marginal cost and increases inflation of home produced goods. Inflation of domestic goods increases overall inflation in what monetary authorities respond by raising the interest rate. Higher economic activity raises the marginal rate of substitution and forces nominal wages to go up. The rise of nominal wages is more than the inflation of CPI, which increases real wages. The tight monetary policy appreciates nominal exchange rate and reduces the marginal cost of imports. The opposite direction of home produced goods and imported goods inflations improves terms of trade. The real exchange rate is appreciated in light of the better terms of trade. Both the real wage and the real exchange rate increase migration costs. Moreover, migration is declined as a result of both high employment and migration costs. Lower migration and appreciated real exchange rate affect real remittances to go down.

Figure 5.16 displays that a monetary policy shock in Armenia initially cuts consumption through Euler equation and appreciates nominal interest rate through uncovered interest parity condition. Firms hire less labor because of lower economic activity. This reduces nominal wage. The effect of nominal wages performs over deflation and real wages goes down, which reduce marginal cost and producers decrease prices for their goods. Appreciated nominal exchange rate reduces the marginal cost of importers and enables them to decrease prices. The latter is much higher to home produced goods deflation. As a result, terms of trade are improved, which appreciates real exchange rate. Migration costs are raised in a response of appreciated real exchange rate. The effect of low employment in Armenia exceed the effect of higher migration costs and households'

members increase migration, but the remittances are declined because of higher appreciated real exchange rate.

As shown in figure 5.17, domestic inflation shock in Armenia increases the terms of trade. Improved terms of trade appreciate real exchange rate. Monetary policy responses to the higher rate of inflation by increasing the interest rate. Higher interest rate leads to falling the consumption. Households become to consume more imported goods because of real exchange rate appreciation. The decline of consumption is supported by home produced goods. In a response to lower economic activity firms decrease their hires of labor. Fall of employment brings to both declines in marginal cost and the marginal rate of substitution. The latter reduces nominal wage. On the other hand, higher inflation supports the real wage to go down. The effect of low real wage performs over the appreciated real exchange rate and migration costs are declined, which affect more on migration than the worse employment does. Real remittances sent to Armenia are increased as a result of high migration.

Figure 5.18 captures responses of the model variables to imported good price mark-up shock in Armenia. Initially, it worsens the terms of trade and affects LOP gap to decline. They affect to real exchange rate to opposite directions and the power of LOP gap exceed the terms of trade by a coefficient of openness of the economy. So, the real exchange rate is appreciated. Imported goods inflation leads to CPI inflation. Central bank reacts to higher inflation by tightening the monetary conditions. In a reaction of interest rate increase, households postpone their consumption in future and decrease today's one. Employment is cut in a response to lower economic activity, which brings to falling of the marginal rate of substitution. MRS declines the nominal wage. The latter together with the high inflation decreases real wage significantly, which forces migration costs to fall. Low migration costs lead the migration to raise, but remittances are a little lower compare to migration because of appreciated real exchange rate.

Figure 5.19 shows that a wage mark-up shock in Armenia increases nominal and real wages. Real wage puts pressure on marginal cost to go up, raising the inflation above target. Monetary authorities react to higher inflation by increasing the interest rate. It declines the aggregate demand. Firms react to lower demand by cutting the employment. Home produced goods inflation brings to the improvement of terms of trade, which appreciate real exchange rate. Real wage together with appreciated exchange rate forces migration costs to go up. The effect of higher migration costs is exceeding over the effect of lower employment and the migration is decreased. As a result of lower migration and appreciated real exchange rate, remittances are declined.

As shown in Figure 5.20, a consumption preference or internal risk premium shock in Russia increases a cost of borrowing of households and decreases consumption. Initially, it decreases consumption in Armenian through international risk sharing condition. Aggregate employment in Russia is decreased as a result of lower economic activity in the non-tradable sector. In a response to the negative output gap, Central bank loses monetary conditions. The lower interest rate depreciates nominal exchange rate, which increases the marginal cost of importers. Higher inflation of imported goods worsens terms of trade and depreciates real exchange rate. Depreciated exchange rate increases export of Russian goods and does not let the output of tradable sector decline. Firms in tradable sector increase their hires of labor and put pressure on marginal costs to increase in a small amount. But prices of non-tradable goods are declined as a result of lower employment in that sector. As a result of higher inflation of imported goods, inflation of CPI is remained positive. Lower consumption and employment decrease the marginal rate of substitution in the economy. A drop of MRS reduces nominal and real wages. The latter rises migration costs of Armenian migrants by its size. Inflation in Russia hits Armenian economy through imported goods inflation and causes a decline in the terms of trade. Bad terms of trade depreciate real exchange rate of Armenian dram against Russian ruble. Firms hire less labor as a result of lower demand in the economy. Negative demand together with lower

employment decreases MRS. As a MRS is dropped, both nominal and real wages are declined. Migration costs are lower two quarters because effects of depreciated exchange rate and lower real wages in Armenia dominate on lower real wages in Russia. Since the third quarter, migration costs increase. The effect of lower employment performs over migration costs and household members increase their migration. Initially, remittances are increased because of higher migration and depreciated real exchange rate. Later the more drop of Russian wages decreases remittances.

Figure 5.21 represents that labor preference shock in Russia increases the disutility of work and force the MRS to go up. Higher MRS rises nominal and real wages. Hence, firms of both tradable and non-tradable sectors face higher costs for production. Rises of marginal costs increase inflation of home produced tradable and non-tradable goods. As a result, aggregate inflation in the economy is increased. Monetary authorities respond to higher inflation by increasing the interest rate. Higher interest rate decreases consumption, which triggers a fall in employment. Inflation in Russia is expressed in a rise of marginal costs of Armenian exporters, which leads to an inflation of imported goods. Imported goods' inflation worsens terms of trade and depreciates real exchange rate. Consumers switch their consumption towards home produced goods as a result of declined terms of trade. Depreciated exchange rate of Armenian dram and higher real wages in Russia decline migration costs. Lower migration costs increase a migration of households. Finally, real remittances sent to Armenian economy from Russia are raised with a support of all the three factors.

Figure 5.22 shows that risk premium shock in Russia initially depreciates both nominal and real exchange rates and brings to the inflation of imported goods. Depreciated real exchange rate increases the demand of foreign households for Russian export. Higher inflation of imported goods worsens terms of trade and Russian households increase their consumption of home produced goods. To provide higher external and internal demand, firms of both sectors rise their hires of labor. Higher economic activity puts pressure on

MRS to go up and increases nominal wages. A positive gap of employment impacts on the marginal cost of two sectors to rise, bringing to higher inflation. Inflation of CPI is higher than the rise in nominal wages, which forces real wage to decline. As a result of the initial depreciation of Russian ruble, Armenian dram is appreciated against Russian ruble nominally because Russian Central bank increases interest rate more than Armenian monetary authorities. Appreciation is lower with a comparison of inflation in Russia, leading to positive LOP gap. Terms of trade become to be worse as a result of higher imported inflation, which brings to the depreciation of real exchange rate. So, consumption is increased by a support of home consumption and firms start to hire more labor to service higher demand in the economy. It increases MRS and puts pressures on nominal wages to rise. Nominal wages are increased more than the observed inflation, leading real wages to become positive. Higher real wages increase marginal cost and hence inflation of home goods is raised. The effect of depreciated real exchange rate performs over the effect of higher wages in Russia and lower wages in Armenia, forcing migration costs to fall. Despite lower migration costs, migration declines as a result of higher employment in Armenia. Real remittances stay high by a support of depreciated real exchange rate.

Productivity shock in Russian tradable sector (Figure 5.23) initially brings to decline of marginal cost and employment of tradable sector. Deflation of home produced tradable goods results to lower terms of trade. In a situation of bad terms of trade, foreign and domestic households increase their demand for Russian tradable goods. Lower prices of tradable goods appreciate internal exchange rate, which forces households to decline their demand for non-tradable goods. The cumulative effect of productivity shock and decline of employment in non-tradable sector brings to decline of employment, putting pressures on MRS to fall. Nominal wages are decreased as a result of lower MRS. The size of deflation dominates over the decline of nominal wages, which result is expressed in higher real wages. The deflation is hitting the Armenian economy through cheap imported goods from Russia, which firstly creates the positive terms of trade. It appreciates real exchange rate.

Central bank reacts to lower inflation by decreasing the interest rate, which rises consumption. Real wages are increased a little bit as a result of good economic conditions, which affect on migration costs is invisible. Migration costs go down because of higher real wages in Russia, which creates a positive migration. As a result, remittances are increased.

As shown in figure 5.24, productivity shock in non-tradable sector decreases marginal cost of that sector's producers and leads them to decline the prices for their goods. Deflation of non-tradable goods depreciates internal exchange rate and switches the households towards consumption of non-tradable goods. Deflation of non-tradable goods bring to deflation of the aggregate good. Central bank decreases interest rate to support the inflation rise to it's target. Employment in the tradable sector is increased as a result of positive external demand for Russian tradable goods. A positive productivity shock in non-tradable sector forces firms to decrease the demand for labor input, the size of which is much higher to the expanded employment in the tradable sector, bringing to the decline of overall employment in the economy. Lower employment reduces MRS, which puts pressure on nominal wages to fall. In an initial period of simulation, real wages remain negative because of lower nominal wages. After the seventh period, it becomes positive as a deflation is going to its steady state slowly than nominal wage is. The negative inflation in Russia brings to deflation of imported goods in Armenia and appreciates the real exchange rate. The effect of this shock on the real sector of Armenian economy is very small and the variables associated with that are not captured in the figure. Migration costs are increased as a result of appreciated exchange rate and lower real wages in Russia. Hence, migration is declined because of higher migration costs for Armenian households. Real remittances are cut with the support of lower wages in Russia and declined migration.

Figure 5.25 demonstrates that the monetary policy shock in Russia forces consumers to decline the consumption. In a response to lower demand, firms both from tradable and non-tradable sectors decline their hiring of labor. Decreased labor puts pressures on marginal costs to go down. With comparison to non-tradable sector, the

marginal cost of tradable sector is falling more because the more capital is used in that sector. As a result, deflation hits the economy. The rise of interest rate appreciates the nominal exchange rate and imported goods become cheaper for Russian households, which leads to the increase of terms of trade. The latter together with LOP gap appreciates real exchange rate. The decline of consumption and employment reduces MRS. The slight decline of MRS affects nominal wages to decrease. The size of lower nominal wage is approximated to deflation size and real wages are not changed significantly. The monetary policy shock in Russia depreciates the nominal exchange rate of Armenian dram, which is higher than observed deflation in Russian and the result is that price of imported good from Russia is increased. It worsens terms of trade and depreciates real exchange rate. Consumers switch their consumption towards home produced goods and decrease demand for imported goods. But the overall consumption is raised in a small amount. As with a previous shock, the impact of the shock on the Armenian real sector has a very small effect. The driver of migration costs is a depreciated real exchange rate. Lower migration costs increase migration of households. Real remittances are increased more by the support of positive migration and depreciated real exchange rate.

Figure 5.26 shows that price mark-up shock on home produced Russian tradable goods brings to higher inflation and initially increases terms of trade. The latter appreciates real exchange rate and cuts the demand of foreign households for Russian export. The internal exchange rate is depreciated as a result of higher inflation of tradable goods. Depreciated internal exchange rate leads consumers to decrease the consumption of tradable goods and increase their demand for non-tradable goods. Higher demand for non-tradable goods rises the employment in that sector. On the other side, employment in the tradable sector is declined higher, which is also supported by the fall of export. As a result, the aggregate employment is reduced and puts pressure on MRS to go down. Lower MRS decreases nominal wage in Russia and with the effect of higher inflation real wages are dropped significantly. Higher inflation in Russia transmits to Armenia through higher

inflation of imported goods, which worsens terms of trade and depreciates real exchange rate. The effect of depreciated real exchange rate is small to the effect of lower real wages in Russia and migration costs go up. The incentive of household members towards migration is declined. Lower migration and wages in Russia open the negative gap of real remittances.

Price mark-up shock on non-tradable goods (Figure 5.27) increases the inflation of non-tradable goods and appreciates internal exchange rate. The real exchange rate is appreciated as a result of appreciated internal exchange rate. Overall inflation in the economy goes up. In a response to higher inflation, Central bank reacts by increasing the interest rate, which cuts the consumption of households. Lower demand decreases employment, which takes place due to the employment of non-tradable sector. Despite the increase in nominal wages, real wages go down because of high inflation. Prices of imported goods to Armenia become higher as a result of positive LOP gap. Increased imported prices worsen terms of trade and depreciate real exchange rate of Armenian dram. Imported inflation increases the overall inflation a little bit in what Central bank answers by tightening monetary conditions. The consumers decline their consumption and firms hire less labor to service the lower demand in the economy. As a result, nominal wages go down. The decline of real wages is higher because of inflation support. The effect of depreciated real exchange rate is covered by lower real wages in Russia and migration costs are raised. Higher migration costs reduce migration. The latter together with lower real wages in Russia decrease real remittances sent to Armenia.

As shown in Figure 5.28, price mark-up shock on imported goods initially worsens Russian terms of trade, which appreciate real exchange rate. But the effect of lower terms of trade performs over the appreciated exchange rate and Russian export is increased. Good external conditions for Russia force households to increase their demand for home-produced tradable and non-tradable goods. To service the higher demand in the economy, firms in both sectors become to hire more labor. A positive employment gap puts pressure

on marginal costs of both sectors to rise. As a result, the economy is hit by inflation of CPI. Improved economic conditions increase MRS. Higher MRS raises the nominal wage. But the observed inflation leads real wages to go down and goes to Armenia through higher inflation on imported goods from Russia. The latter worsens terms of trade and depreciates real exchange rate of Armenian dram against Russian ruble. Lower terms of trade rise the consumption of home-produced goods. To provide the overall supply, firms increase their hires of labor and put pressure on marginal cost to go up. Inflation of CPI is raised due to home and imported goods inflation. Improved economic activity increases nominal wage, which is lower than inflation. So, real wages are cut. The effect of lower real wages in Russia dominates on depreciated exchange rate of Armenian dram and migration costs are increased. Higher migration costs lead households to decline their migration. Lower migration and wages of migrants decrease real remittances sent to their families.

Figure 5.29 shows that wage mark-up shock in Russia initially increases nominal and real wages. This shock also impacts on migration costs to decline. The hiring of labor becomes costly, which generates higher marginal cost both in tradable and non-tradable sectors. Overall inflation goes up. Monetary authorities react to positive disturbance of inflation from its target by raising the interest rate. Tighter monetary conditions decrease consumption of households. Firms answer to lower demand in the economy by cutting the demand for employment, which impacts the inflation to go to the target through marginal costs. Higher inflation in Russia increases the marginal cost of Armenian importers and import becomes costly. It worsens terms of trade of Armenia and depreciates real exchange rate. Consumers switch their consumption from imported to home produced goods. The effect of worse terms of trade on consumption is totally positive, which raises the MRS in the economy. But the observed inflation is higher than the increase of nominal wages, so real wage goes down. All three factors impacting on migration costs decrease the latter. As a result, people in Armenia increase migration and the inflow of remittances is raised.

As demonstrated in Figure 5.30, foreign demand shock increases Russian export directly and consumption through international risk sharing condition. Firms expand production by hiring more labor to provide more internal and external demand. Higher employment increases MRS and upwards pressure on nominal wages. Nominal wages raise real wages and impact on marginal costs to increase from the target. It brings to inflation, which is not raised a lot because of cheap import. Central bank reacts to inflation and positive output gap by increasing the interest rate, which appreciates nominal exchange rate of the Russian ruble and depreciates Armenian dram. Inflation in Russia and depreciated exchange rate of Armenian national currency increase marginal cost of importers. Inflation of imported goods worsens terms of trade and depreciates real exchange rate. Consumption in Armenia is increased as a result of the foreign demand shock, which comes to Armenian economy through Russia. Firms generate positive output gap by hiring more workers. Nominal wages are increased, leading real wages to follow them because of relatively lower inflation. Migration costs are declined as a result of higher real wages in Russian and depreciated real exchange rate. But the effect of higher employment in Armenia performs over the lower migration costs and households cut migration. Despite the lower inflation, higher wages of migrants and depreciated exchange rate keep remittances above it's steady state.

Foreign interest rate shock (Figure 5.31) initially depreciates nominal exchange rate of the Russian ruble and makes the positive LOP gap. Higher marginal costs of importers let them increase prices, which worsen terms of trade. Terms of trade depreciate real exchange rate and increase the consumption of tradable and non-tradable goods. Depreciated real exchange rate increases the demand for foreign households to Russian export. Firms of both tradable and non-tradable sectors hire more labor to service the excess demand and put pressure on marginal costs to rise. Expanded economic activity increases nominal wages, which is less than inflation and brings to lower real wages. Observed higher inflation in Russia impacts on the marginal cost of Armenian importers

and price of import goes up. It worsens terms of trade and depreciates real exchange rate of Armenian dram. Russian positive consumption and expectations of the depreciation of real exchange rate increase consumption. Expansion of employment brings to the higher MRS and increase nominal wages. The impact on real wages is positive because of lower inflation. Higher employment and real wages upward pressure on marginal cost and enable domestic firms to increase prices on their goods. During this shock Armenian dram is nominally depreciated against Russian ruble because Central bank of Armenia increases interest rate less compare to the increase of interest rate in Russia. The total effect of wages in Armenia and Russia is compensated and does not have a significant effect on migration costs. Depreciated real exchange rate makes it to decline. Despite the cheap migration, migration falls as a result of expanded employment in Armenia. The effect of depreciated real exchange rate performs over lower migration and real wages in Russia, so the real remittances are increased.

5.6. Variance decomposition

Table 5 in Appendix 5 gives forecast error variance decomposition of migration costs and migration based on the posterior mean of model's posterior distribution. In a short run within 2 quarters, SW consumption preference shock explains about 40% variation of migration costs. Later this contribution becomes higher. Relative risk premium shock of Armenia against Russia in short period affects more on migration costs and this contribution lowers to about 8.7% in long horizons. Risk premium shock in Russia also has a contribution to the variation of migration costs. Its contribution is reducing over time. On the other hand, the contribution of price mark-ups and wage mark-ups shocks on migration costs in both countries rises over time. Together mark-up shocks explain 15% of the variation in first periods. The contribution of that shocks rise over time and counts to be 32%. The effects of monetary policy shocks in both countries are declined (from 11.5% in a first period to 2.8% in a long run horizon). Migration policy shock counts about 50%

variance of migration in the first period and declines to be 43.5% for longer periods. Consumption preference shock explains 16% variation of migration in the first period and it's contribution increases to 25% in 20 quarter. Another driving forces of migration are productivity and demand shocks, which contributions decrease over time. Risk premium and consumption preference shocks in Russia make an influence on migration by a 3% of the variation.

Table 6 provides variance decomposition of Armenian employment and real remittances. The main driving force of employment in Armenia is productivity shock, which contribution counts to be 43%. Consumption preference and demand shock explains about 20% variation of employment. Another important shocks are risk premium and monetary policy shocks, which totally contribute the 20% variation of employment. Russian structural shocks (consumption preference and risk premium) account for about 10% of the forecast error variance of Armenian employment in the first quarter. Later their contributions rise to 13% in longer horizons. Driving forces of the variance of real remittances are SW internal risk premium, the relative risk premium of Armenian against Russia and political factor shocks. The contribution of last two shocks decreases from 30% in first quarter to 17% in 20 quarter. As one can see, the contribution of political shock on remittances are declining significantly compare to it's decreasing effect on migration. On the other hand, effects of mark-up shocks on remittances are increasing over time. Price mark-up of domestically produced and wage mark-up shocks in Russia count more than 16% variation of remittances.

As shown in Table 7, the main forces of real wage variation in Armenia are price mark-ups and wage mark-up shocks, which together explain more than 85% of the variation. Russian structural shocks (consumption preference and risk premium) account the variation of Armenian real wages from 2.5% in a first quarter to 6% in 100 quarter. On the other hand, in the first period mark-up shocks drive 90% of the variation of Russian

real wages. Their contribution decreases to 60% over time. In long term horizons demand shock (consumption preference) becomes a dominant driver of real wages.

Table 8 reports forecast error variance decomposition of nominal wages in Armenia and Russia. The variation of Armenian nominal wage is mostly driven by wage mark-up shock, which contribution is higher than 58% over time. All other Armenian structural shocks explain a significant value of variance. The contribution of Russian structural shocks counts to be about 10%. In Russia, the 75% variation of nominal wage in a first period is driven by wage mark-up shock. In long horizons, the effect of it decreases to 54%. Other significant forces of variation are risk premium and productivity shocks, which together explain more than 28% of the variation in 20 quarter.

As shown in Table 9, employment in Russian tradable sector is mostly determined by productivity shock in tradable sector in a short term horizon. Later its contribution to the variation of employment decreases to 13.8%. On the other hand, the contribution of domestically produced tradable good price mark-up shock decreases from 29% to 60%. Another important driving forces of employment in tradable sector are consumption preference and risk premium shocks. Shocks from the rest of the world explain about 9.5% variation in the first period. The 63% variation of non-tradable sector's employment over time is governed by consumption preference shock and productivity shock in the non-tradable sector. The effect of risk premium shock decreases from 17% to 11%. Another important shocks are monetary policy and foreign.

6. Use of the Model in Central Bank of Armenia

This section shortly presents the use of the model in Armenian Central Bank's forecasting and policy analysis system (FPAS). Besides being the analytical tool for business cycle analysis, this model gives forecasts and provides the story about labor market developments in Armenia and Russia. This DSGE model is used as a satellite model in FPAS system by the forecasting team of CBA. One of the aims of the model is to forecast the real remittances gap, which is then used in the core model (QPM) for policy design. This model gets medium-term forecasts (3-year time horizon) of the main macroeconomic variables representing Armenian (real consumption, GDP, inflation, interest rate and real exchange rate), Russian (GDP, inflation and interest rate) and foreign (GDP, inflation and interest rate) economies from QPM and the model of external sector as an input and gives forecasts of labor market developments. The medium term forecast of real remittances gap goes back to QPM.

Figure 6.1 in Appendix 6 provides forecasts of the main variables up to the third quarter of 2018. The information about this scenario is presented in the CBA's Inflation report (fourth quarter 2015)[1]. Given the macroeconomic scenario for a medium-term horizon, DSGE model generates the forecast of the remaining variables of the model. The important variables are captured in Figure 6.2. As it is mentioned in the Inflation report, recovery of the Russian economy will only be expected at the end of 2016. The economic growth will reach 0.8% and the inflation will reach its target at the end of the forecast horizon. Under this scenario, the model predicts a depreciated real exchange rate of Russian ruble for the whole forecast horizon, which will affect the tradable sector to recover while the non-tradable sector's output will continue to be negative. As a result, the significant negative real consumption gap will be prolonged in the medium term horizon. The recovery of the Russian economy will be supported mostly by the export sector. Figure

[1] The link to the Inflation report on the Internet: https://www.cba.am/EN/pperiodicals/Gnach%204-15eng.pdf

6.2 shows that employment in the tradable sector will be positive at the forecast horizon while the non-tradable sector's employment will decline by reaching the steady state. Negative wage mark-up shock and higher inflation in Russia will keep real wages negative, which will put pressures on migration costs to be higher.

The fall of Armenian nominal wage growth from 9.2% in 2015 to 5.4% in 2016 will generate a negative real wage gap, which will affect on migration costs to decline by its size, but the latter's effect is about three-time lower compared to the effect of Russian wage. From the second quarter of 2016, the real exchange rate of Armenian dram against Russian ruble will depreciate because of higher inflation in Russia and lower inflation in Armenia. The effect of the depreciated exchange rate and negative Armenian real wages will exceed over Russian negative wages at the end of 2016 and migration costs will become negative. Since the beginning of 2017, migration costs will start to rise because of prolonged higher decline of Russian wages and the faster close of real exchange rate gap, but will remain close to the steady state level. The Armenian effective employment, captured in Figure 6.2, is the number of employed times the average weekly worked hours. The low observed effective employment is the result of the positive productivity shock, which took place in the second quarter of 2015. Employment will be negative until the second quarter of 2017, reflecting the negative output gap in Armenia. The decreasing migration costs and lower employment will force migration to raise and reach positive values from the second quarter of 2016. At the end of the forecast horizon, migration will fall to the negative field because of the higher employment and balanced migration costs that will be observed in that period. The gap of real remittances will be closing faster by the support of the depreciated real exchange rate. After being at the point around zero for two periods until the first quarter of 2017, it will start to decline because of prolonged lower real wages in Russia.

7. Conclusion

This paper develops the DSGE model with the migration of labor force from Armenia to Russia. The model is based on New Keynesian framework with the imperfect mobility of labor across countries. The imperfect mobility is discussed by introducing migration costs. Because Russia is the main destination country for the Armenian migrants, equations characterizing the Russian economy are derived from microeconomic foundations to better understand the transmission mechanism of Russian structural shocks on migration and real remittances.

The resulting model is estimated by Bayesian technique using Armenian and Russian data from the first quarter of 2003 to the second quarter of 2015. Posterior distributions of the model parameters are got by running the Metropolis-Hastings algorithm. Overall, the diagnostic measures of the estimation show that the estimation quality is quite well.

A number of key results emerge from the analysis. First, the volatility of migration is based more on extensive margin. On the other hand, the volatility of Armenian employment is mainly explained by an intensive margin. Secondly, the relative disutility of working as a migrant is lower compared to home employment. Thirdly, the model estimates the unobserved political factor, which is in line with the Russian migration policy and explains a huge amount of the Armenian migration volatility. Fourthly, impulse responses analysis show that the real exchange rate of Armenian dram against Russian ruble plays a significant role in the transmission mechanism of structural shocks to the migration and real remittances. Fifthly, the estimation points that the significant forces of the decline of migration in 2009 were the cut of work permissions in Russia, the negative productivity shock in Armenia and the positive risk premium shock in Russia. The second huge decline of migration took place in the period of recent Russian crisis, which was mostly driven by negative internal risk premium shock in Armenia (Smets-Wouters consumption preference shock), negative wage mark-up shock in Russia and positive risk premium shock in Russia.

The Smets-Wouters shock plays a significant role in driving the business cycle, which is a good topic for further research by constructing the model with financial frictions for the Armenian economy. Finally, historical decomposition of the model generated real remittances data and conditional variance decomposition capture that all the shocks of the model directly impact on real remittances volatility, which is a good instrument to study real remittances fluctuations deeply.

The estimated model replicates Armenian and Russian data. Moreover, the estimated structural shocks of the model capture the main stylized facts of the Armenian and Russian economies. Historical simulations show that the model is able to forecast the main variables of labor markets both in Armenian and Russia. Testing the ability of the model as a story telling instrument and the ability of the model to forecast labor market developments, the forecasting team of the Central Bank of Armenia considers this model appropriate to use in the forecasting and policy analysis system.

Appendix 1. Log-linearized model equations

Armenian block of the model

1. Marginal rate of substitution between consumption and labor

$$mrs_t^{arm} = c_t^{arm} + \varphi_{arm} n_t^{arm}$$

2. Consumption Euler equation with international risk sharing condition

$$c_t^{arm} = c_{t+1}^{rus} + q_{t+1}^{arm} - (r_t^{arm} - \pi_{t+1}^{arm} + \varepsilon_{C,t}^{arm})$$

3. Uncovered interest parity

$$r_t^{arm} = r_t^{rus} + \Delta e_{t+1}^{arm} + \varepsilon_{prem,t}^{arm}$$

4. Terms of trade

$$s_t^{arm} - s_{t-1}^{arm} = \pi_{H,t}^{arm} - \pi_{F,t}^{arm}$$

5. Deviation from the law of one price

$$\psi_t^{arm} - \psi_{t-1}^{arm} = \Delta e_t^{arm} + \pi_t^{rus} - \pi_{F,t}^{arm}$$

6. Real exchange rate

$$q_t^{arm} = \psi_t^{arm} - (1 - \gamma_1) s_t^{arm}$$

7. Production function

$$y_t^{arm} = a_t^{arm} + (1 - \alpha_1) n_t^{arm}$$

8. Real marginal cost

$$mc_t^{arm} = w_{r,t}^{arm} + \alpha_1 n_t^{arm} - \gamma_1 s_t^{arm} - a_t^{arm}$$

9. New Keynesian Phillips curve for home produced goods

$$\pi_{H,t}^{arm} = \beta \pi_{H,t+1}^{arm} + \lambda_H^{arm} mc_t^{arm} + \varepsilon_{\pi_H,t}^{arm}$$

10. New Keynesian Phillips curve for importers

$$\pi_{F,t}^{arm} = \beta\pi_{F,t+1}^{arm} + \lambda_F^{arm}\psi_t^{arm} + \varepsilon_{\pi_F,t}^{arm}$$

11. Consumption price index inflation

$$\pi_t^{arm} = (1-\gamma_1)\pi_{H,t}^{arm} + \gamma_1\pi_{F,t}^{arm}$$

12. New Keynesian wage inflation equation

$$\pi_{W,t}^{arm} = \beta\pi_{W,t+1}^{arm} - \lambda_W^{arm}(w_{r,t}^{arm} - mrs_t^{arm}) + \varepsilon_{\pi_W,t}^{arm}$$

13. Real wage

$$w_{r,t}^{arm} = w_{r,t-1}^{arm} + \pi_{W,t}^{arm} - \pi_t^{arm}$$

14. Migration equation

$$n_t^m = -\frac{1}{\chi_m\varphi_m}\left(1+\frac{COST^{SS}}{C^{SS}}\right)cost_t - \frac{\varphi_{arm}}{\varphi_m}n_t^{arm} - \frac{COST^{SS}POLITICS^{SS}}{\chi_m C^{SS}}politics_t$$

15. Demand for home produced goods

$$c_{H,t}^{arm} = -\eta_1\gamma_1 s_t^{arm} + c_t^{arm}$$

16. Demand for imported goods

$$c_{F,t}^{arm} = \eta_1(1-\gamma_1)s_t^{arm} + c_t^{arm}$$

17. Market clearing condition

$$y_t^{arm} = (1-\gamma_1)C^{SS}(c_t^{arm} - \eta_1\gamma_1 s_t^{arm}) + COST^{SS}N_M^{SS}(cost_t + n_t^m) + \varepsilon_{Y,t}^{arm}$$

18. Taylor rule

$$r_t^{arm} = \rho_r^{arm}r_{t-1}^{arm} + (1-\rho_r^{arm})\left(\mu_\pi^{arm}\pi_{t+1}^{arm} + \mu_y^{arm}y_{t+1}^{arm}\right) + \sigma_{r,t}^{arm}$$

19. Migration costs

$$cost_t = w_{r,t}^{arm} - w_{r,t}^{rus} - q_t^{arm}$$

20. Real remittances equation

$$tr_t = n_t^m + w_{r,t}^{rus} + q_t^{arm}$$

21. Consumption preference shock

$$\varepsilon_{C,t}^{arm} = \rho_C^{arm} \varepsilon_{C,t-1}^{arm} + \sigma_{C,t}^{arm}$$

22. Risk premium shock

$$\varepsilon_{prem,t}^{arm} = \rho_{prem}^{arm} \varepsilon_{prem,t-1}^{arm} + \sigma_{prem,t}^{arm}$$

23. Productivity shock

$$a_t^{arm} = \rho_a^{arm} a_{t-1}^{arm} + \sigma_{a,t}^{arm}$$

24. Price mark-up shock of home produced goods

$$\varepsilon_{\pi_H,t}^{arm} = \rho_{\pi_H}^{arm} \varepsilon_{\pi_H,t-1}^{arm} + \sigma_{\pi_H,t}^{arm}$$

25. Price mark-up shock of imported goods

$$\varepsilon_{\pi_F,t}^{arm} = \rho_{\pi_F}^{arm} \varepsilon_{\pi_F,t-1}^{arm} + \sigma_{\pi_F,t}^{arm}$$

26. Wage mark-up shock

$$\varepsilon_{\pi_W,t}^{arm} = \rho_{\pi_W}^{arm} \varepsilon_{\pi_W,t}^{arm} + \sigma_{\pi_W,t}^{arm}$$

27. Migration legislation shock

$$politics_t = \rho_{politics} politics_{t-1} + \sigma_{politics,t}$$

28. Demand shock

$$\varepsilon_{Y,t}^{arm} = \rho_Y^{arm} \varepsilon_{Y,t-1}^{arm} + \sigma_{Y,t}^{arm}$$

Russian block of the model

29. Marginal rate of substitution

$$mrs_t^{rus} = c_t^{rus} + (\varphi_{rus} n_t^{rus} + \varepsilon_{N,t}^{rus})$$

30. Employment of Russian households

$$n_t^{rus} = (1 - \gamma_4) n_{T,t}^{rus} + \gamma_4 n_{NT,t}^{rus}$$

31. Uncovered interest parity

$$r_t^{rus} = r_t^* + \Delta e_t^{rus} + \varepsilon_{prem,t}^{rus}$$

32. Consumption Euler equation with international risk sharing condition

$$c_t^{rus} = y_{t+1}^* + q_{t+1}^{rus} - (r_t^{rus} - \pi_{t+1}^{rus} + \varepsilon_{C,t}^{rus})$$

33. Deviation from the law of one price

$$\psi_t^{rus} - \psi_{t-1}^{rus} = \Delta e_t^{rus} + \pi_t^* - \pi_{F,t}^{rus}$$

34. Terms of trade

$$s_t^{rus} - s_{t-1}^{rus} = \pi_{H,t}^{rus} - \pi_{F,t}^{rus}$$

35. Internal exchange rate

$$s_{I,t}^{rus} - s_{I,t-1}^{rus} = \pi_{NT,t}^{rus} - \pi_{H,t}^{rus}$$

36. Real effective exchange rate

$$q_t^{rus} = \psi_t^{rus} - (1 - \gamma_3 + \gamma_2 \gamma_3) s_t^{rus} - \gamma_2 s_{I,t}^{rus}$$

37. Production function of tradable sector

$$y_{T,t}^{rus} = a_{T,t}^{rus} + (1 - \alpha_2) n_{T,t}^{rus}$$

38. Real marginal cost in tradable sector

$$mc_{T,t}^{rus} = w_t^{rus} - \gamma_3 (1 - \gamma_2) s_t^{rus} + \gamma_2 s_{I,t}^{rus} + \alpha_2 n_{T,t}^{rus} - a_{T,t}^{rus}$$

39. Production function of non-tradable sector

$$y_{NT,t}^{rus} = a_{NT,t}^{rus} + (1-\alpha_3)n_{N,t}^{rus}$$

40. Employment in non-tradable sector

$$n_{N,t}^{rus} = (1-\gamma_5)n_{NT,t}^{rus} + \gamma_5 n_t^m$$

41. Real marginal cost in non-tradable sector

$$mc_{NT,t}^{rus} = w_t^{rus} - \gamma_3(1-\gamma_2)s_t^{rus} - (1-\gamma_2)s_{I,t}^{rus} + \alpha_3 n_{N,t}^{rus} - a_{NT,t}^{rus}$$

42. New Keynesian Phillips curve for home produced tradable goods

$$\pi_{H,t}^{rus} = \beta\pi_{H,t+1}^{rus} + \lambda_H^{rus} mc_{T,t}^{rus} + \varepsilon_{\pi_H,t}^{rus}$$

43. New Keynesian Phillips curve for non-tradable goods

$$\pi_{NT,t}^{rus} = \beta\pi_{NT,t+1}^{rus} + \lambda_{NT}^{rus} mc_{NT,t}^{rus} + \varepsilon_{\pi_{NT},t}^{rus}$$

44. New Keynesian Phillips curve of importers

$$\pi_{F,t}^{rus} = \beta\pi_{F,t+1}^{rus} + \lambda_F^{rus}\psi_t^{rus} + \varepsilon_{\pi_F,t}^{rus}$$

45. Tradable goods' inflation equation

$$\pi_{T,t}^{rus} = (1-\gamma_3)\pi_{H,t}^{rus} + \gamma_3\pi_{F,t}^{rus}$$

46. Consumer price index inflation

$$\pi_t^{rus} = (1-\gamma_2)\pi_{T,t}^{rus} + \gamma_3\pi_{NT,t}^{rus}$$

47. New Keynesian wage inflation equation

$$\pi_{W,t}^{rus} = \beta\pi_{W,t+1}^{rus} - \lambda_W^{rus}(w_t^{rus} - mrs_t^{rus}) + \varepsilon_{\pi_W,t}^{rus}$$

48. Real wage

$$w_t^{rus} = w_{t-1}^{rus} + \pi_{W,t}^{rus} - \pi_t^{rus}$$

49. Demand of Russian households for home produced tradable goods

$$c_{H,t}^{rus} = -\gamma_3(\eta_3 - \eta_2\gamma_2)s_t^{rus} + \eta_2\gamma_2 s_{I,t}^{rus} + c_t^{rus}$$

50. Russian export

$$y_{EX,t}^{rus} = -\eta_3\gamma_3(1-\gamma_2)s_t^{rus} + \eta_3\gamma_2 s_{I,t}^{rus} + \eta_3 q_t^{rus} + y_t^*$$

51. Demand of Russian households for non-tradable goods

$$c_{NT,t}^{rus} = -\eta_2\gamma_3(1-\gamma_2)s_t^{rus} - \eta_2(1-\gamma_2)s_{I,t}^{rus} + c_t^{rus}$$

52. All produced non-tradable goods are consumed by Russian households

$$c_{NT,t}^{rus} = y_{NT,t}^{rus}$$

53. Home produced tradable goods are consumed by Russian and foreign households

$$y_{T,t}^{rus} = (1-\gamma_6)c_{H,t}^{rus} + \gamma_6 y_{EX,t}^{rus}$$

54. Market clearing condition

$$y_t^{rus} = (1-\gamma_7)y_{T,t}^{rus} + \gamma_7 y_{NT,t}^{rus}$$

55. Taylor rule

$$r_t^{rus} = \rho_r^{rus} r_{t-1}^{rus} + (1-\rho_r^{rus})\left(\mu_\pi^{rus}\pi_{t+1}^{rus} + \mu_y^{rus} y_{t+1}^{rus}\right) + \sigma_{r,t}^{rus}$$

56. Labor supply shock

$$\varepsilon_{N,t}^{rus} = \rho_N^{rus}\varepsilon_{N,t-1}^{rus} + \sigma_{N,t}^{rus}$$

57. Risk premium shock

$$\varepsilon_{prem,t}^{rus} = \rho_{prem}^{rus}\varepsilon_{prem,t-1}^{rus} + \sigma_{prem,t}^{rus}$$

58. Consumption preference shock

$$\varepsilon_{C,t}^{rus} = \rho_C^{rus}\varepsilon_{C,t-1}^{rus} + \sigma_{C,t}^{rus}$$

59. Productivity in tradable sector

$$a_{T,t}^{rus} = \rho_{a_T}^{rus} a_{T,t-1}^{rus} + \sigma_{a_T,t}^{rus}$$

60. Productivity in non-tradable sector

$$a_{NT,t}^{rus} = \rho_{a_{NT}}^{rus} a_{NT,t-1}^{rus} + \sigma_{a_{NT},t}^{rus}$$

61. Price mark-up shock of home produced tradable goods

$$\varepsilon_{\pi_H,t}^{rus} = \rho_{\pi_H}^{rus} \varepsilon_{\pi_H,t-1}^{rus} + \sigma_{\pi_H,t}^{rus}$$

62. Price mark-up shock of non-tradable goods

$$\varepsilon_{\pi_{NT},t}^{rus} = \rho_{\pi_{NT}}^{rus} \varepsilon_{\pi_{NT},t-1}^{rus} + \sigma_{\pi_{NT},t}^{rus}$$

63. Price mark-up shock of imported goods

$$\varepsilon_{\pi_F,t}^{rus} = \rho_{\pi_F}^{rus} \varepsilon_{\pi_F,t-1}^{rus} + \sigma_{\pi_F,t}^{rus}$$

64. Wage mark-up shock

$$\varepsilon_{\pi_W,t}^{rus} = \rho_{\pi_W}^{rus} \varepsilon_{\pi_W,t}^{rus} + \sigma_{\pi_W,t}^{rus}$$

External world

65. Foreign demand

$$y_t^* = \rho_y^* y_{t-1}^* + \sigma_{y,t}^*$$

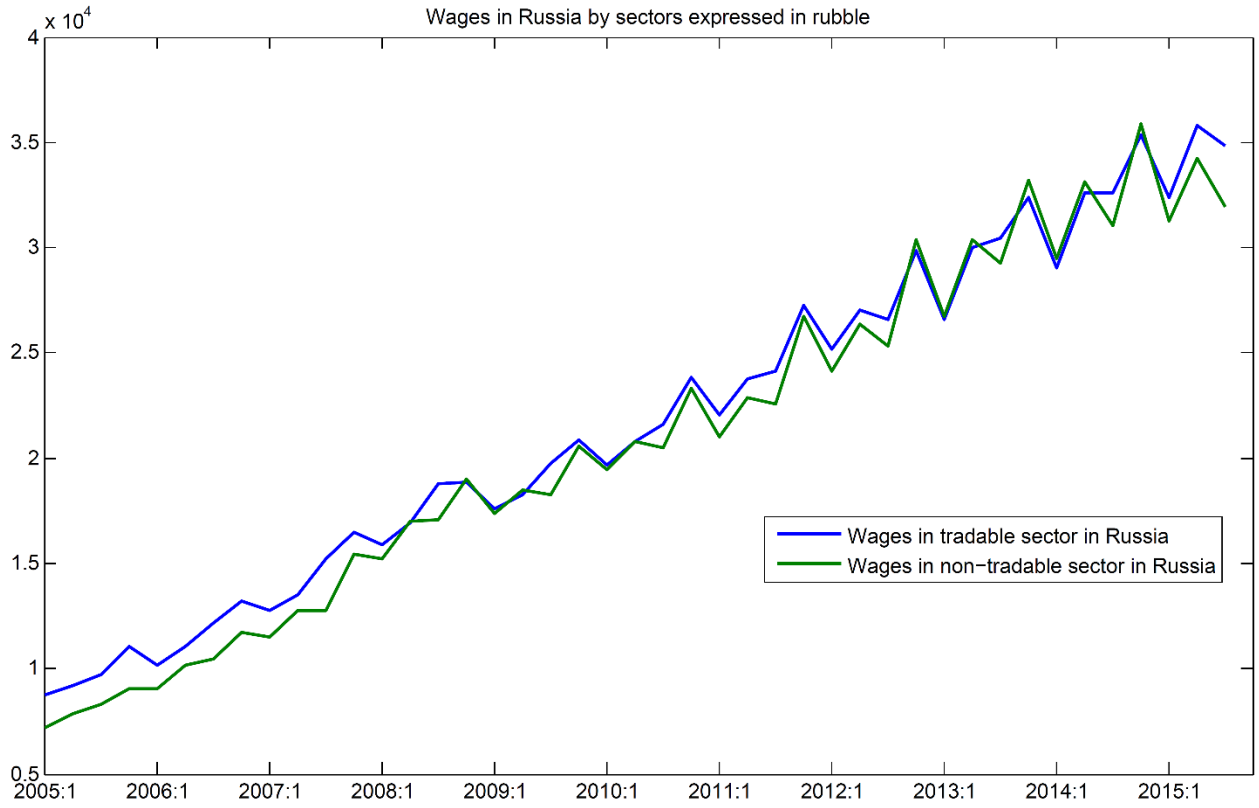
66. Foreign inflation

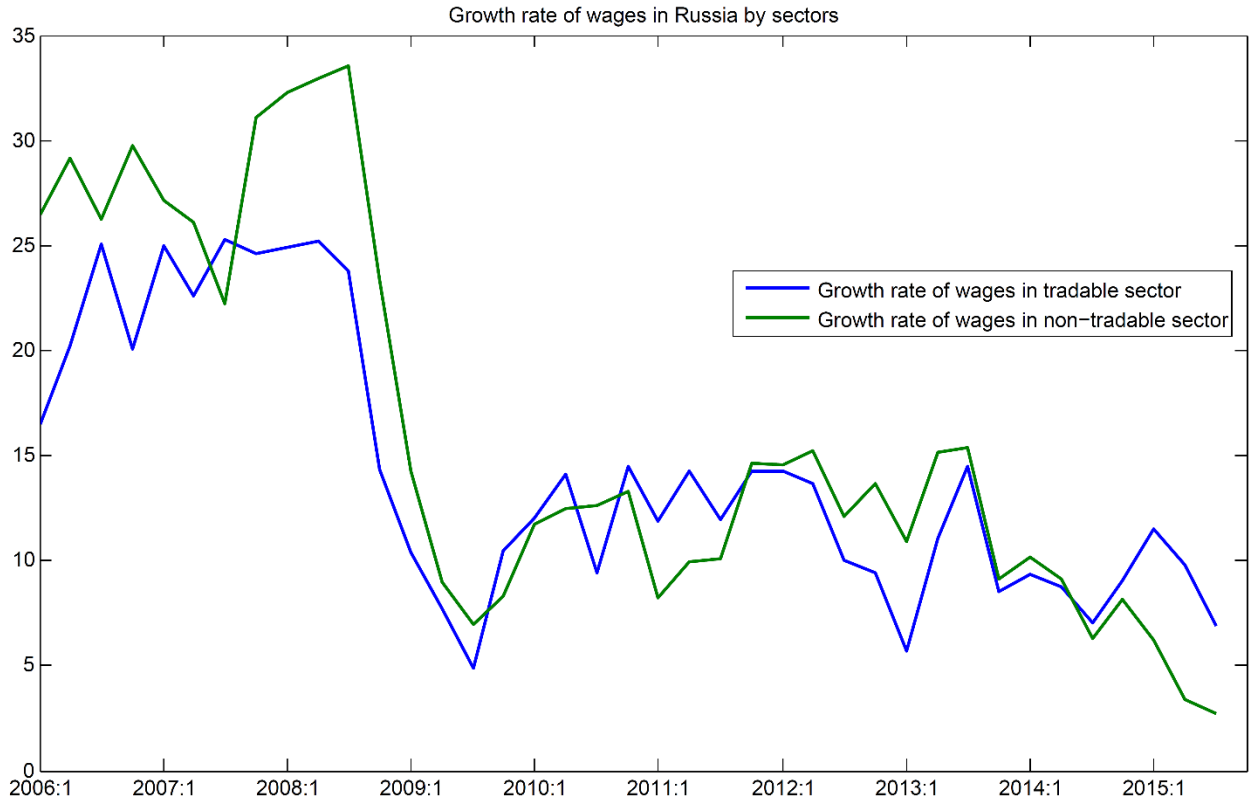
$$\pi_t^* = \rho_\pi^* \pi_{t-1}^* + \sigma_{\pi,t}^*$$

67. Foreign interest rate

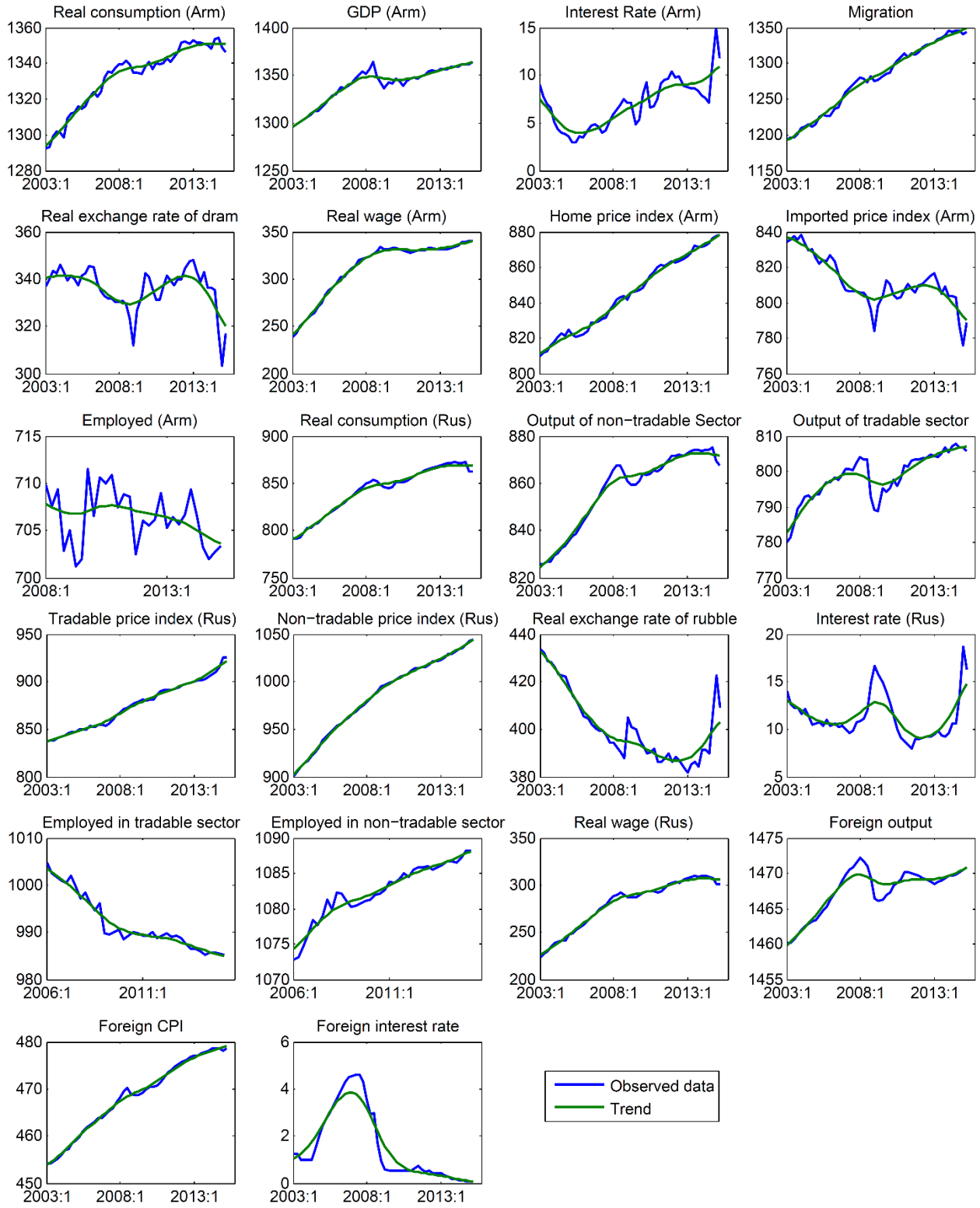
$$r_t^* = \rho_r^* r_{t-1}^* + \sigma_{r,t}^*$$

Appendix 2. Wages in Russia

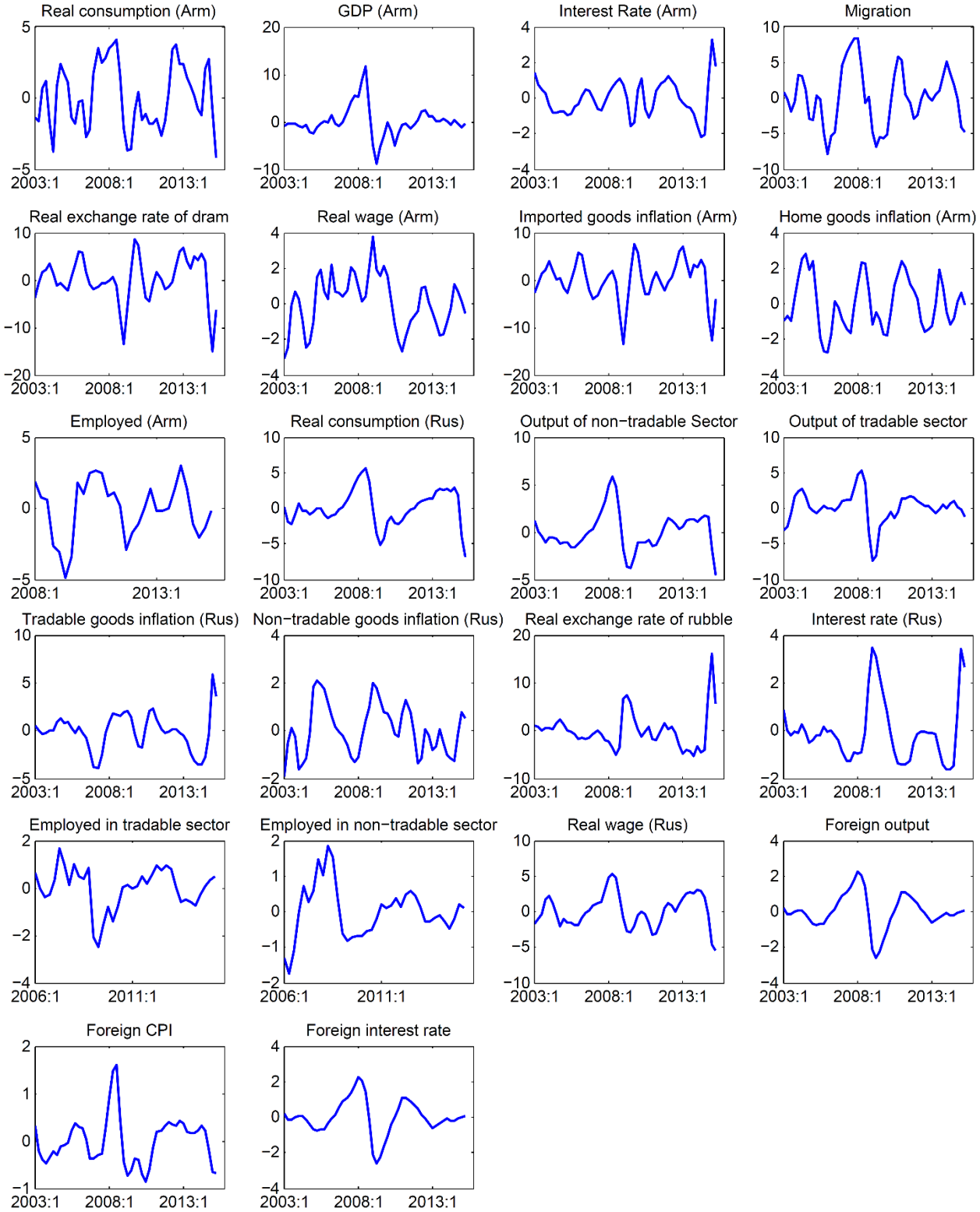




Appendix 3.1. Observed Data and Removed Trends



Appendix 3.2. Model Input Data



Appendix 4. Estimation Results

Table 1: Fixed parameters

Parameters	Description	Values
β	Discount factor	0.99
γ_1	Share of imported goods from Russia in Armenian CPI	0.1
α_1	Share of capital in production function of Armenian firms	0.35
C^{SS}	Share of private consumption in Armenian GDP	0.85
N_M^{SS}	Share of migrants in Armenian labor force	0.15
τ	Share of income that migrants spend in Russia	0.3
$W_{SS}^{ARM}/(W_{SS}^{RUS} * E_{SS}^{ARM})$	Relative wages of workers	0.5
σ	Coefficient of migration costs	0.12
$POLITICS^{SS}$	Political factor	3
γ_2	Share of non-tradable goods in Russian CPI	0.35
γ_3	Share of imported goods in tradable basket of Russian household	0.7
γ_5	Share of Armenian migrants in Russian non-tradable employment	0.004
γ_4	Share of employment of non-tradable sector in overall employment	0.7
γ_6	Share of export of Russian tradable goods	0.65
γ_7	Share of non-tradable sector in total output in Russia	0.6
η_1	Elasticity of substitution between home produced and imported goods from Russia for Armenian household	1.1
α_2	Share of capital in production function of Russian tradable sector	0.5
α_3	Share of capital in production function of Russian non-tradable sector	0.3

Table 2: Prior and posterior distributions of structural parameters

	Description	Prior mean	Posterior mean	5%	95%	Prior dist.	Prior SD
θ_H^{arm}	Price stickiness coefficient of home produced goods (Armenia)	0.75	0.86	0.77	0.97	Beta	0.15
θ_F^{arm}	Price stickiness coefficient of imported goods from Russia	0.65	0.54	0.46	0.63	Beta	0.15
θ_H^{rus}	Price stickiness coefficient of home produced tradable goods (Russia)	0.75	0.69	0.5	0.89	Beta	0.15
θ_{NT}^{rus}	Price stickiness coefficient of home produced non-tradable goods (Russia)	0.75	0.82	0.74	0.89	Beta	0.15
θ_F^{rus}	Price stickiness coefficient of imported goods (Russia)	0.65	0.67	0.58	0.76	Beta	0.15
χ_m	Relative disutility working as a migrant	1.3	0.7	0.44	0.96	Gamma	0.35
θ_W^{arm}	Stickiness coefficient of wages in Armenia	0.75	0.79	0.68	0.89	Beta	0.15
θ_W^{rus}	Stickiness coefficient of wages in Russia	0.75	0.62	0.51	0.72	Beta	0.15
φ_{arm}	Inverse of the elasticity of labor supply with wage for Armenian households	2.0	1.28	0.85	1.71	Gamma	0.4
φ_m	Inverse of the elasticity of labor supply with wage for Armenian migrants	2.0	2.81	2.13	3.48	Gamma	0.4
φ_{rus}	Inverse of the elasticity of labor supply with wage for Russian households	2.0	1.9	1.26	2.51	Gamma	0.4
ϵ_W^{arm}	Elasticity of substitution between varieties of labor in Armenia	4.0	3.89	3.11	4.70	Gamma	0.5
ϵ_W^{rus}	Elasticity of substitution between varieties of labor in Russia	4.0	3.76	2.97	4.53	Gamma	0.5
η_2	Elasticity of substitution between tradable and non-tradable goods	1.4	1.03	0.78	1.26	Gamma	0.25
η_3	Elasticity of substitution between home produced and imported goods for Russian household	1.4	0.56	0.42	0.71	Gamma	0.25
ρ_r^{arm}	Persistence coefficient of interest rate in Armenia	0.6	0.67	0.58	0.76	Beta	0.2
ρ_r^{rus}	Persistence coefficient of interest rate in Russia	0.6	0.46	0.38	0.55	Beta	0.2
μ_π^{arm}	Reaction of interest rate to inflation expectations in a Taylor rule of Armenian Central Bank	1.5	1.45	1.13	1.75	Gamma	0.25
μ_π^{rus}	Reaction of interest rate to inflation expectations in a Taylor rule of Russian Central Bank	1.5	1.78	1.5	2.05	Gamma	0.25
μ_y^{arm}	Reaction of interest rate to future output gap in a Taylor rule of Armenian Central Bank	0.5	0.59	0.44	0.73	Gamma	0.1
μ_y^{rus}	Reaction of interest rate to future output gap in a Taylor rule of Russian Central Bank	0.5	0.59	0.46	0.72	Gamma	0.1

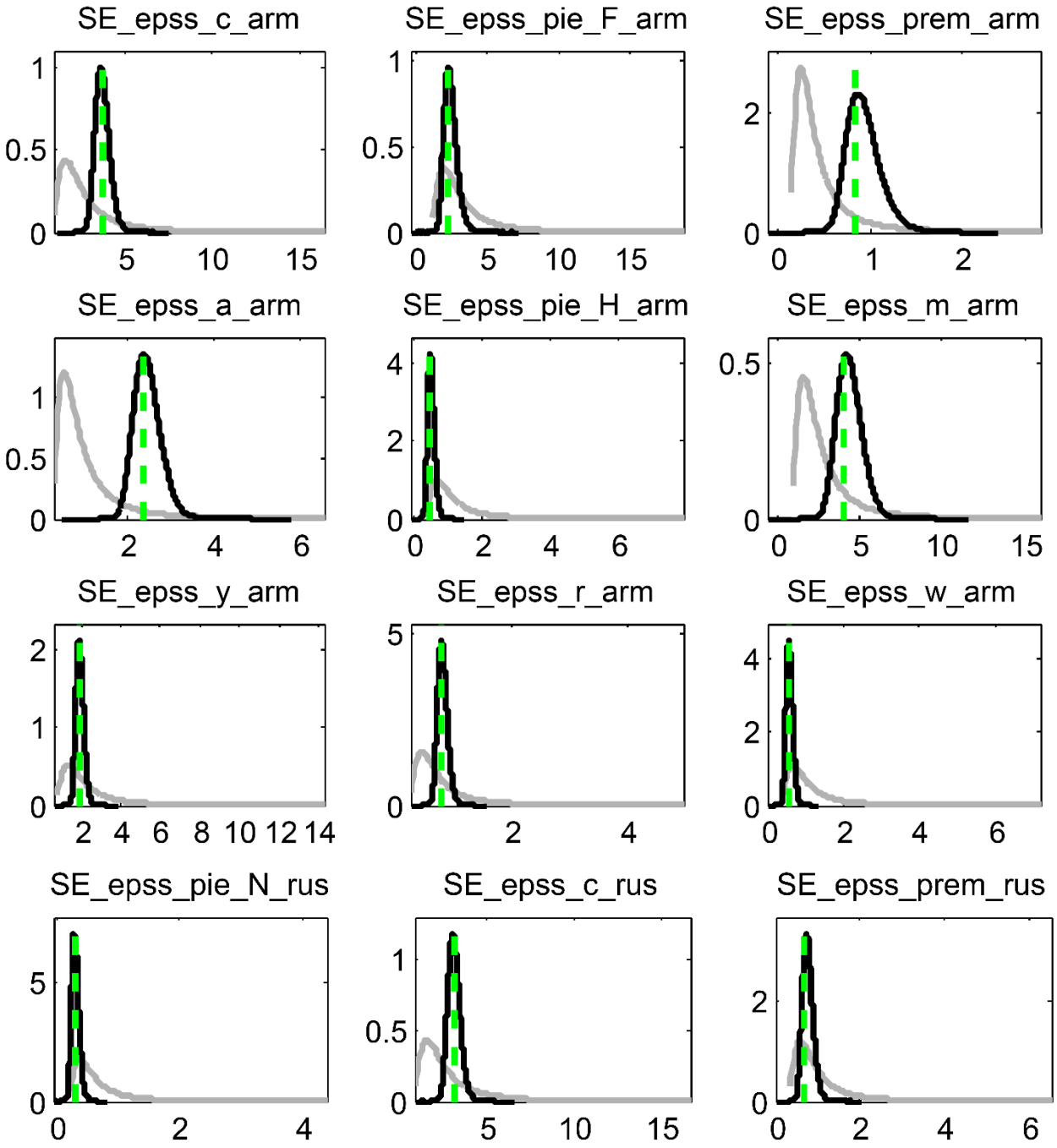
Table 3: Prior and posterior distribution of autoregressive parameters of the shocks

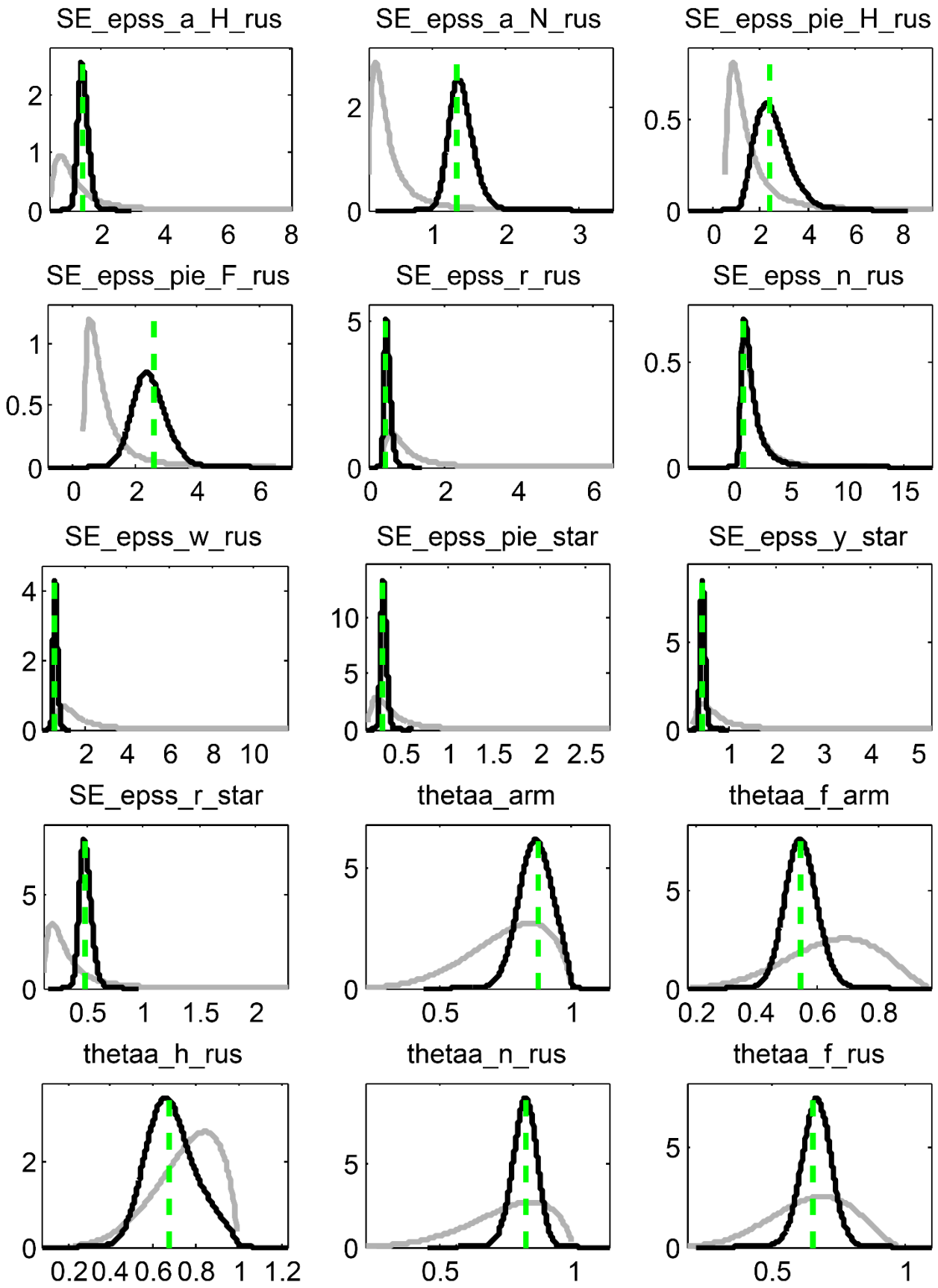
	Description	Prior mean	Posterior mean	5%	95%	Prior dist.	Prior SD
ρ_c^{arm}	Consumption preference (Armenia)	0.72	0.85	0.79	0.92	Beta	0.1
ρ_{prem}^{arm}	Risk premium (Armenia)	0.72	0.76	0.67	0.86	Beta	0.1
ρ_a^{arm}	Productivity (Armenia)	0.72	0.64	0.53	0.75	Beta	0.1
$\rho_{\pi_H}^{arm}$	Price mark-up of domestically produced goods (Armenia)	0.72	0.6	0.49	0.72	Beta	0.1
$\rho_{\pi_F}^{arm}$	Price mark-up of imported goods from Russia	0.72	0.81	0.72	0.91	Beta	0.1
$\rho_{\pi_W}^{arm}$	Wage mark-up (Armenia)	0.72	0.62	0.49	0.74	Beta	0.1
$\rho_{politics}$	Political factor	0.72	0.73	0.62	0.85	Beta	0.1
ρ_y^{arm}	Demand (Armenia)	0.72	0.59	0.47	0.71	Beta	0.1
ρ_c^{rus}	Consumption preference (Russia)	0.72	0.88	0.82	0.95	Beta	0.1
ρ_{prem}^{rus}	Risk premium (Russia)	0.72	0.83	0.77	0.89	Beta	0.1
ρ_N^{rus}	Labor supply (Russia)	0.72	0.70	0.54	0.86	Beta	0.1
$\rho_{a_T}^{rus}$	Productivity in tradable sector (Russia)	0.72	0.78	0.67	0.90	Beta	0.1
$\rho_{a_{NT}}^{rus}$	Productivity in non-tradable sector (Russia)	0.72	0.86	0.81	0.93	Beta	0.1
$\rho_{\pi_H}^{rus}$	Price mark-up of domestically produced tradable goods (Russia)	0.72	0.73	0.59	0.87	Beta	0.1
$\rho_{\pi_{NT}}^{rus}$	Price mark-up of domestically produced non-tradable goods (Russia)	0.72	0.67	0.56	0.79	Beta	0.1
$\rho_{\pi_F}^{rus}$	Price mark-up of imported goods (Russia)	0.72	0.35	0.24	0.46	Beta	0.1
$\rho_{\pi_W}^{rus}$	Wage mark-up (Russia)	0.72	0.53	0.42	0.64	Beta	0.1
ρ_y^*	Foreign demand	0.72	0.85	0.77	0.93	Beta	0.1
ρ_π^*	Foreign inflation	0.72	0.78	0.69	0.86	Beta	0.1
ρ_r^*	Foreign interest rate	0.72	0.71	0.62	0.80	Beta	0.1

Table 4: Prior and posterior distribution of standard errors of the shocks

	Description	Prior mean	Posterior mean	5%	95%	Prior dist.	Prior SD
σ_c^{arm}	Consumption preference (Armenia)	3.2	3.71	3.03	4.35	Inv. gamma	6.0
σ_{prem}^{arm}	Risk premium (Armenia)	0.5	0.93	0.63	1.21	Inv. Gamma	2.0
σ_r^{arm}	Monetary policy (Armenia)	0.9	0.8	0.66	0.94	Inv. Gamma	3.0
σ_a^{arm}	Productivity (Armenia)	1.2	2.48	1.98	2.96	Inv. Gamma	4.0
$\sigma_{\pi_H}^{arm}$	Price mark-up of domestically produced goods (Armenia)	1.4	0.52	0.36	0.67	Inv. Gamma	5.0
$\sigma_{\pi_F}^{arm}$	Price mark-up of imported goods from Russia	3.8	2.43	1.71	3.13	Inv. Gamma	6.0
$\sigma_{\pi_W}^{arm}$	Wage mark-up (Armenia)	1.3	0.55	0.41	0.7	Inv. Gamma	4.0
$\sigma_{politics}$	Political factor	3.1	4.33	3.09	5.55	Inv. Gamma	6.0
σ_y^{arm}	Demand (Armenia)	2.7	1.94	1.63	2.25	Inv. Gamma	6.0
σ_c^{rus}	Consumption preference (Russia)	3.2	3.18	2.61	3.72	Inv. Gamma	6.0
σ_{prem}^{rus}	Risk premium (Russia)	1.2	0.75	0.54	0.94	Inv. Gamma	4.0
σ_r^{rus}	Monetary policy (Russia)	1.2	0.47	0.33	0.61	Inv. Gamma	4.0
σ_N^{rus}	Labor supply (Russia)	2.2	1.77	0.54	3.14	Inv. Gamma	6.0
$\sigma_{a_T}^{rus}$	Productivity in tradable sector (Russia)	1.5	1.46	1.19	1.72	Inv. Gamma	4.0
$\sigma_{a_{NT}}^{rus}$	Productivity in non-tradable sector (Russia)	0.5	1.39	1.13	1.66	Inv. Gamma	2.0
$\sigma_{\pi_H}^{rus}$	Price mark-up of domestically produced tradable goods (Russia)	1.7	2.53	1.36	3.63	Inv. Gamma	4.0
$\sigma_{\pi_{NT}}^{rus}$	Price mark-up of domestically produced non-tradable goods (Russia)	0.8	0.31	0.22	0.41	Inv. Gamma	3.0
$\sigma_{\pi_F}^{rus}$	Price mark-up of imported goods (Russia)	1.2	2.48	1.6	3.36	Inv. Gamma	4.0
$\sigma_{\pi_W}^{rus}$	Wage mark-up (Russia)	2.1	0.65	0.49	0.81	Inv. Gamma	6.0
σ_y^*	Foreign demand	0.9	0.47	0.39	0.55	Inv. Gamma	3.0
σ_π^*	Foreign inflation	0.5	0.31	0.26	0.36	Inv. Gamma	2.0
σ_r^*	Foreign interest rate	0.4	0.48	0.4	0.56	Inv. Gamma	2.0

Figure 4.1. Prior and posterior densities

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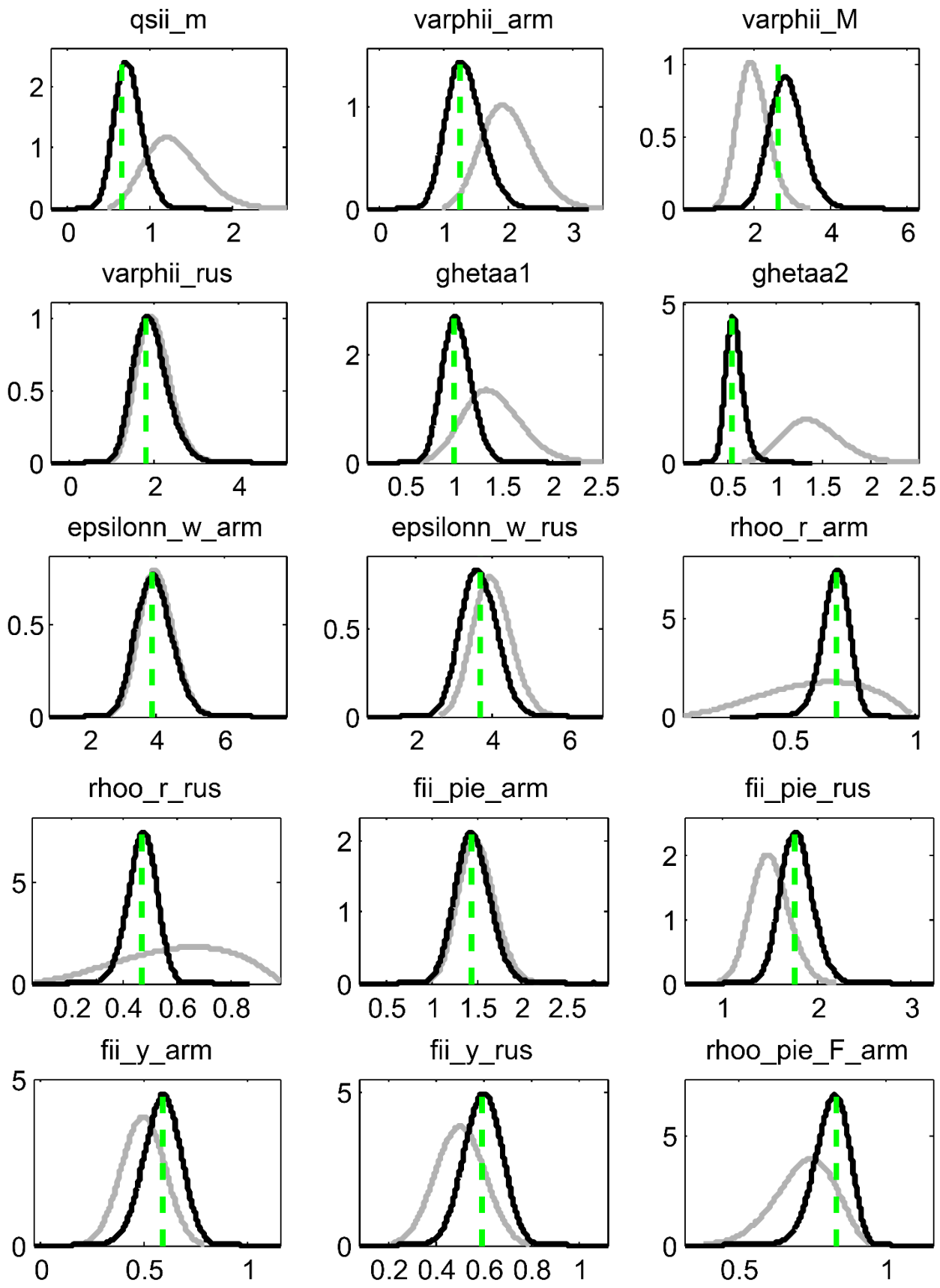
thetaa_f_rus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varphii_rus
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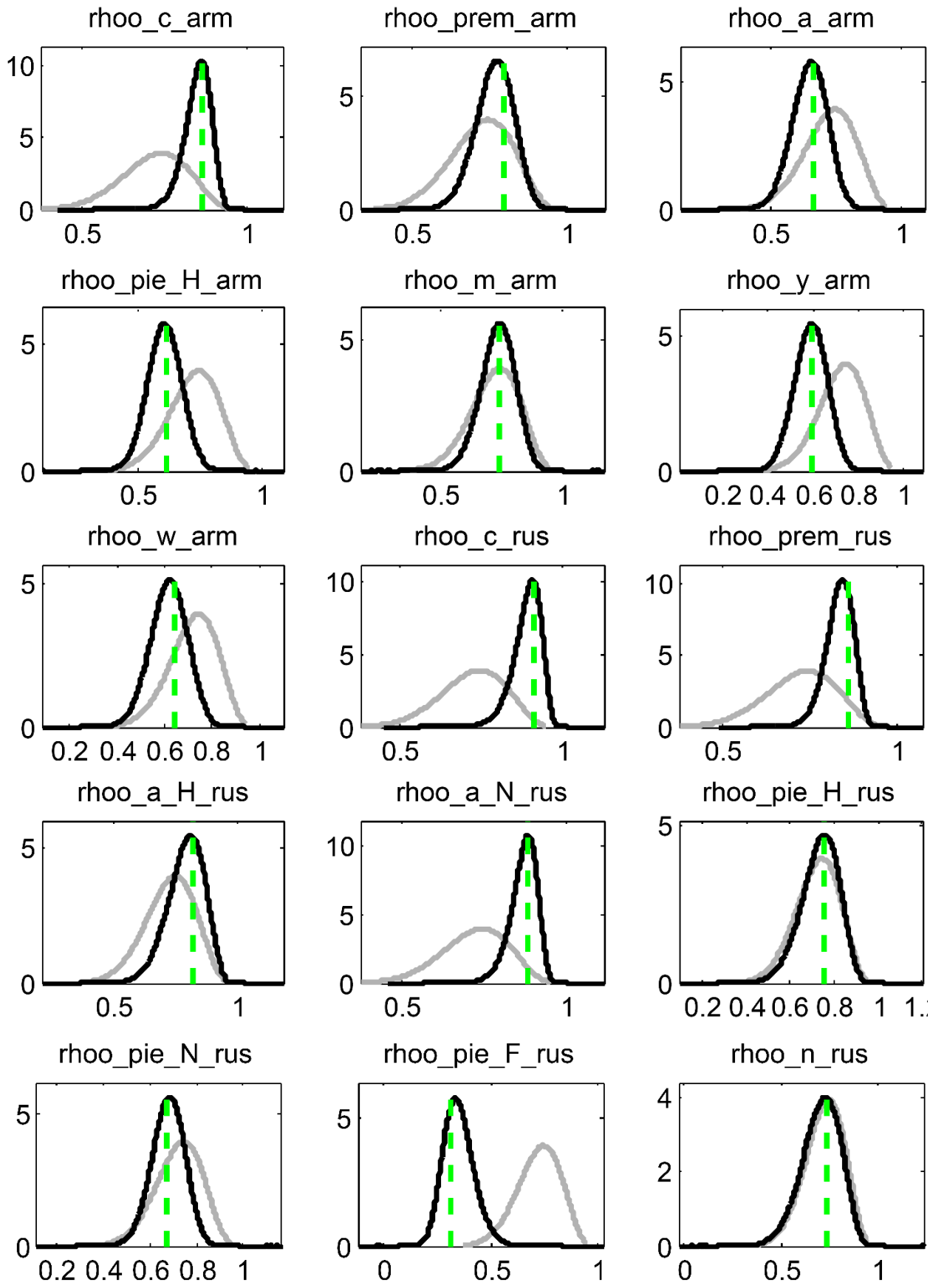
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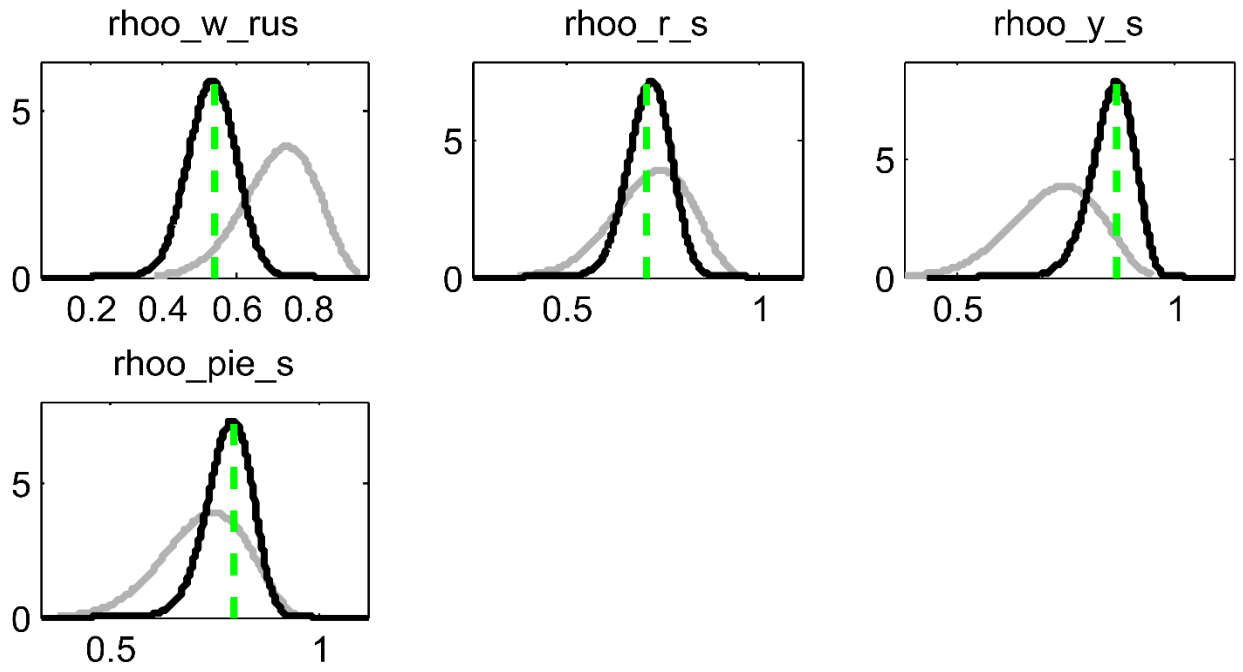
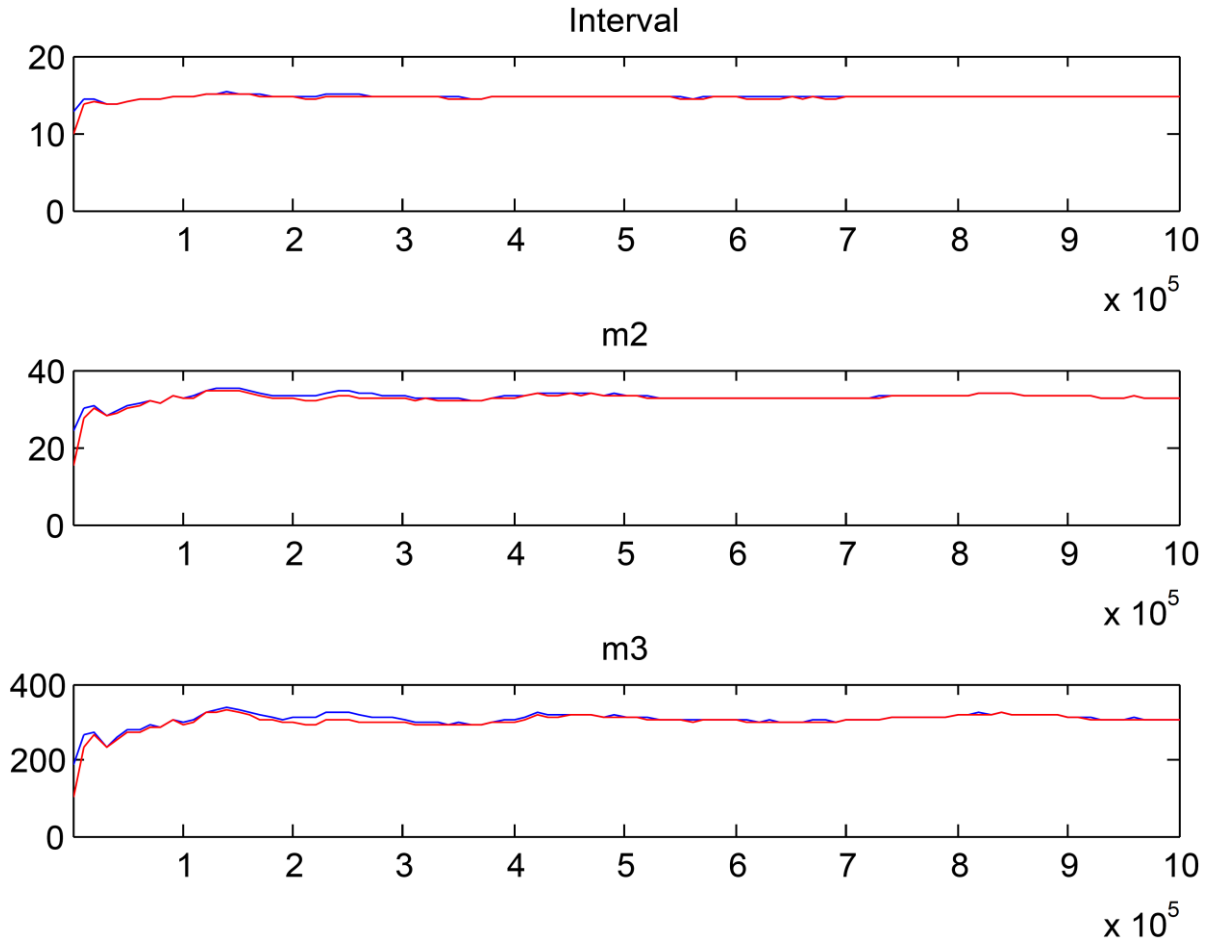


Figure 4.2. Multivariate convergence diagnostic



Appendix 5. Model Properties

Figure 5.1. Observed and model generated data

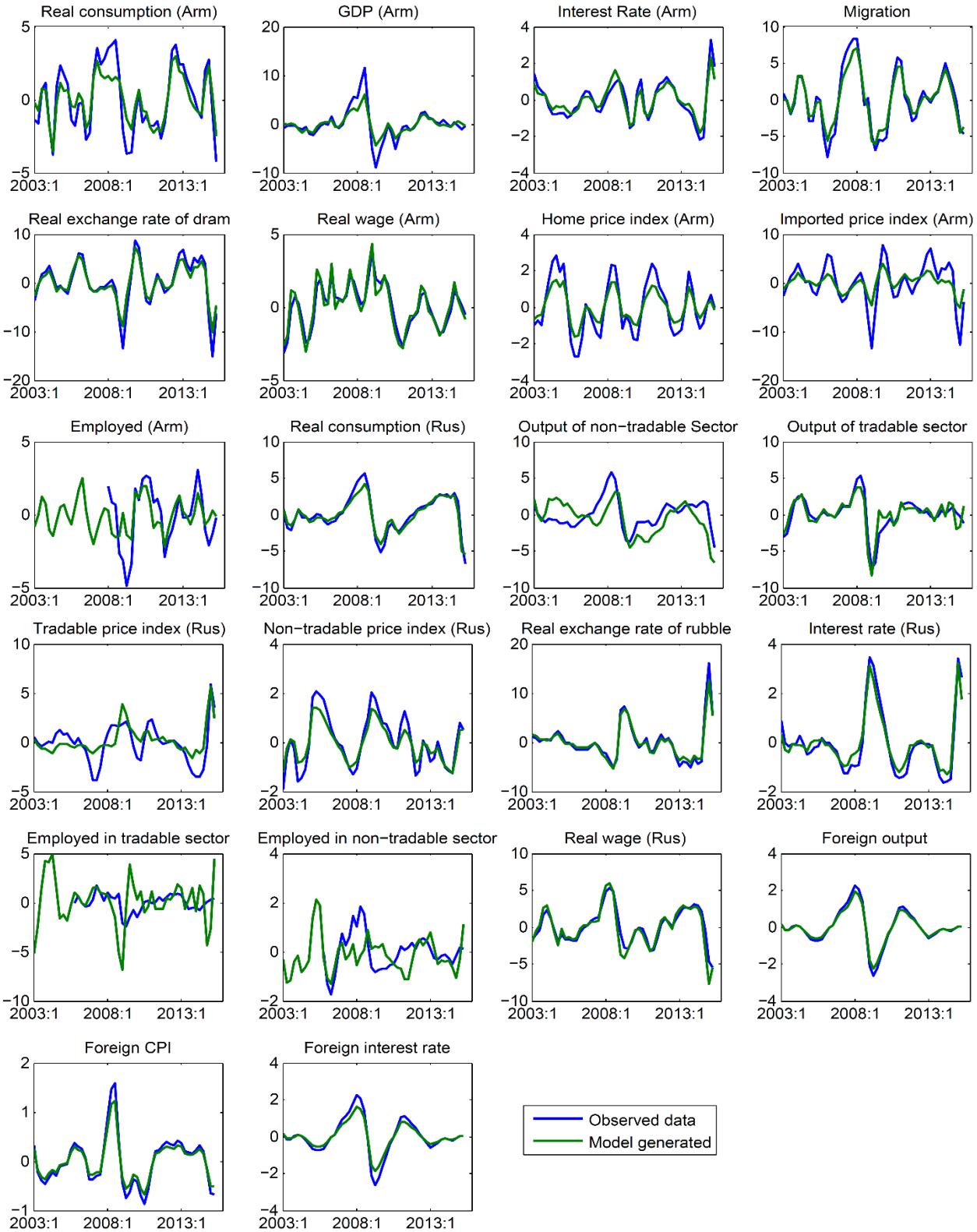


Figure 5.2. Estimated structural shocks of the model

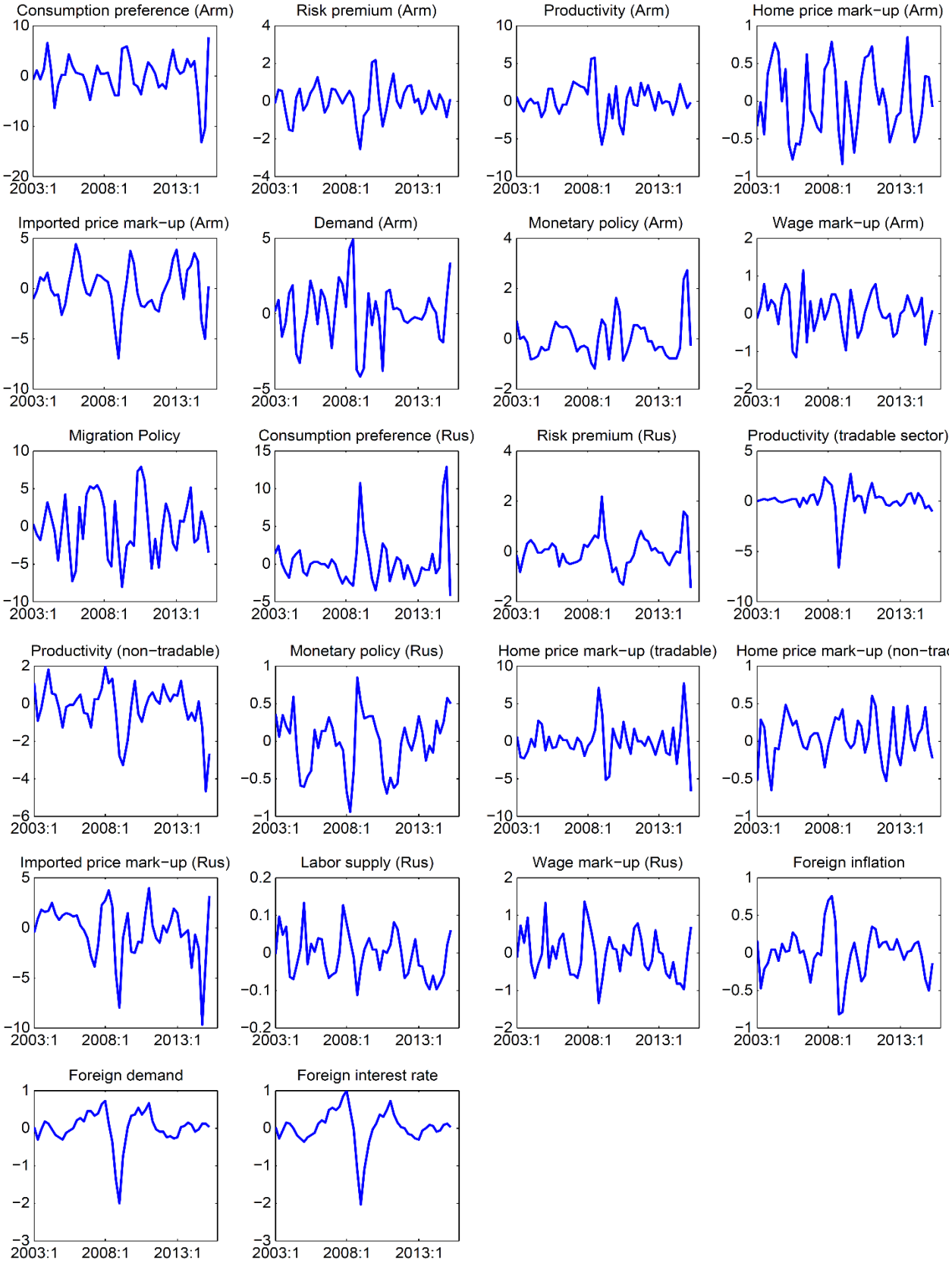


Figure 5.3. Migration policy of Russian Federation (estimated political factor)

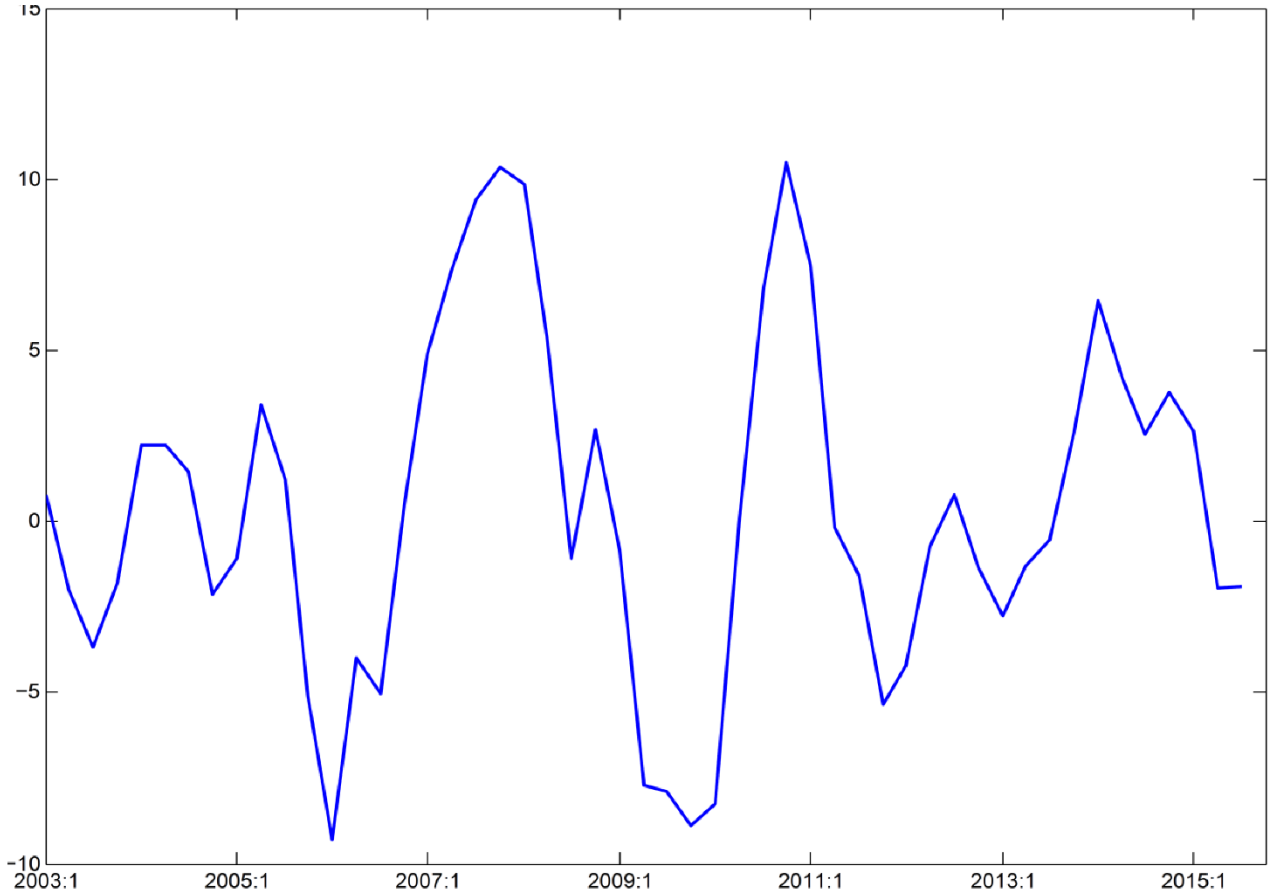


Figure 5.4. Generation of real remittances data

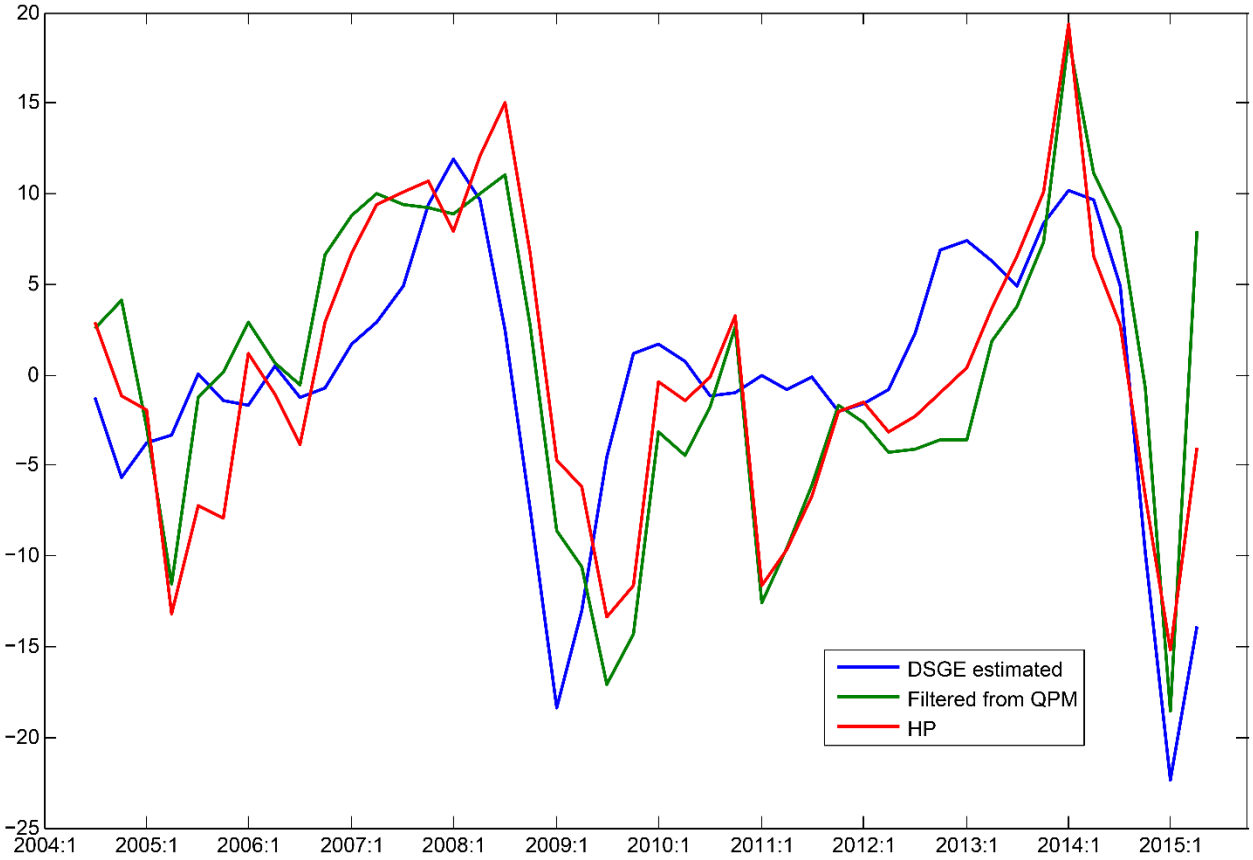


Figure 5.5. Historical simulations (migration, real wages in Armenia and Russia, employment in Armenia)

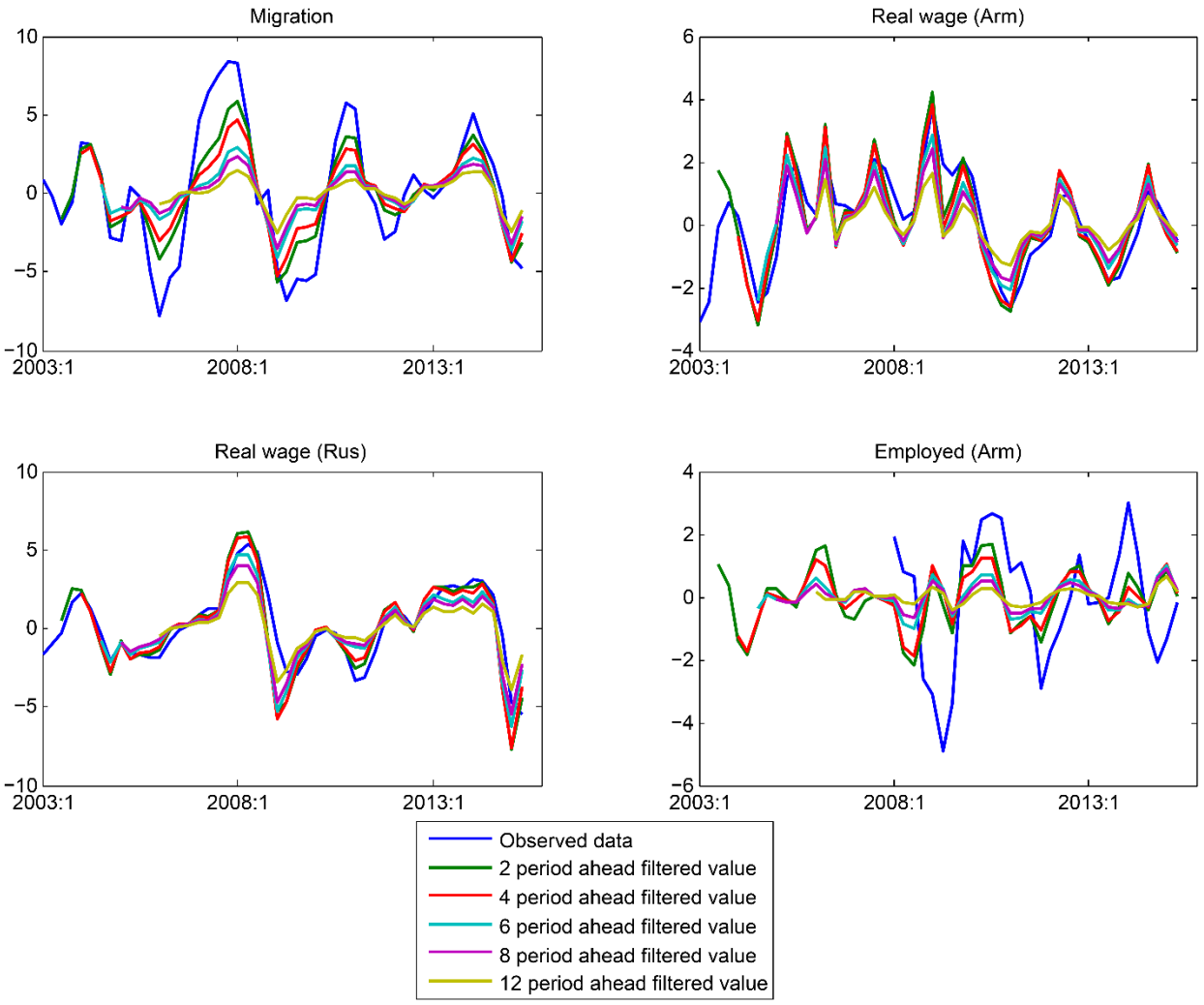


Figure 5.6. Historical simulations (real exchange rate of Armenian dram against Russian ruble, ruble real effective exchange rate, interest rates in Armenia and Russia)

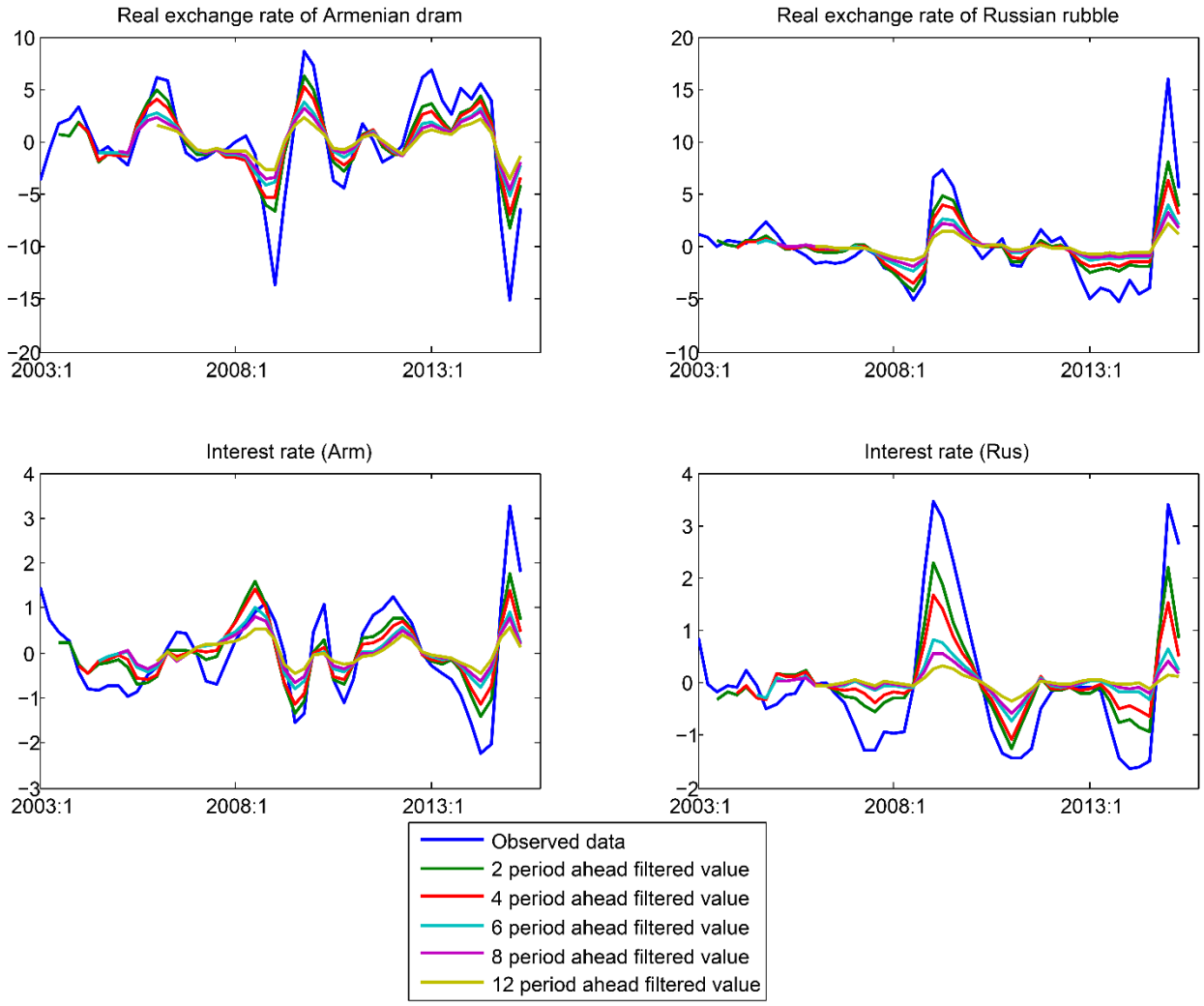


Figure 5.7. Historical decomposition of migration

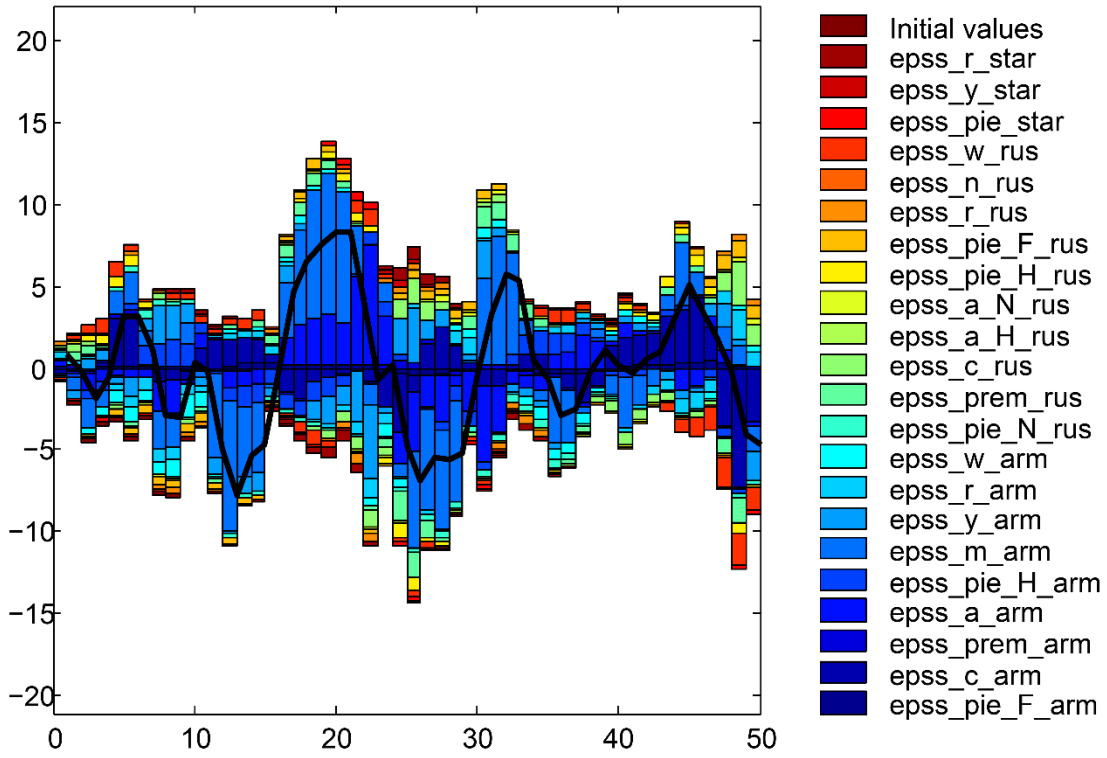


Figure 5.8. Historical decomposition of migration costs

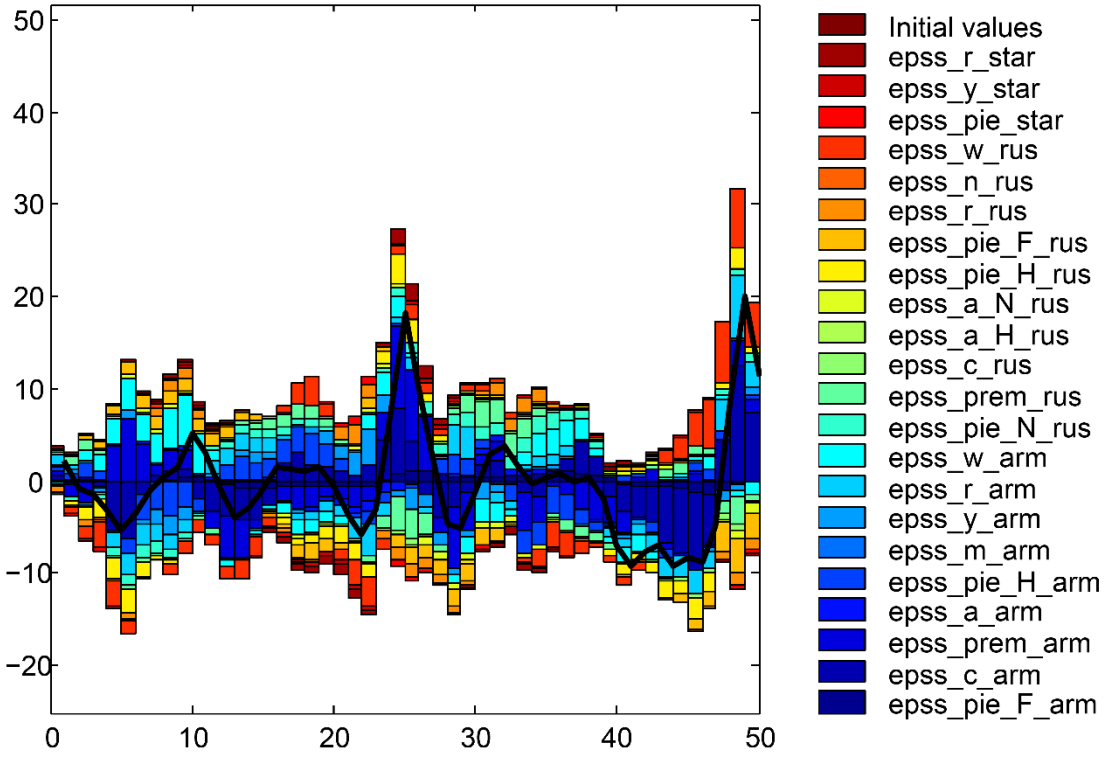


Figure 5.9. Historical decomposition of real remittances

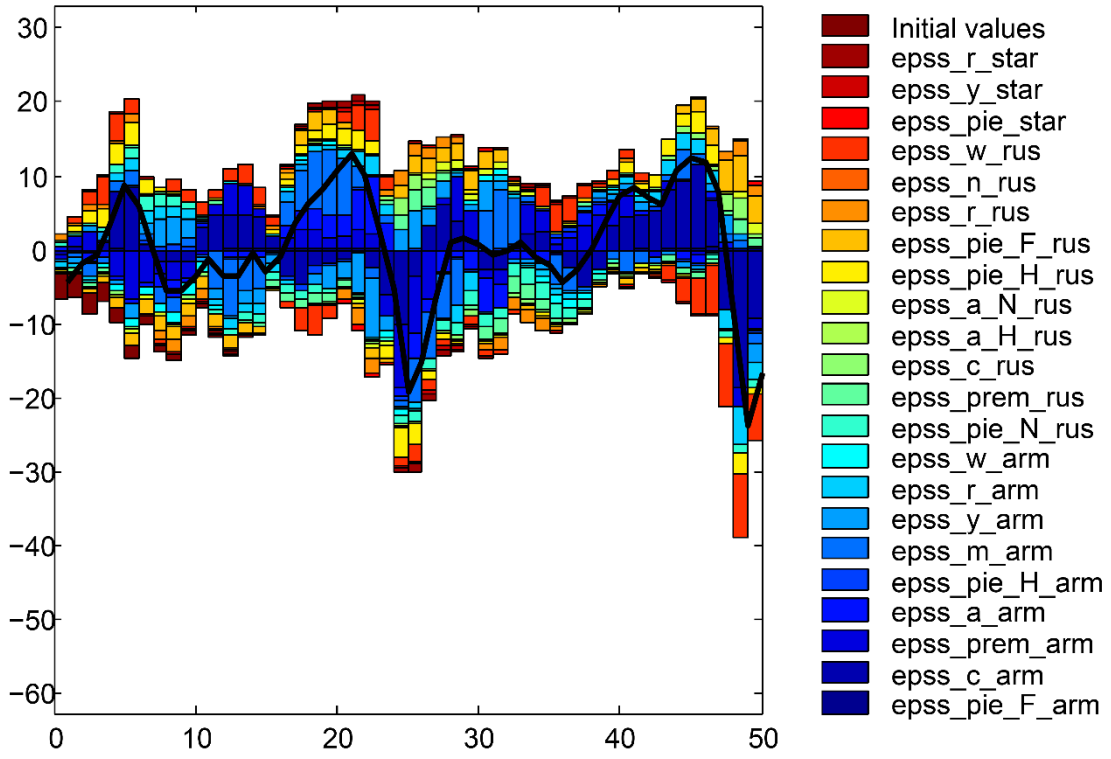


Figure 5.10. Historical decomposition of Armenian real wage

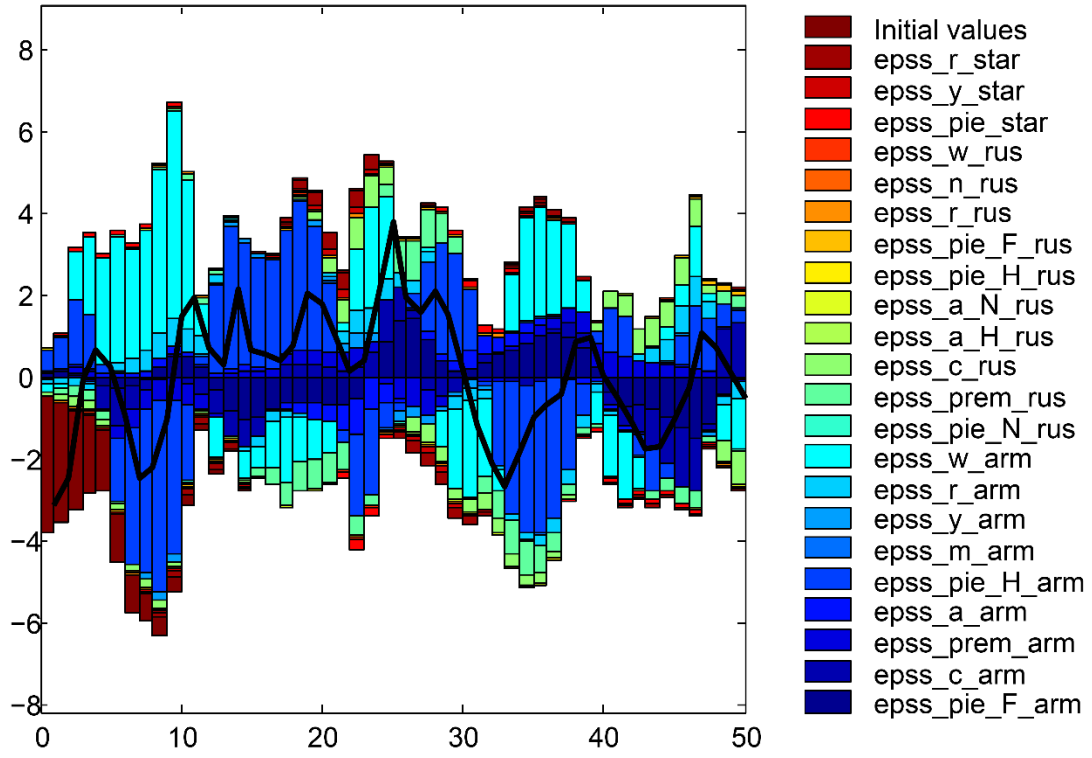


Figure 5.11. Historical decomposition of Russian real wage

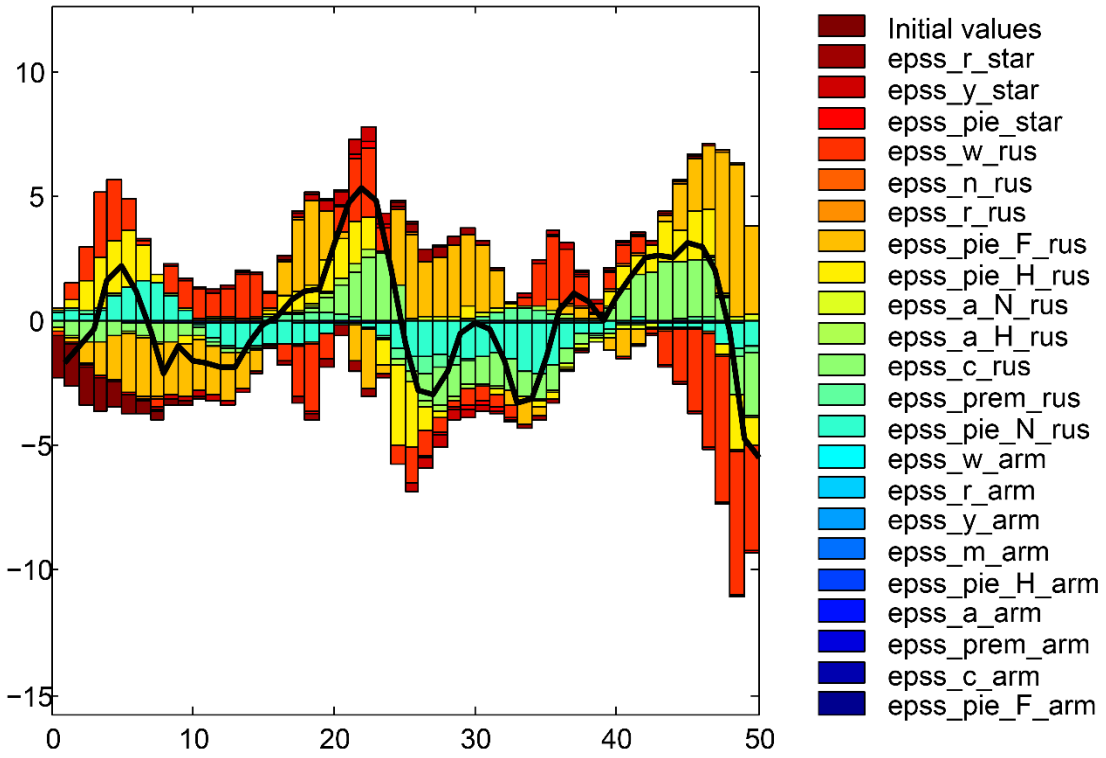


Figure 5.12. Impulse responses to a consumption preference shock in Armenia

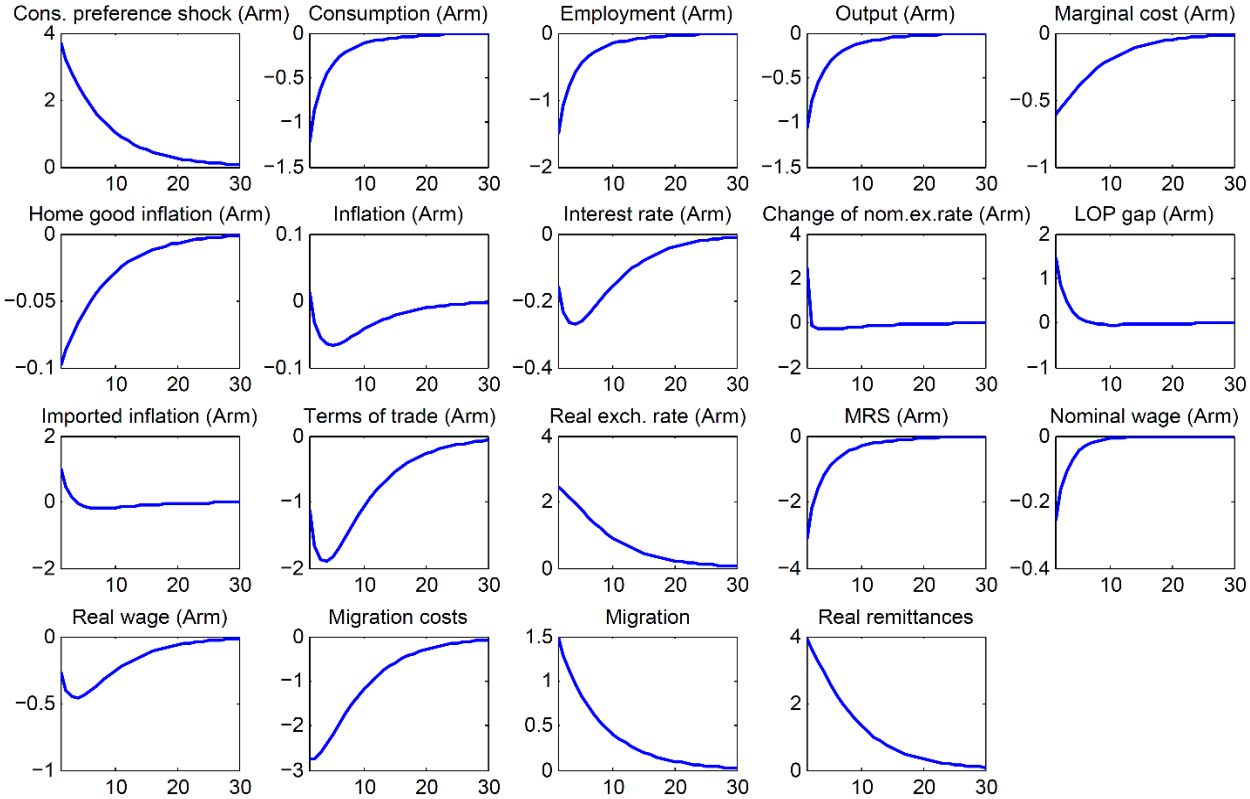


Figure 5.13. Impulse responses to a risk premium shock in Armenia

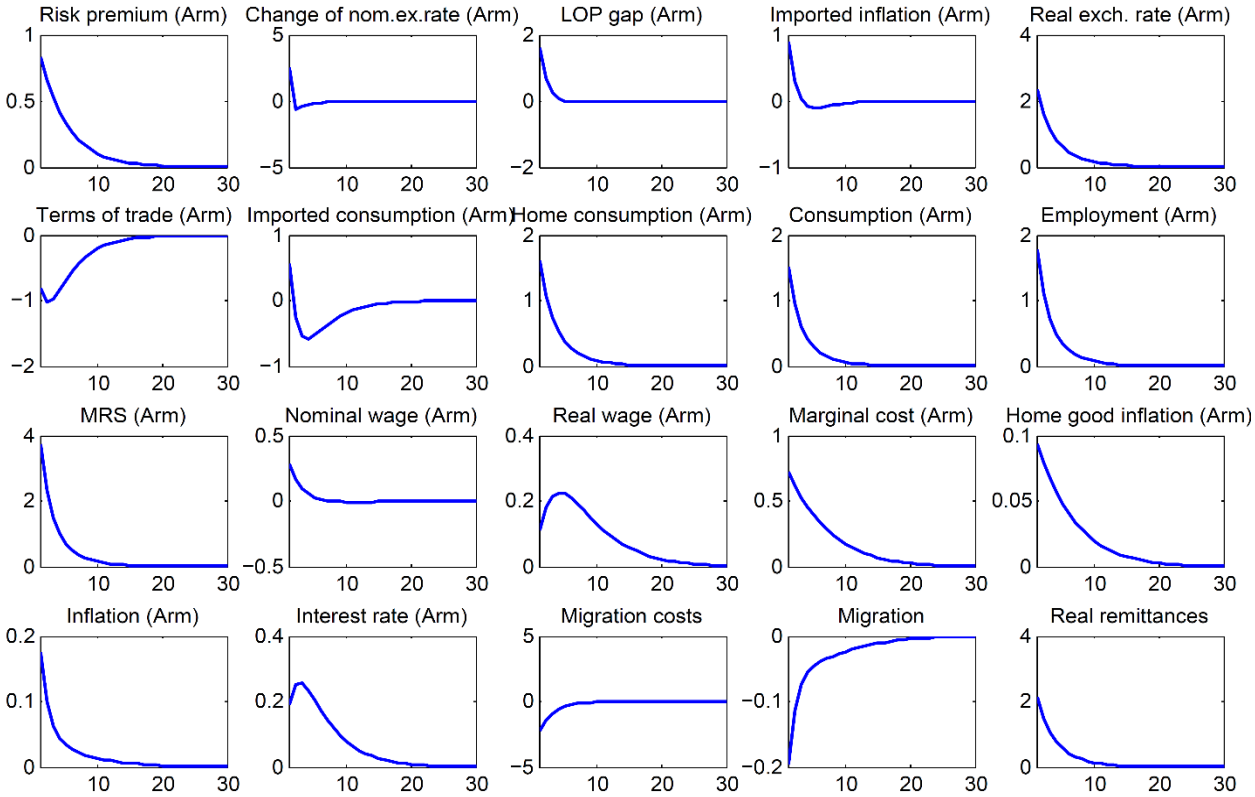


Figure 5.14. Impulse responses to a productivity shock in Armenia

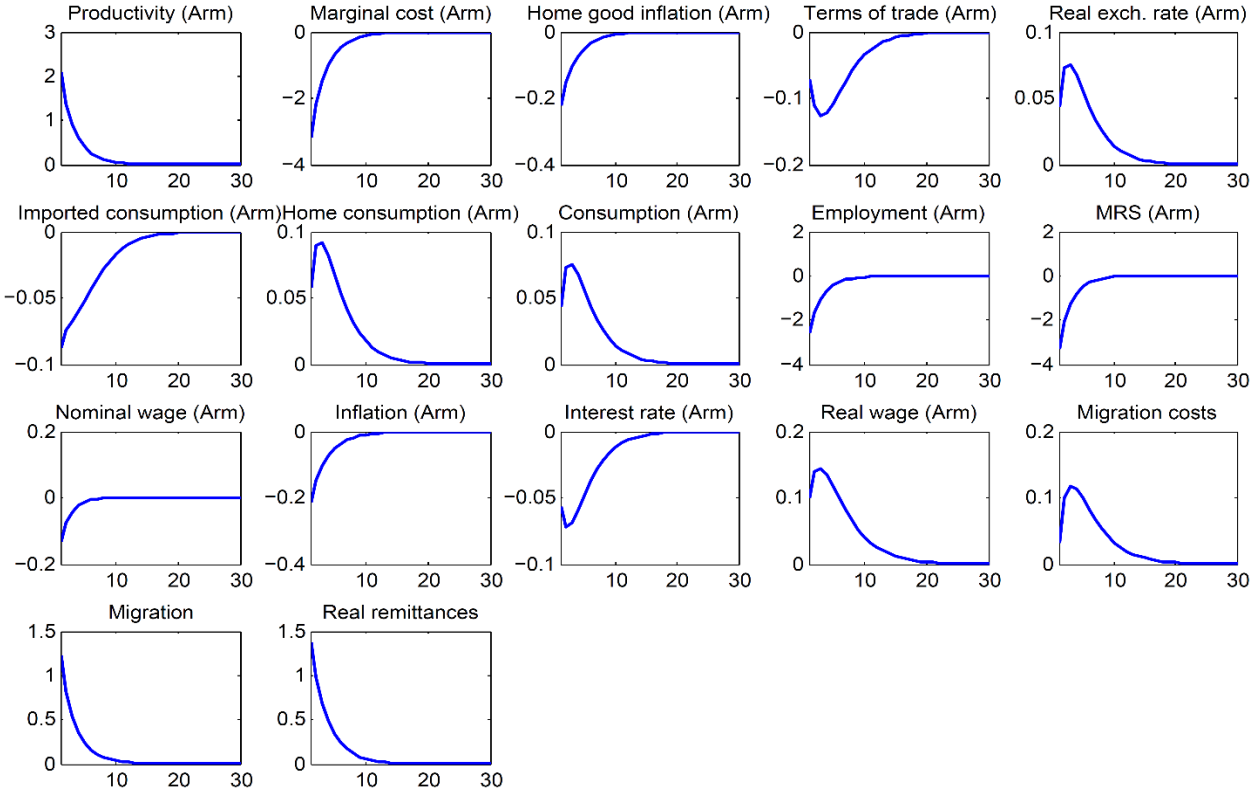


Figure 5.15. Impulse responses to a demand shock in Armenia

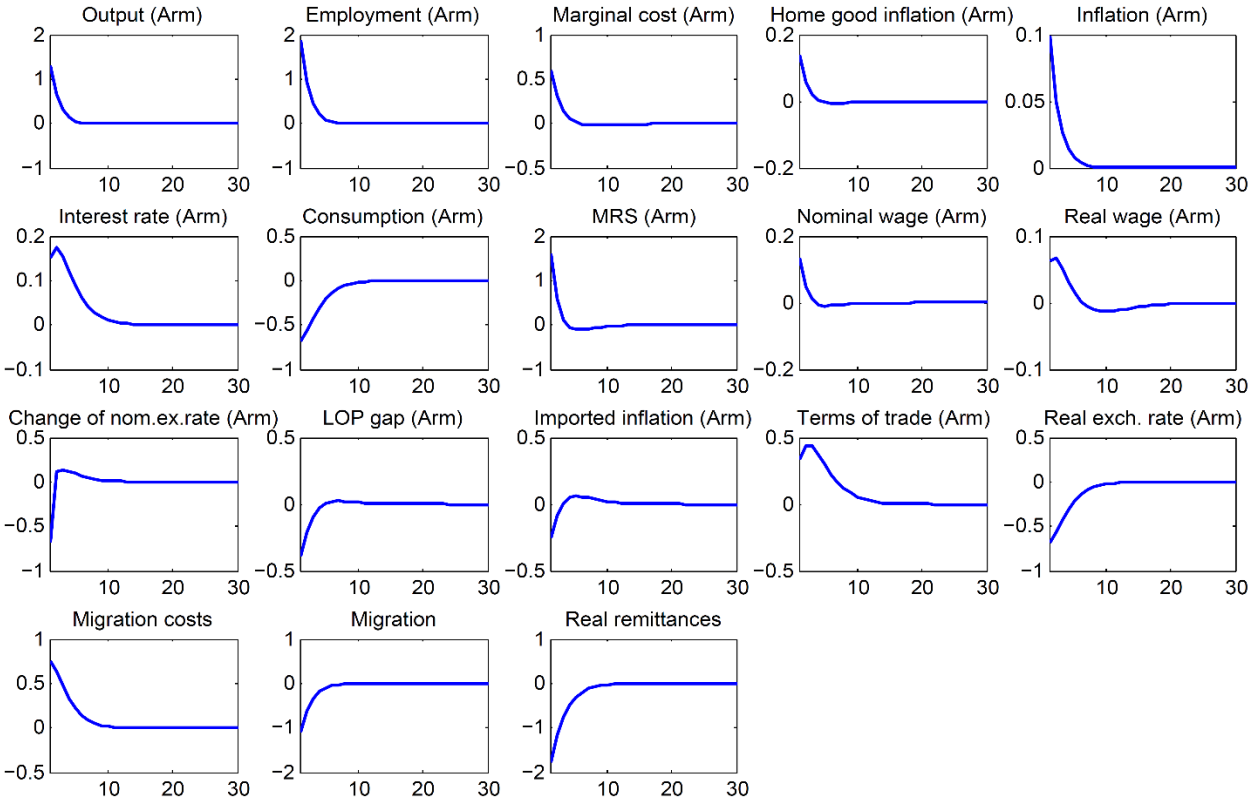


Figure 5.16. Impulse responses to a monetary policy shock in Armenia

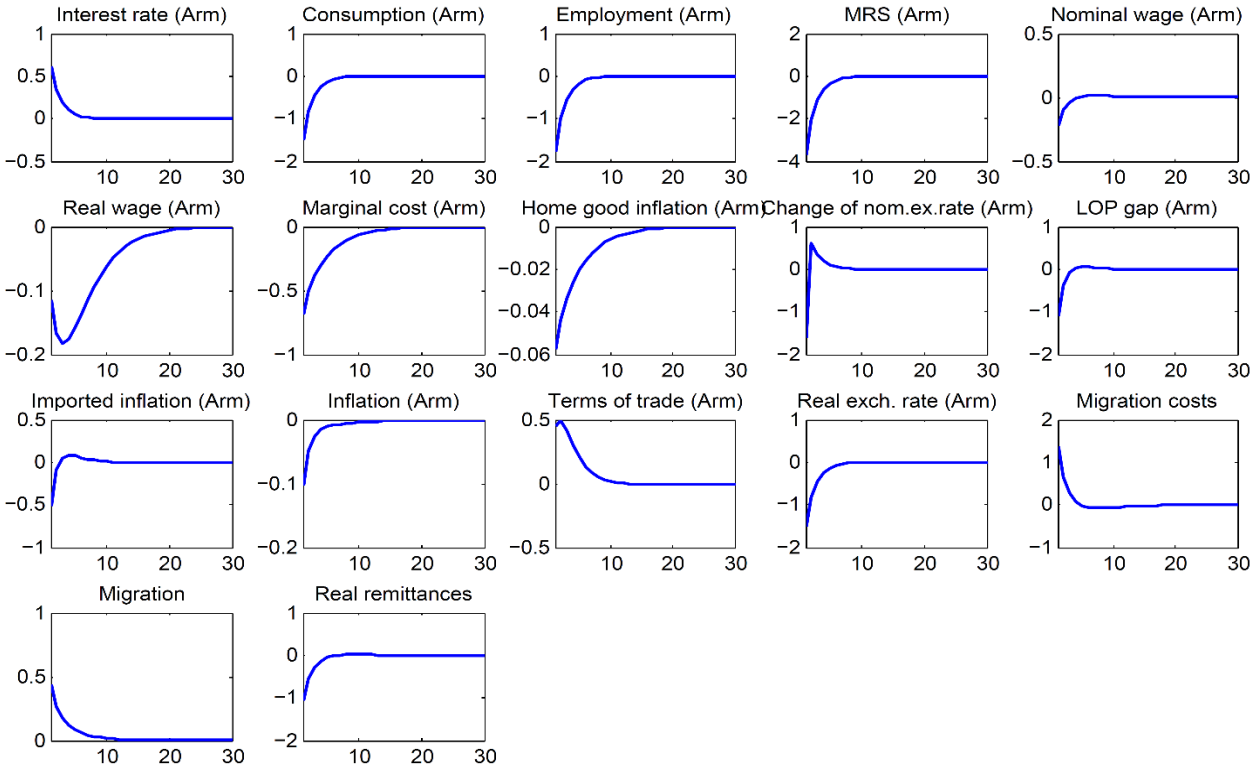


Figure 5.17. Impulse responses to a home produced goods price mark-up shock in Armenia

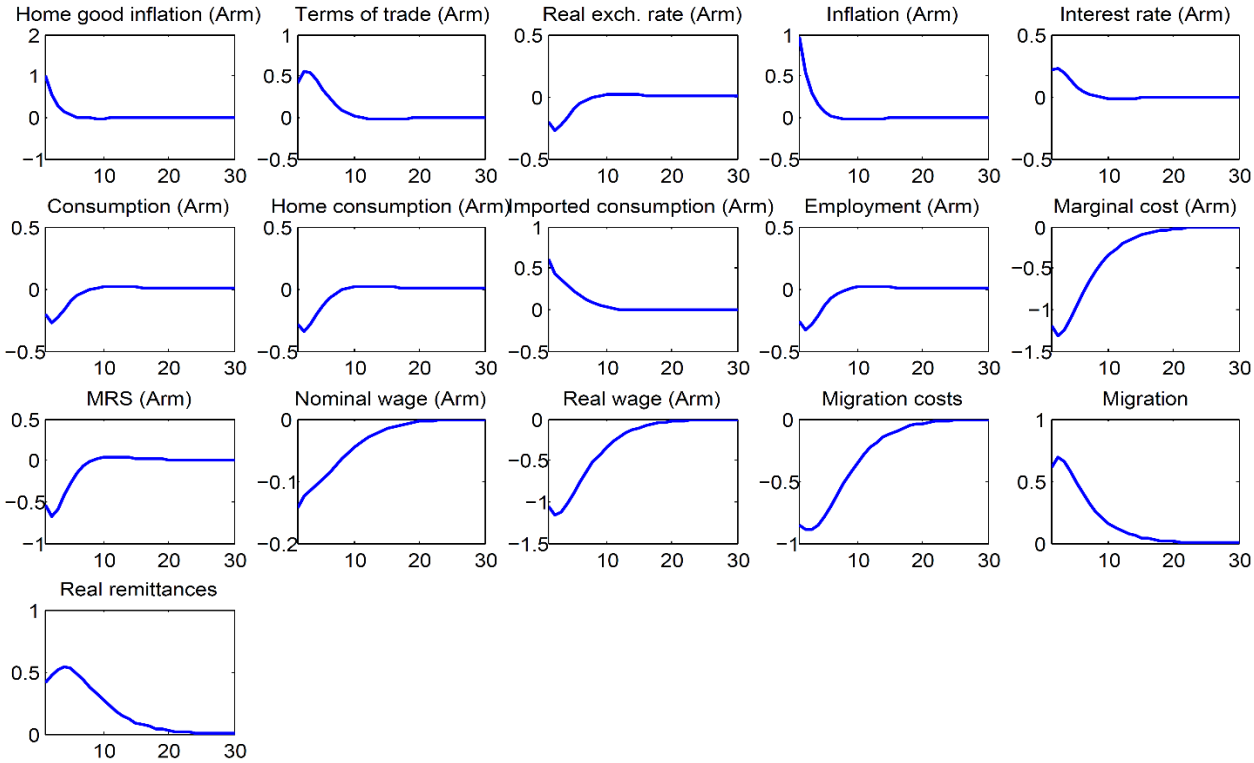


Figure 5.18. Impulse responses to an imported goods price mark-up shock in Armenia

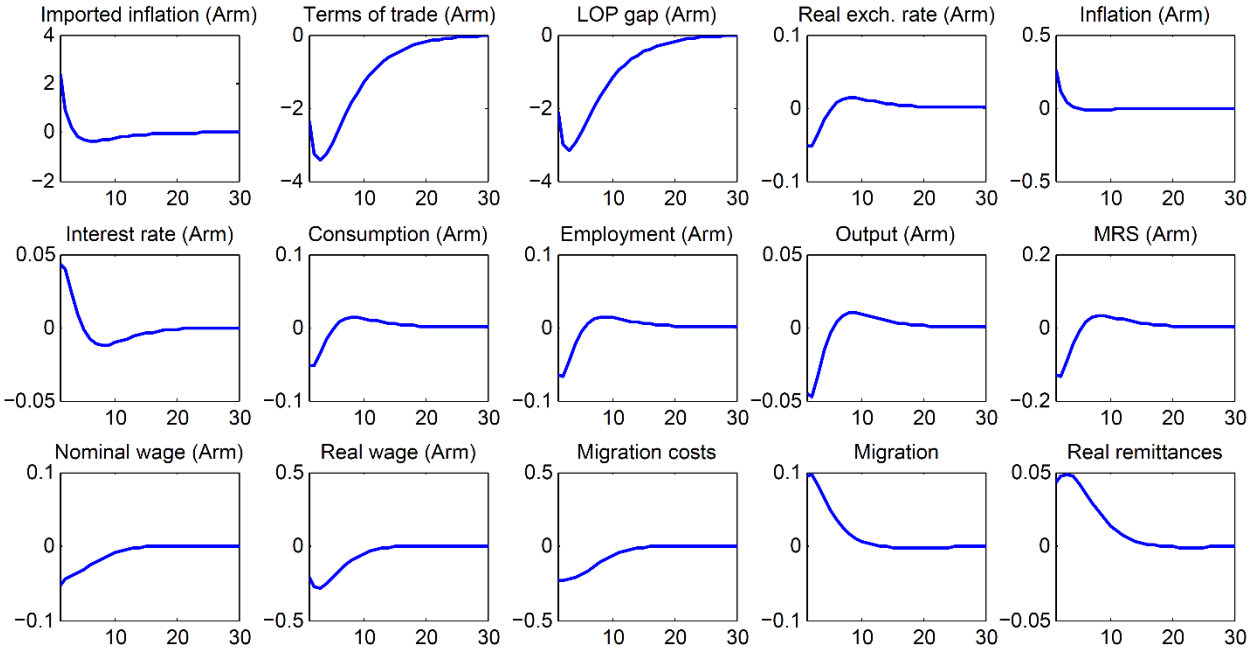


Figure 5.19. Impulse responses to a wage mark-up shock in Armenia

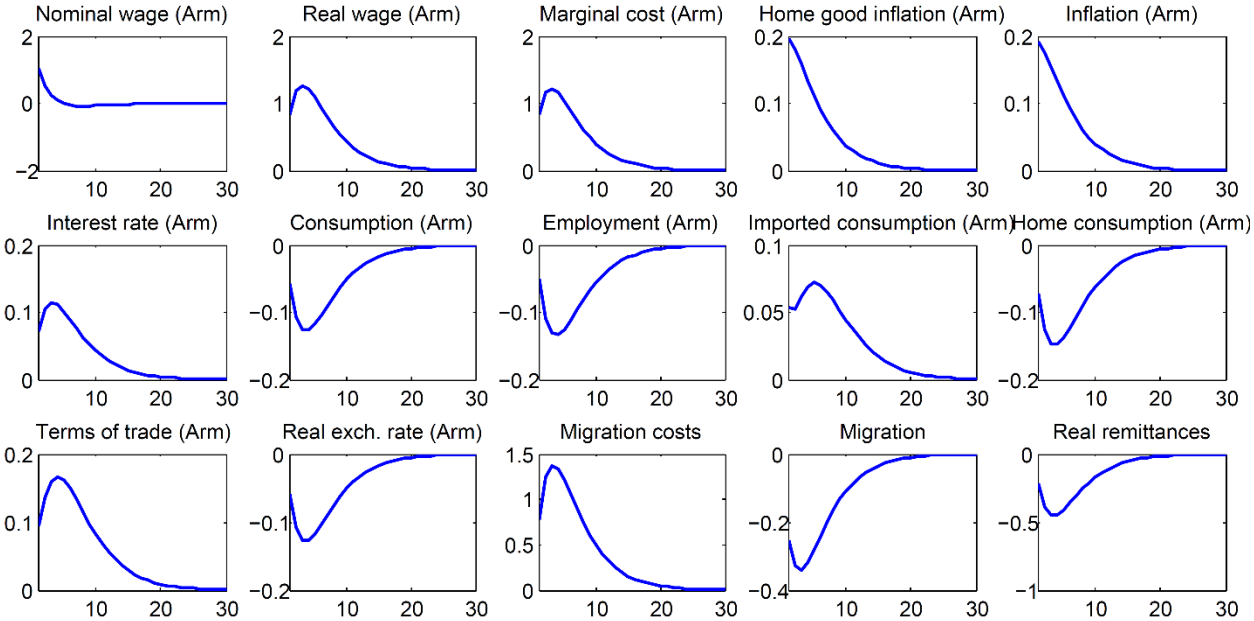


Figure 5.20. Impulse responses to a consumption preference shock in Russia

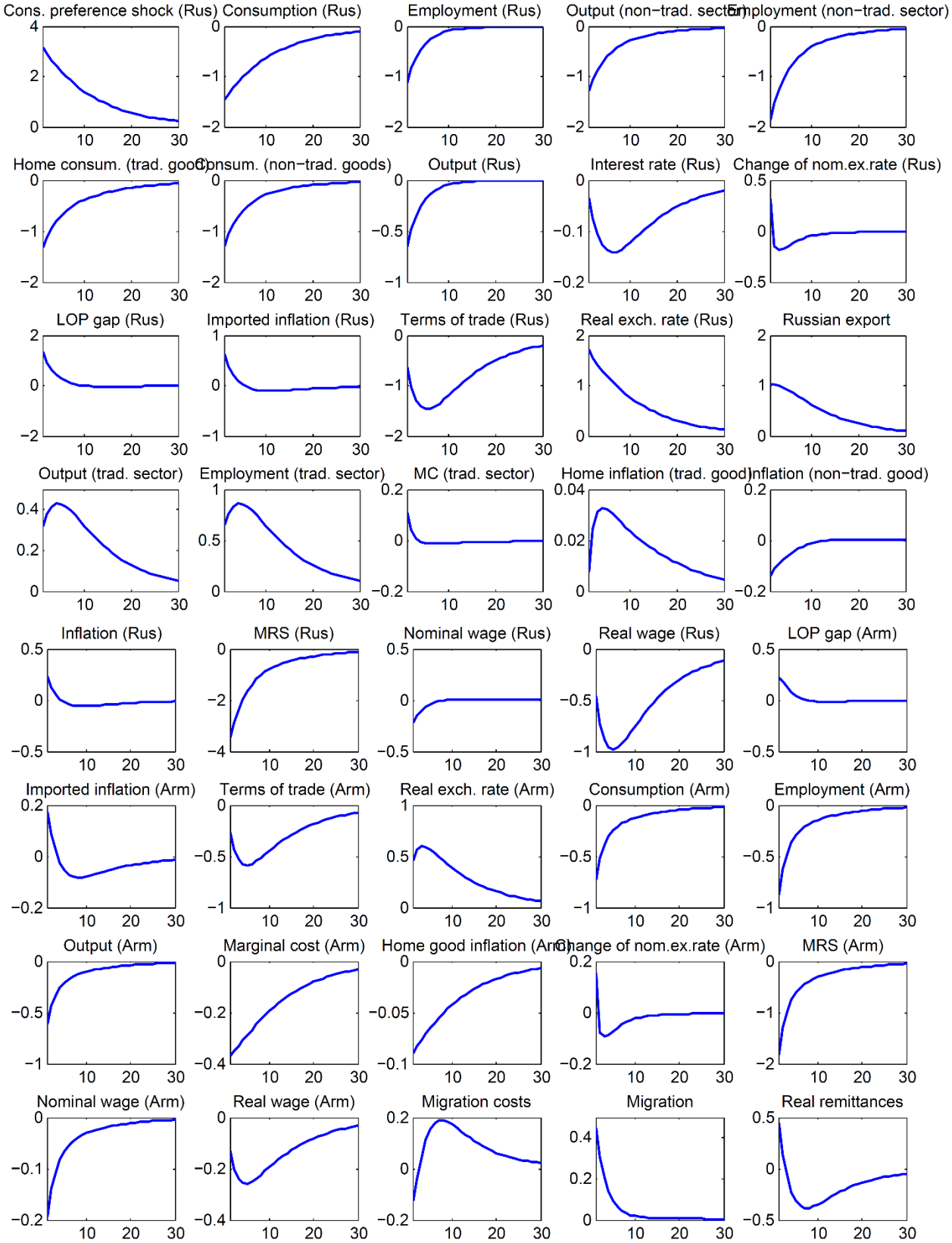


Figure 5.21. Impulse responses to a labor supply shock in Russia

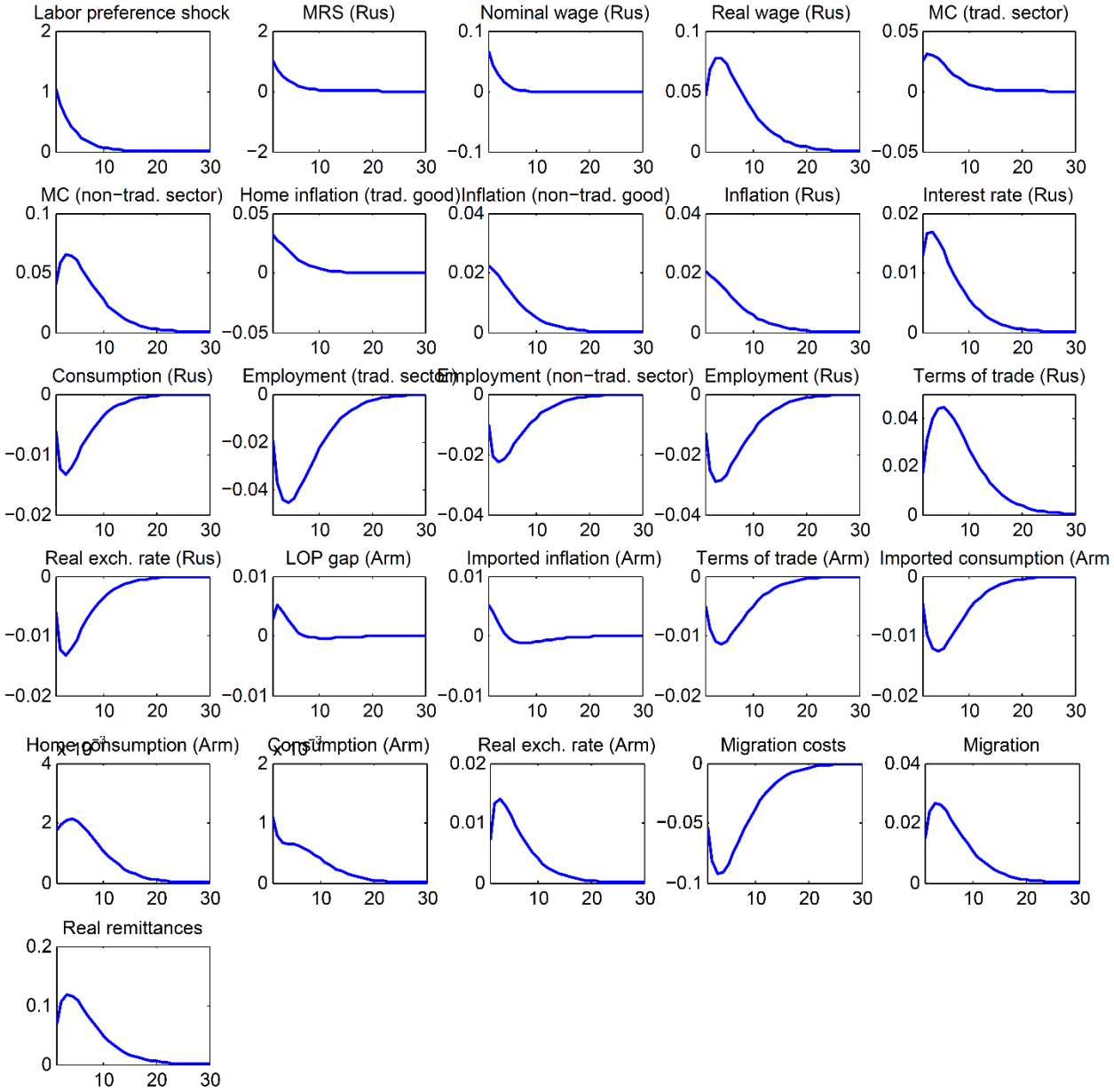


Figure 5.22. Impulse responses to a risk premium shock in Russia

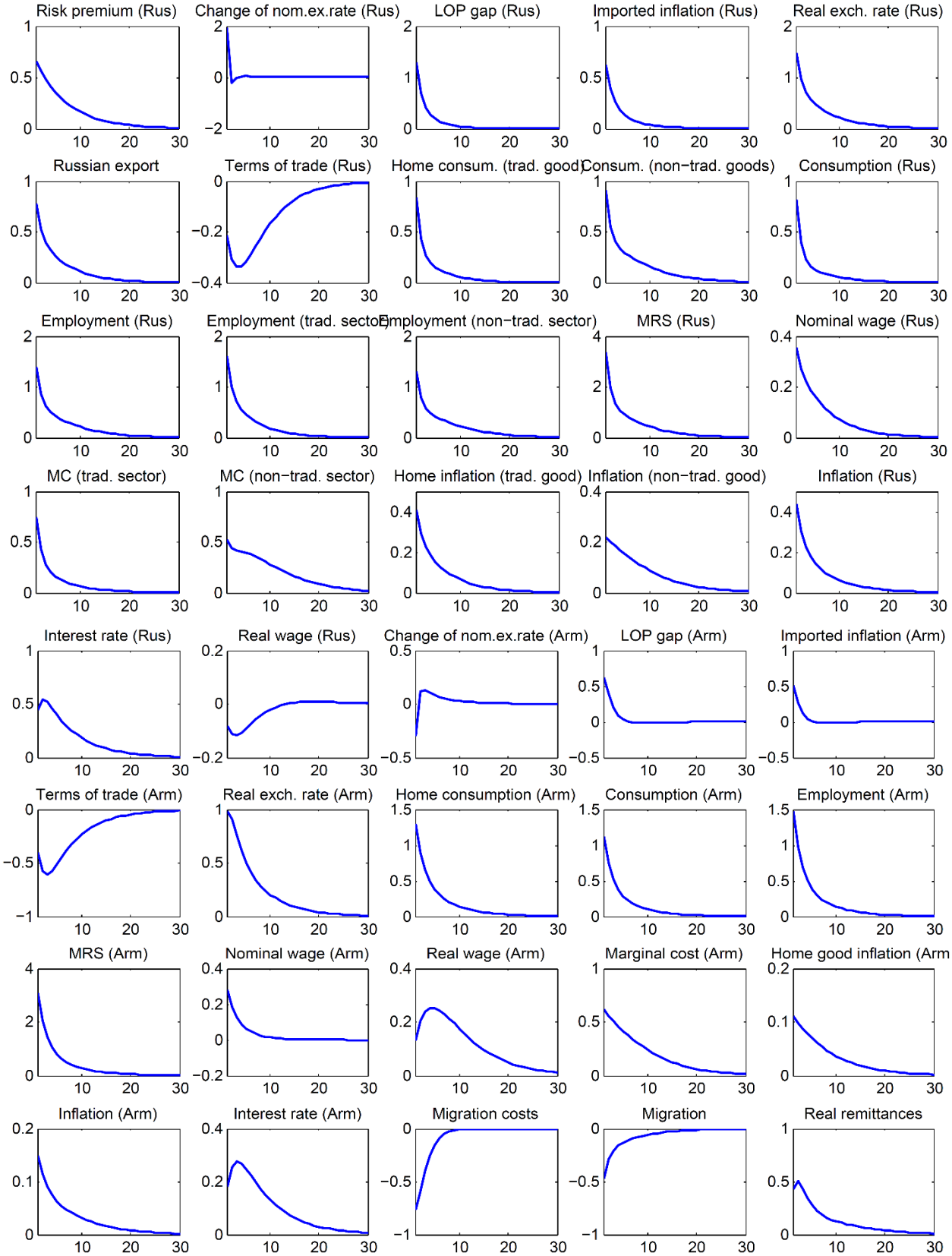


Figure 5.23. Impulse responses to a productivity shock of tradable sector in Russia

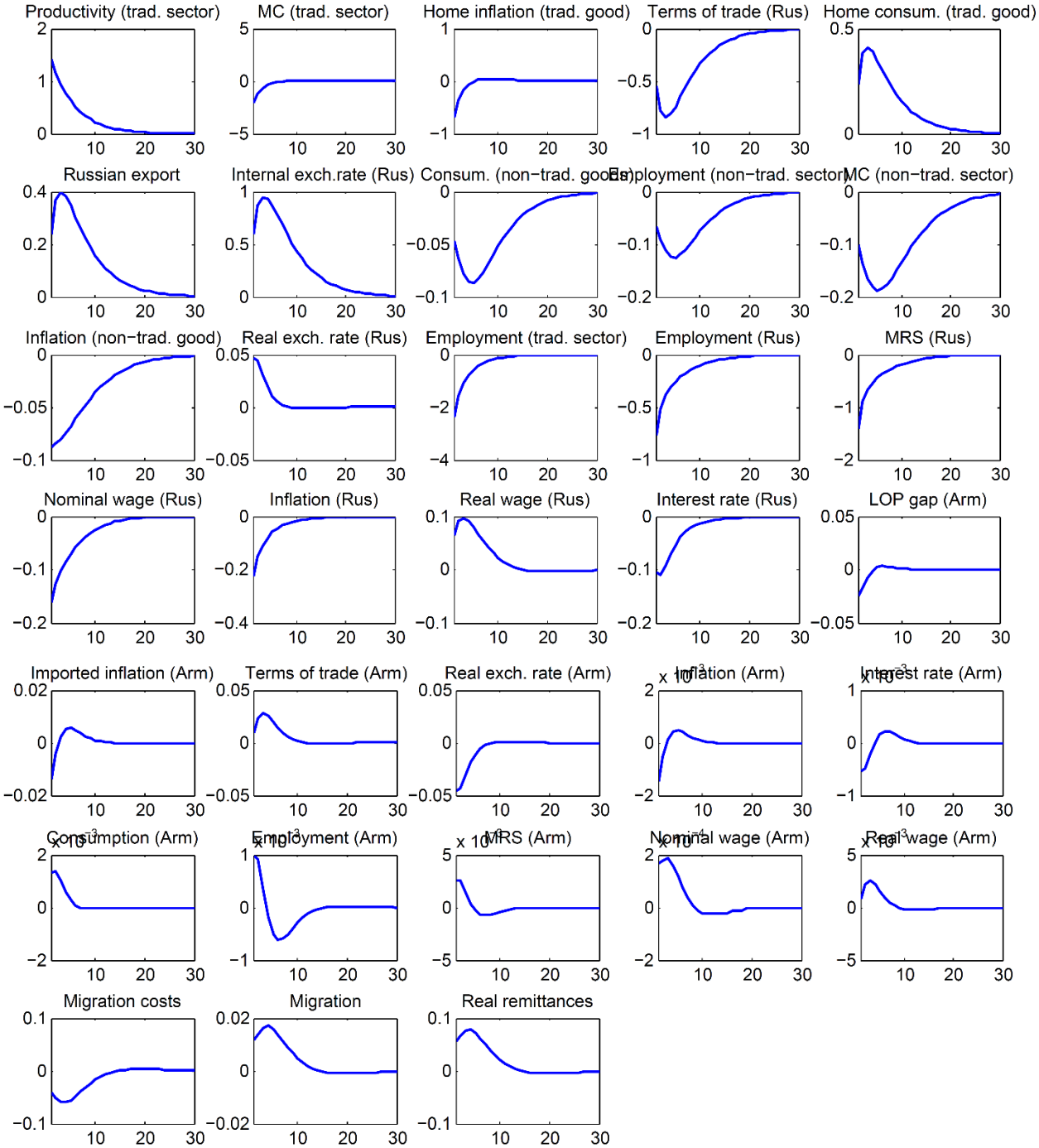


Figure 5.24. Impulse responses to a productivity shock of non-tradable sector in Russia

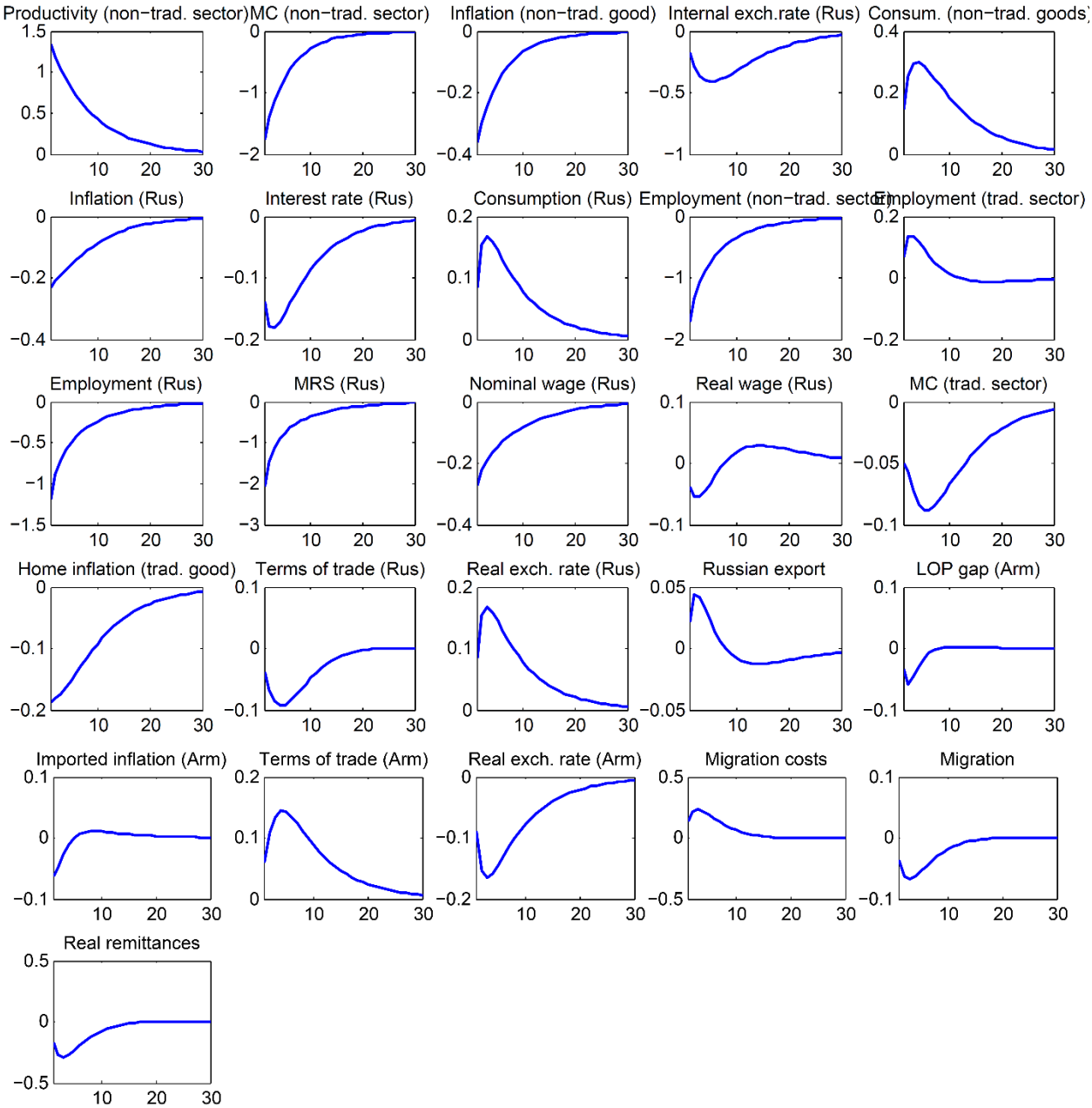


Figure 5.25. Impulse responses to a monetary policy shock in Russia

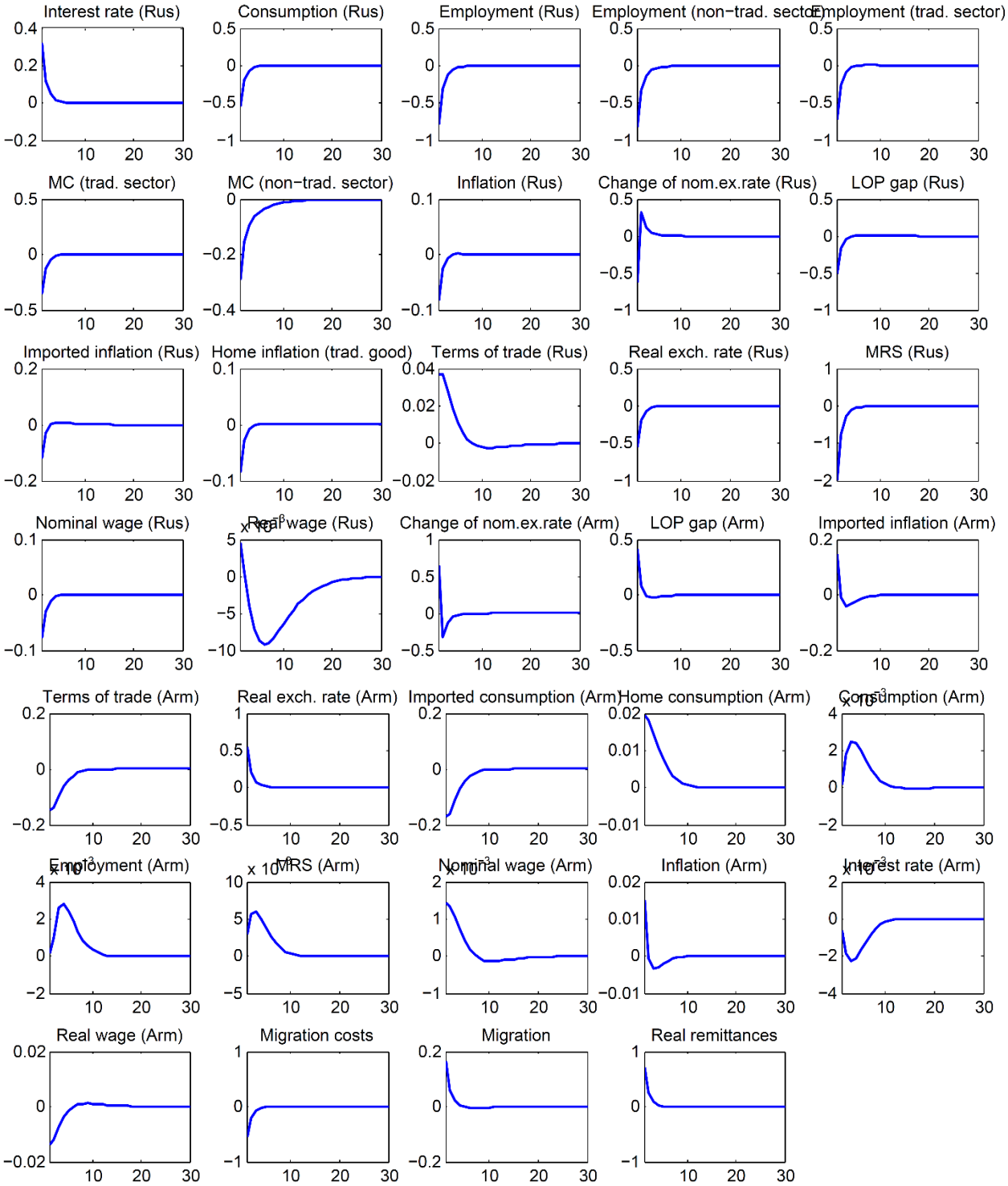


Figure 5.26. Impulse responses to a home produced tradable goods price mark-up shock in Russia

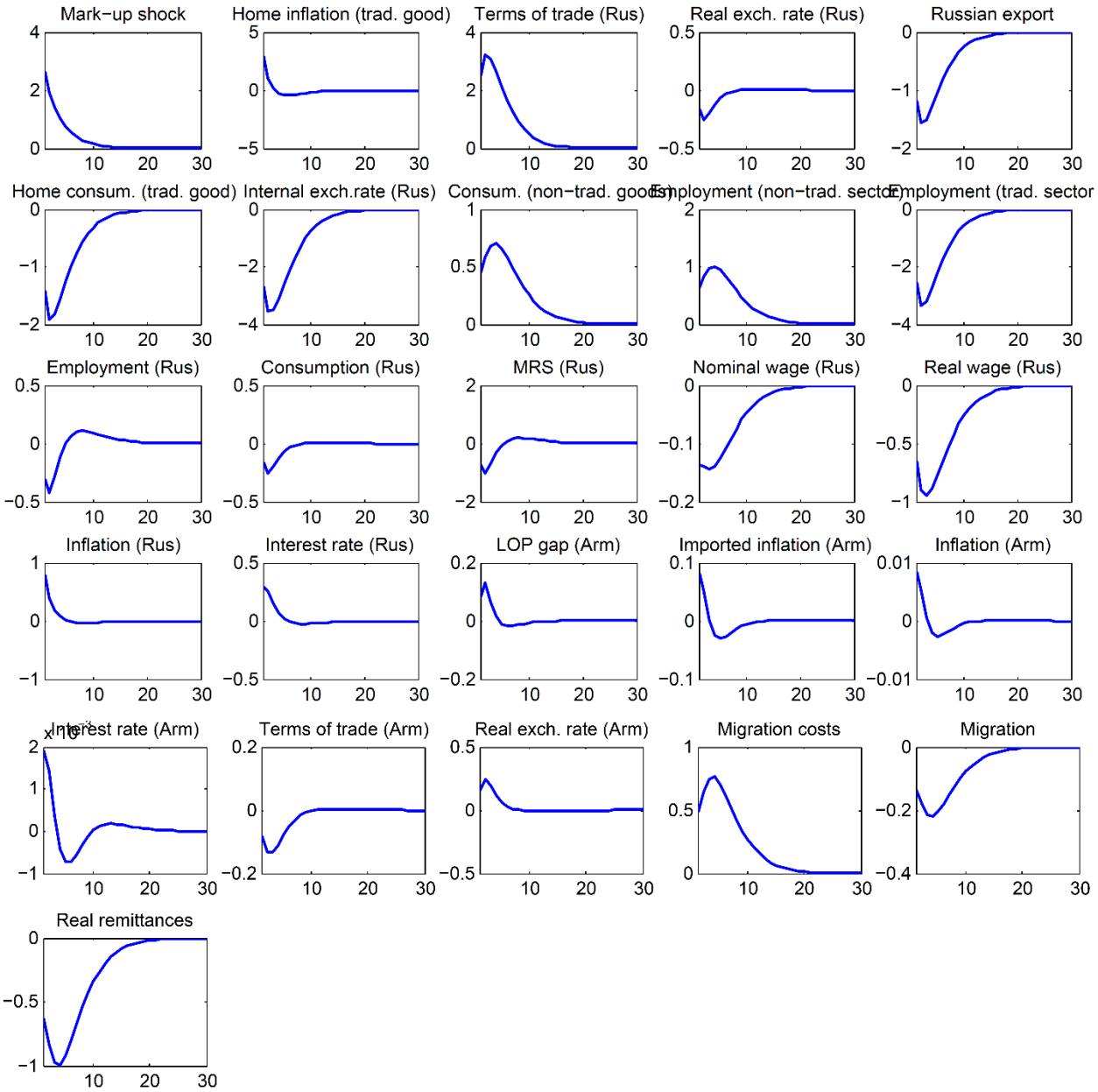


Figure 5.27. Impulse responses to a non-tradable goods price mark-up shock in Russia

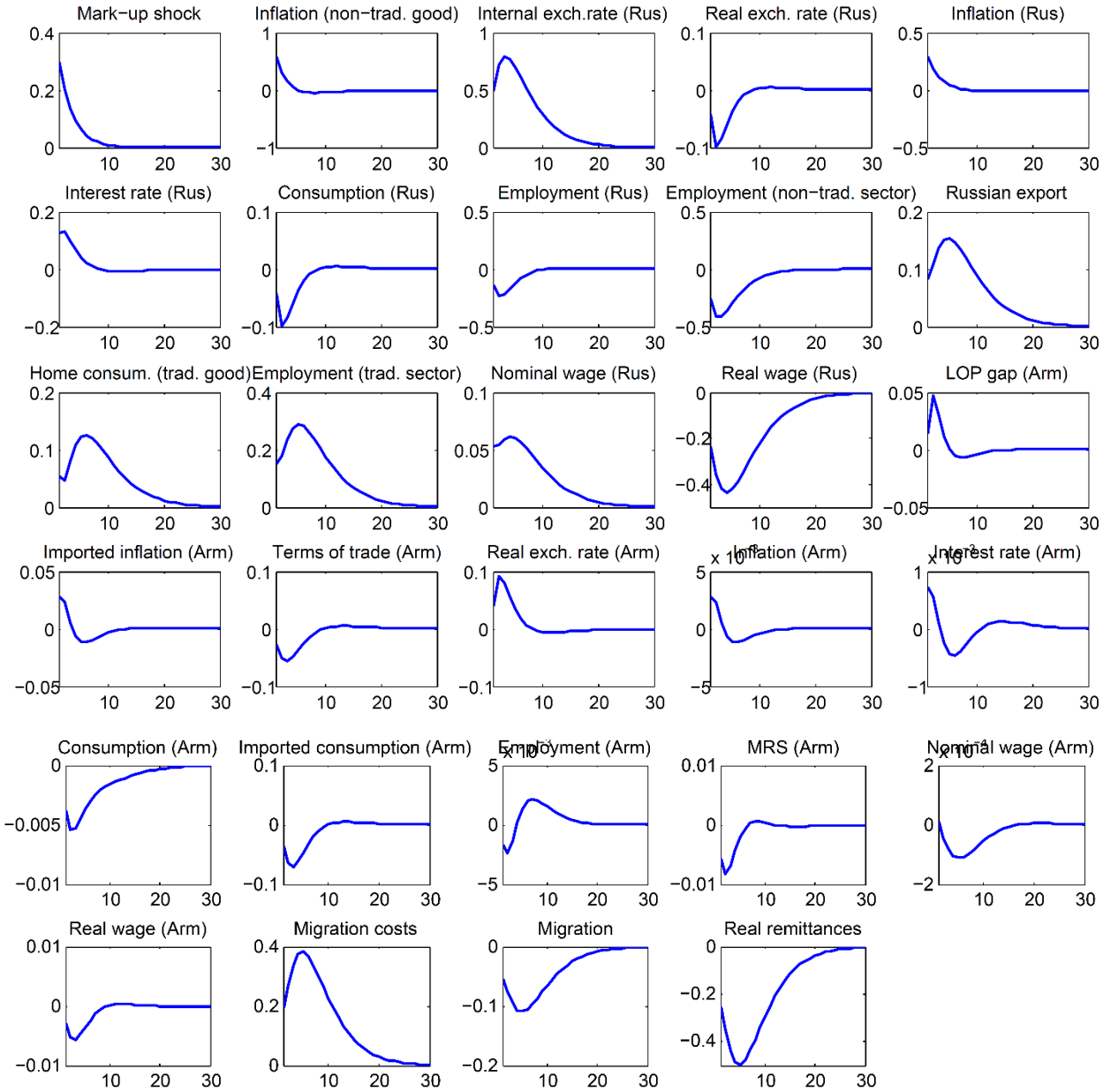


Figure 5.28. Impulse responses to an imported goods price mark-up shock in Russia

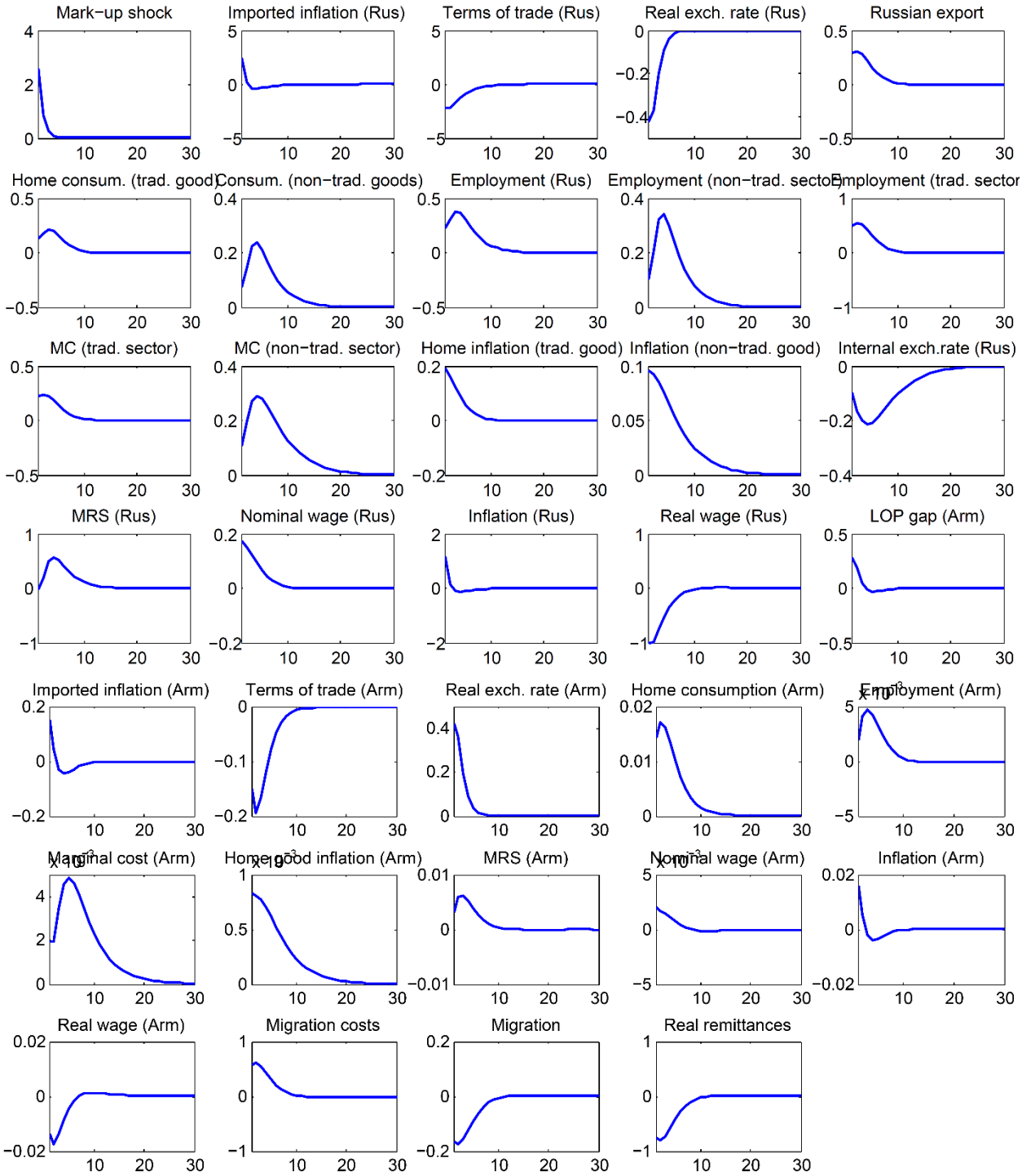


Figure 5.29. Impulse responses to a wage mark-up shock in Russia

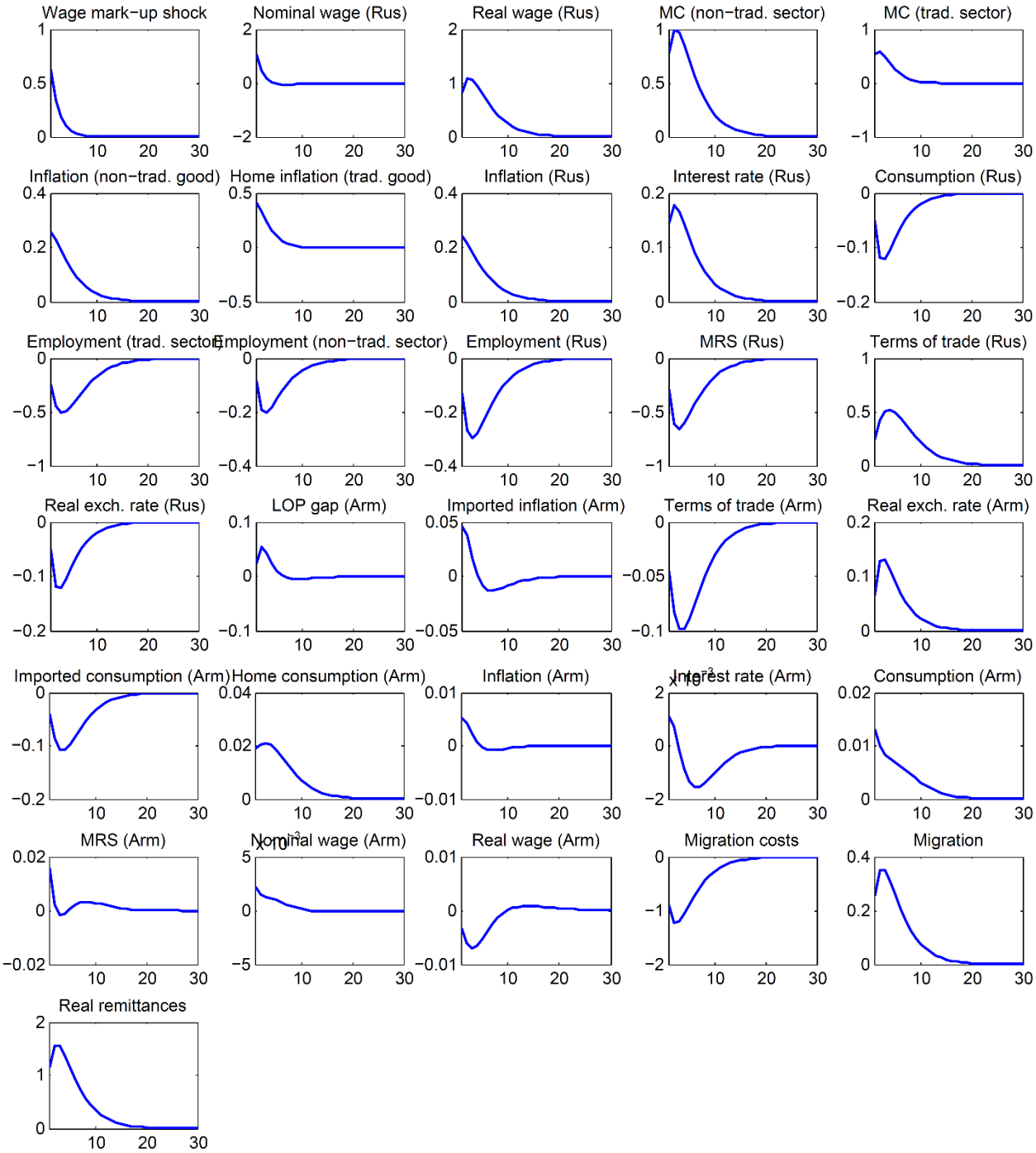


Figure 5.30. Impulse responses to a foreign demand shock

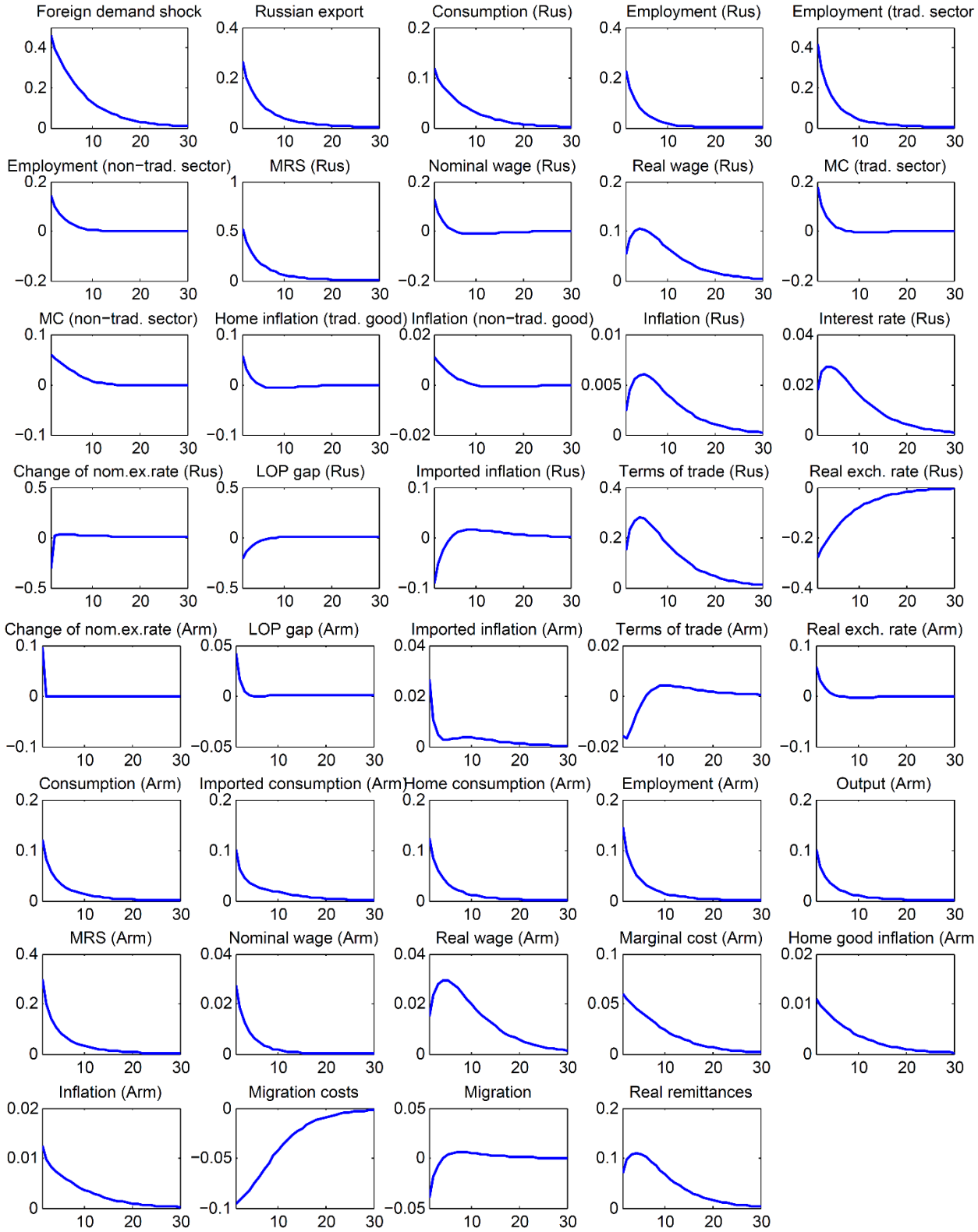


Figure 5.31. Impulse responses to a foreign interest rate shock

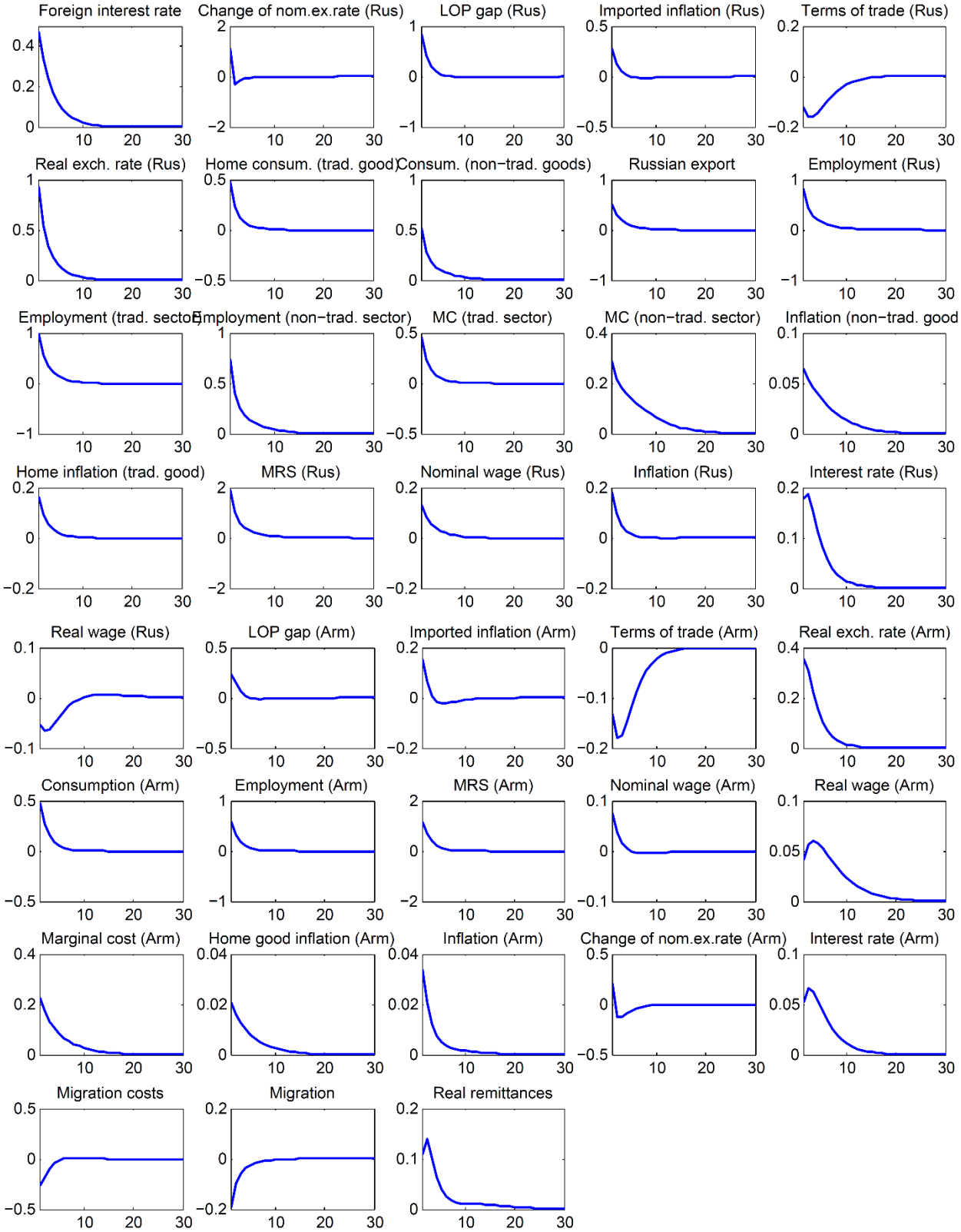


Table 5: Conditional variance decomposition: migration costs and migration

		Migration costs					Migration				
	Disturbance	1Q	2Q	4Q	20Q	100Q	1Q	2Q	4Q	20Q	100Q
σ_c^{arm}	Consumption preference (Armenia)	38.84	42.41	45.18	49.86	49.98	16.03	18.00	20.84	25.47	25.54
σ_{prem}^{arm}	Risk premium (Armenia)	26.06	19.94	13.49	8.69	8.66	0.28	0.24	0.21	0.20	0.20
σ_r^{arm}	Monetary policy (Armenia)	9.85	6.57	3.92	2.48	2.47	1.37	1.22	1.06	0.93	0.93
σ_a^{arm}	Productivity (Armenia)	0.02	0.12	0.25	0.27	0.27	19.09	17.65	15.67	13.66	13.65
$\sigma_{\pi_H}^{arm}$	Price mark-up of domestically produced goods (Armenia)	3.79	4.12	4.96	6.17	6.15	0.24	0.78	1.68	2.43	2.43
$\sigma_{\pi_F}^{arm}$	Price mark-up of imported goods from Russia	0.27	0.29	0.33	0.32	0.32	0.02	0.06	0.10	0.11	0.11
$\sigma_{\pi_W}^{arm}$	Wage mark-up (Armenia)	3.21	6.10	9.53	11.01	10.98	0.47	0.81	1.36	1.88	1.88
σ_{politi}	Political factor	0.00	0.00	0.00	0.00	0.00	49.17	48.93	47.28	43.55	43.51
σ_y^{arm}	Demand (Armenia)	2.90	2.70	2.11	1.40	1.39	8.57	7.23	5.88	4.96	4.96
σ_c^{rus}	Consumption preference (Russia)	0.09	0.05	0.05	0.35	0.37	1.45	1.36	1.23	1.08	1.08
σ_{prem}^{rus}	Risk premium (Russia)	3.10	2.67	1.91	1.23	1.22	1.61	1.43	1.30	1.28	1.28
σ_r^{rus}	Monetary policy (Russia)	1.65	1.02	0.60	0.38	0.38	0.20	0.14	0.11	0.09	0.09
σ_N^{rus}	Labor supply (Russia)	0.01	0.03	0.04	0.05	0.05	0.00	0.00	0.01	0.01	0.01
$\sigma_{a_T}^{rus}$	Productivity in tradable sector (Russia)	0.02	0.02	0.02	0.02	0.02	0.00	0.00	0.00	0.01	0.01
$\sigma_{a_{NT}}^{rus}$	Productivity in non-tradable sector (Russia)	0.09	0.18	0.27	0.27	0.27	0.01	0.03	0.05	0.07	0.07
$\sigma_{\pi_H}^{rus}$	Price mark-up of domestically produced tradable goods (Russia)	3.43	4.51	6.45	7.59	7.57	0.38	0.60	1.12	1.77	1.77
$\sigma_{\pi_{NT}}^{rus}$	Price mark-up of domestically produced non-tradable goods (Russia)	0.19	0.31	0.59	1.10	1.10	0.02	0.04	0.10	0.26	0.26
$\sigma_{\pi_F}^{rus}$	Price mark-up of imported goods (Russia)	1.70	1.98	1.94	1.38	1.38	0.20	0.27	0.34	0.32	0.32
$\sigma_{\pi_W}^{rus}$	Wage mark-up (Russia)	4.15	6.45	8.00	7.17	7.15	0.47	0.87	1.40	1.68	1.68
σ_y^*	Foreign demand	0.05	0.05	0.05	0.06	0.06	0.01	0.01	0.01	0.01	0.01
σ_π^*	Foreign inflation	0.21	0.18	0.12	0.08	0.08	0.13	0.11	0.10	0.09	0.09
σ_r^*	Foreign interest rate	0.36	0.29	0.19	0.12	0.12	0.27	0.22	0.18	0.16	0.15

Table 6: Conditional variance decomposition: Armenian employment and real remittances

		Employment (Arm)					Real remittances				
	Disturbance	1Q	2Q	4Q	20Q	100Q	1Q	2Q	4Q	20Q	100Q
σ_c^{arm}	Consumption preference (Armenia)	8.35	9.08	9.93	10.68	10.69	41.32	44.15	47.24	52.25	52.33
σ_{prem}^{arm}	Risk premium (Armenia)	11.65	11.67	11.67	11.58	11.58	12.30	10.56	8.47	6.41	6.40
σ_r^{arm}	Monetary policy (Armenia)	11.45	10.78	10.07	9.65	9.64	2.96	2.19	1.50	1.05	1.04
σ_a^{arm}	Productivity (Armenia)	42.98	43.54	43.35	42.32	42.31	6.56	5.73	4.59	3.41	3.40
$\sigma_{\pi_H}^{arm}$	Price mark-up of domestically produced goods (Armenia)	0.07	0.23	0.64	0.76	0.76	0.24	0.18	0.16	0.25	0.25
$\sigma_{\pi_F}^{arm}$	Price mark-up of imported goods from Russia	0.00	0.01	0.03	0.03	0.03	0.02	0.01	0.01	0.01	0.01
$\sigma_{\pi_W}^{arm}$	Wage mark-up (Armenia)	0.02	0.02	0.08	0.24	0.24	0.12	0.30	0.59	0.81	0.81
σ_{politi}	Political factor	0.00	0.00	0.00	0.00	0.00	17.63	15.83	13.20	10.13	10.10
σ_y^{arm}	Demand (Armenia)	12.77	11.44	10.29	9.79	9.78	8.31	6.93	5.25	3.77	3.76
σ_c^{rus}	Consumption preference (Russia)	2.82	3.07	3.42	4.01	4.04	0.52	0.33	0.27	1.09	1.15
σ_{prem}^{rus}	Risk premium (Russia)	8.00	8.31	8.73	9.19	9.19	0.50	0.68	0.73	0.70	0.70
σ_r^{rus}	Monetary policy (Russia)	0.00	0.00	0.00	0.00	0.00	1.35	0.89	0.58	0.40	0.40
σ_N^{rus}	Labor supply (Russia)	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.06	0.06
$\sigma_{a_T}^{rus}$	Productivity in tradable sector (Russia)	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.02	0.03	0.03
$\sigma_{a_{NT}}^{rus}$	Productivity in non-tradable sector (Russia)	0.00	0.00	0.00	0.00	0.00	0.07	0.16	0.25	0.28	0.28
$\sigma_{\pi_H}^{rus}$	Price mark-up of domestically produced tradable goods (Russia)	0.00	0.00	0.00	0.00	0.00	2.87	4.04	6.42	8.42	8.40
$\sigma_{\pi_{NT}}^{rus}$	Price mark-up of domestically produced non-tradable goods (Russia)	0.00	0.00	0.00	0.00	0.00	0.17	0.28	0.60	1.24	1.24
$\sigma_{\pi_F}^{rus}$	Price mark-up of imported goods (Russia)	0.00	0.00	0.00	0.00	0.00	1.49	1.85	2.02	1.60	1.60
$\sigma_{\pi_W}^{rus}$	Wage mark-up (Russia)	0.00	0.00	0.00	0.00	0.00	3.47	5.76	7.93	7.95	7.92
σ_y^*	Foreign demand	0.08	0.08	0.08	0.09	0.09	0.01	0.02	0.04	0.07	0.07
σ_π^*	Foreign inflation	0.62	0.62	0.62	0.61	0.61	0.03	0.04	0.04	0.03	0.03
σ_r^*	Foreign interest rate	1.21	1.15	1.08	1.04	1.04	0.03	0.05	0.05	0.03	0.03

Table 7: Conditional variance decomposition: Armenian and Russian real wages

		Real wage (Armenia)					Real wage (Russia)				
	Disturbance	1Q	2Q	4Q	20Q	100Q	1Q	2Q	4Q	20Q	100Q
σ_c^{arm}	Consumption preference (Armenia)	5.14	5.40	5.91	7.70	7.73	0.00	0.00	0.00	0.00	0.00
σ_{prem}^{arm}	Risk premium (Armenia)	0.88	1.02	1.28	1.83	1.83	0.00	0.00	0.00	0.00	0.00
σ_r^{arm}	Monetary policy (Armenia)	0.95	0.96	0.96	0.90	0.90	0.00	0.00	0.00	0.00	0.00
σ_a^{arm}	Productivity (Armenia)	0.89	0.85	0.78	0.67	0.67	0.00	0.00	0.00	0.00	0.00
$\sigma_{\pi_H}^{arm}$	Price mark-up of domestically produced goods (Armenia)	37.57	38.01	38.43	37.45	37.36	0.00	0.00	0.00	0.00	0.00
$\sigma_{\pi_F}^{arm}$	Price mark-up of imported goods from Russia	3.06	2.77	2.36	1.77	1.76	0.00	0.00	0.00	0.00	0.00
$\sigma_{\pi_W}^{arm}$	Wage mark-up (Armenia)	48.55	47.86	46.64	43.38	43.27	0.00	0.00	0.00	0.00	0.00
σ_{politi}	Political factor	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
σ_y^{arm}	Demand (Armenia)	0.27	0.19	0.11	0.06	0.06	0.00	0.00	0.00	0.00	0.00
σ_c^{rus}	Consumption preference (Russia)	1.21	1.35	1.66	3.21	3.36	8.56	11.93	18.21	36.72	37.74
σ_{prem}^{rus}	Risk premium (Russia)	1.24	1.36	1.64	2.76	2.79	0.29	0.32	0.33	0.25	0.25
σ_r^{rus}	Monetary policy (Russia)	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
σ_N^{rus}	Labor supply (Russia)	0.00	0.00	0.00	0.00	0.00	0.09	0.11	0.14	0.15	0.15
$\sigma_{a_T}^{rus}$	Productivity in tradable sector (Russia)	0.00	0.00	0.00	0.00	0.00	0.17	0.19	0.22	0.18	0.18
$\sigma_{a_{NT}}^{rus}$	Productivity in non-tradable sector (Russia)	0.00	0.00	0.00	0.00	0.00	0.06	0.07	0.07	0.07	0.08
$\sigma_{\pi_H}^{rus}$	Price mark-up of domestically produced tradable goods (Russia)	0.00	0.00	0.00	0.00	0.00	19.13	22.90	26.71	23.94	23.55
$\sigma_{\pi_{NT}}^{rus}$	Price mark-up of domestically produced non-tradable goods (Russia)	0.00	0.00	0.00	0.00	0.00	2.25	2.98	4.09	5.02	4.94
$\sigma_{\pi_F}^{rus}$	Price mark-up of imported goods (Russia)	0.01	0.01	0.01	0.00	0.00	41.33	31.73	21.04	11.69	11.50
$\sigma_{\pi_W}^{rus}$	Wage mark-up (Russia)	0.00	0.00	0.00	0.00	0.00	27.86	29.45	28.82	21.56	21.21
σ_y^*	Foreign demand	0.02	0.02	0.02	0.04	0.04	0.12	0.16	0.23	0.34	0.33
σ_π^*	Foreign inflation	0.08	0.08	0.09	0.11	0.11	0.03	0.04	0.03	0.02	0.02
σ_r^*	Foreign interest rate	0.12	0.11	0.11	0.10	0.10	0.11	0.11	0.10	0.06	0.06

Table 8: Conditional variance decomposition: Armenian and Russian nominal wages

		Nominal wage (Armenia)					Nominal wage (Russia)				
	Disturbance	1Q	2Q	4Q	20Q	100Q	1Q	2Q	4Q	20Q	100Q
σ_c^{arm}	Consumption preference (Armenia)	4.31	4.63	4.88	4.70	4.70	0.00	0.00	0.00	0.00	0.00
σ_{prem}^{arm}	Risk premium (Armenia)	5.32	5.44	5.45	5.12	5.12	0.00	0.00	0.00	0.00	0.00
σ_r^{arm}	Monetary policy (Armenia)	3.11	2.87	2.65	2.51	2.51	0.00	0.00	0.00	0.00	0.00
σ_a^{arm}	Productivity (Armenia)	6.98	7.11	7.09	6.64	6.64	0.00	0.00	0.00	0.00	0.00
$\sigma_{\pi_H}^{arm}$	Price mark-up of domestically produced goods (Armenia)	3.85	5.14	7.80	11.50	11.50	0.00	0.00	0.00	0.00	0.00
$\sigma_{\pi_F}^{arm}$	Price mark-up of imported goods from Russia	0.18	0.24	0.34	0.43	0.43	0.00	0.00	0.00	0.00	0.00
$\sigma_{\pi_W}^{arm}$	Wage mark-up (Armenia)	67.74	65.27	61.49	58.48	58.47	0.00	0.00	0.00	0.00	0.00
σ_{politi}	Political factor	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
σ_y^{arm}	Demand (Armenia)	0.19	0.15	0.15	0.16	0.16	0.00	0.00	0.00	0.00	0.00
σ_c^{rus}	Consumption preference (Russia)	2.46	2.84	3.33	3.77	3.80	3.31	3.70	3.79	3.39	3.39
σ_{prem}^{rus}	Risk premium (Russia)	5.18	5.66	6.15	6.06	6.06	8.37	10.24	12.65	14.65	14.65
σ_r^{rus}	Monetary policy (Russia)	0.00	0.00	0.00	0.00	0.00	0.41	0.36	0.31	0.28	0.28
σ_N^{rus}	Labor supply (Russia)	0.00	0.00	0.00	0.00	0.00	0.29	0.31	0.31	0.28	0.28
$\sigma_{a_T}^{rus}$	Productivity in tradable sector (Russia)	0.00	0.00	0.00	0.00	0.00	1.73	2.15	2.65	2.88	2.88
$\sigma_{a_{NT}}^{rus}$	Productivity in non-tradable sector (Russia)	0.00	0.00	0.00	0.00	0.00	4.88	6.30	8.26	11.05	11.12
$\sigma_{\pi_H}^{rus}$	Price mark-up of domestically produced tradable goods (Russia)	0.00	0.00	0.00	0.00	0.00	1.96	2.86	4.75	6.84	6.83
$\sigma_{\pi_{NT}}^{rus}$	Price mark-up of domestically produced non-tradable goods (Russia)	0.00	0.00	0.00	0.00	0.00	0.19	0.30	0.58	1.17	1.17
$\sigma_{\pi_F}^{rus}$	Price mark-up of imported goods (Russia)	0.01	0.01	0.01	0.00	0.00	1.99	2.70	3.38	3.25	3.24
$\sigma_{\pi_W}^{rus}$	Wage mark-up (Russia)	0.00	0.00	0.00	0.00	0.00	75.07	69.10	61.21	54.21	54.16
σ_y^*	Foreign demand	0.05	0.05	0.06	0.06	0.06	0.05	0.05	0.05	0.04	0.04
σ_π^*	Foreign inflation	0.27	0.28	0.28	0.26	0.26	0.60	0.69	0.78	0.78	0.78
σ_r^*	Foreign interest rate	0.36	0.34	0.32	0.30	0.30	1.15	1.24	1.28	1.18	1.18

Table 9: Conditional variance decomposition: Employment in Russian tradable and non-tradable sectors

		Tradable sector employment					Non-tradable sector employment				
	Disturbance	1Q	2Q	4Q	20Q	100Q	1Q	2Q	4Q	20Q	100Q
σ_c^{arm}	Consumption preference (Armenia)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
σ_{prem}^{arm}	Risk premium (Armenia)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
σ_r^{arm}	Monetary policy (Armenia)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
σ_a^{arm}	Productivity (Armenia)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma_{\pi_H}^{arm}$	Price mark-up of domestically produced goods (Armenia)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma_{\pi_F}^{arm}$	Price mark-up of imported goods from Russia	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma_{\pi_W}^{arm}$	Wage mark-up (Armenia)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
σ_{politi}	Political factor	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
σ_y^{arm}	Demand (Armenia)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
σ_c^{rus}	Consumption preference (Russia)	2.70	3.29	4.56	10.09	10.46	34.34	36.75	36.88	35.44	35.58
σ_{prem}^{rus}	Risk premium (Russia)	16.80	11.83	8.27	6.71	6.68	17.24	14.92	12.72	11.55	11.53
σ_r^{rus}	Monetary policy (Russia)	3.43	1.97	1.14	0.80	0.79	6.62	4.88	3.45	2.48	2.47
σ_N^{rus}	Labor supply (Russia)	0.00	0.01	0.01	0.02	0.02	0.00	0.00	0.01	0.01	0.01
$\sigma_{a_T}^{rus}$	Productivity in tradable sector (Russia)	36.47	26.68	18.44	13.86	13.80	0.08	0.08	0.15	0.40	0.40
$\sigma_{a_{NT}}^{rus}$	Productivity in non-tradable sector (Russia)	0.03	0.08	0.11	0.10	0.10	29.43	29.96	29.07	27.42	27.40
$\sigma_{\pi_H}^{rus}$	Price mark-up of domestically produced tradable goods (Russia)	28.99	46.83	59.56	61.08	60.83	3.87	4.94	8.58	14.29	14.23
$\sigma_{\pi_{NT}}^{rus}$	Price mark-up of domestically produced non-tradable goods (Russia)	0.14	0.18	0.35	0.81	0.81	0.64	1.50	2.35	2.36	2.35
$\sigma_{\pi_F}^{rus}$	Price mark-up of imported goods (Russia)	1.56	1.75	1.83	1.50	1.50	0.10	0.34	1.19	1.57	1.56
$\sigma_{\pi_W}^{rus}$	Wage mark-up (Russia)	0.39	0.87	1.45	1.87	1.86	0.07	0.28	0.52	0.56	0.56
σ_y^*	Foreign demand	1.14	0.87	0.63	0.50	0.50	0.21	0.19	0.16	0.12	0.12
σ_π^*	Foreign inflation	1.96	1.36	0.92	0.69	0.69	1.89	1.61	1.33	1.09	1.09
σ_r^*	Foreign interest rate	6.39	4.27	2.74	1.97	1.97	5.50	4.54	3.57	2.71	2.70

Appendix 6. Use of the Model in Central Bank of Armenia

Figure 6.1. Forecasts from QPM and the Model of External sector

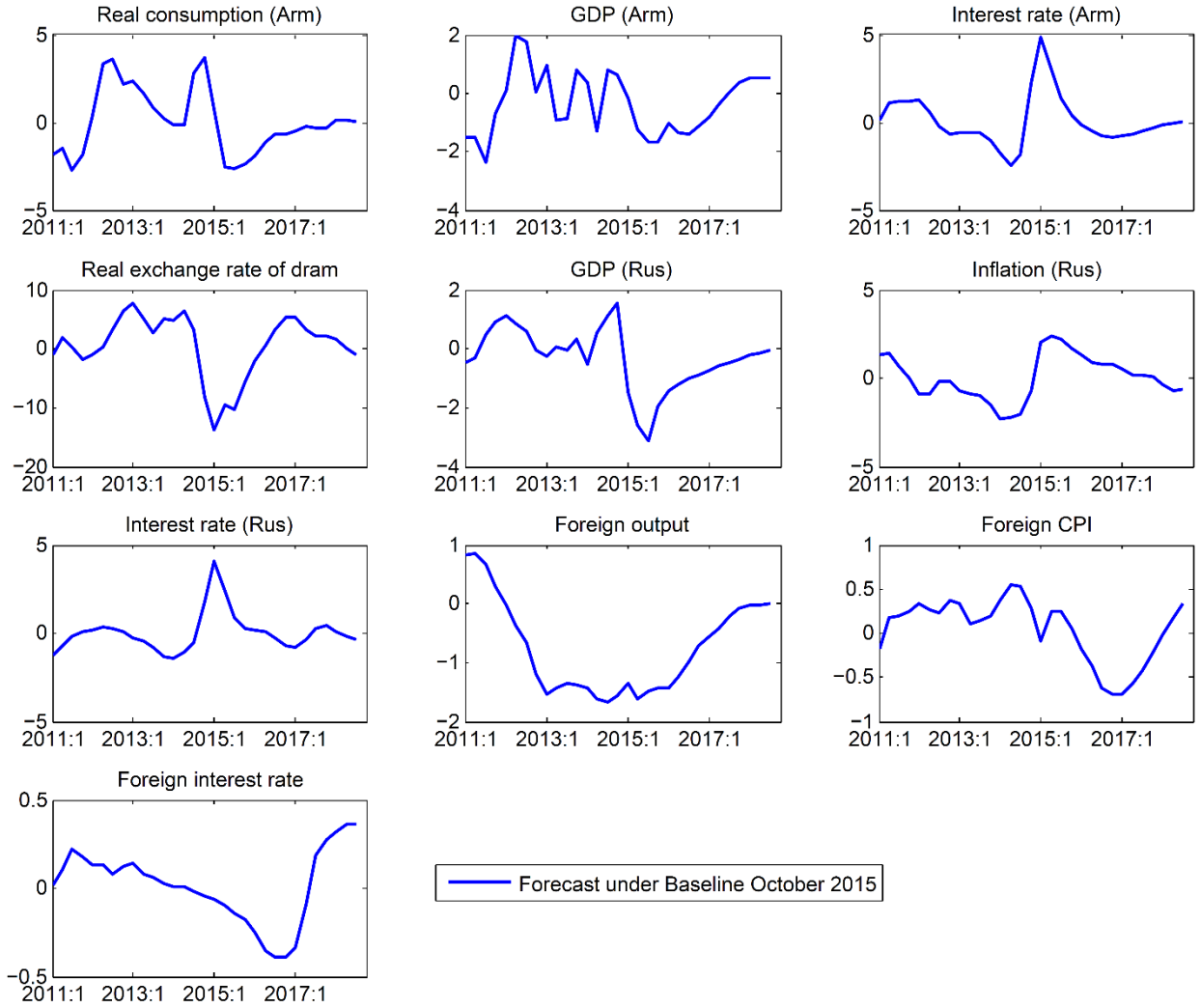
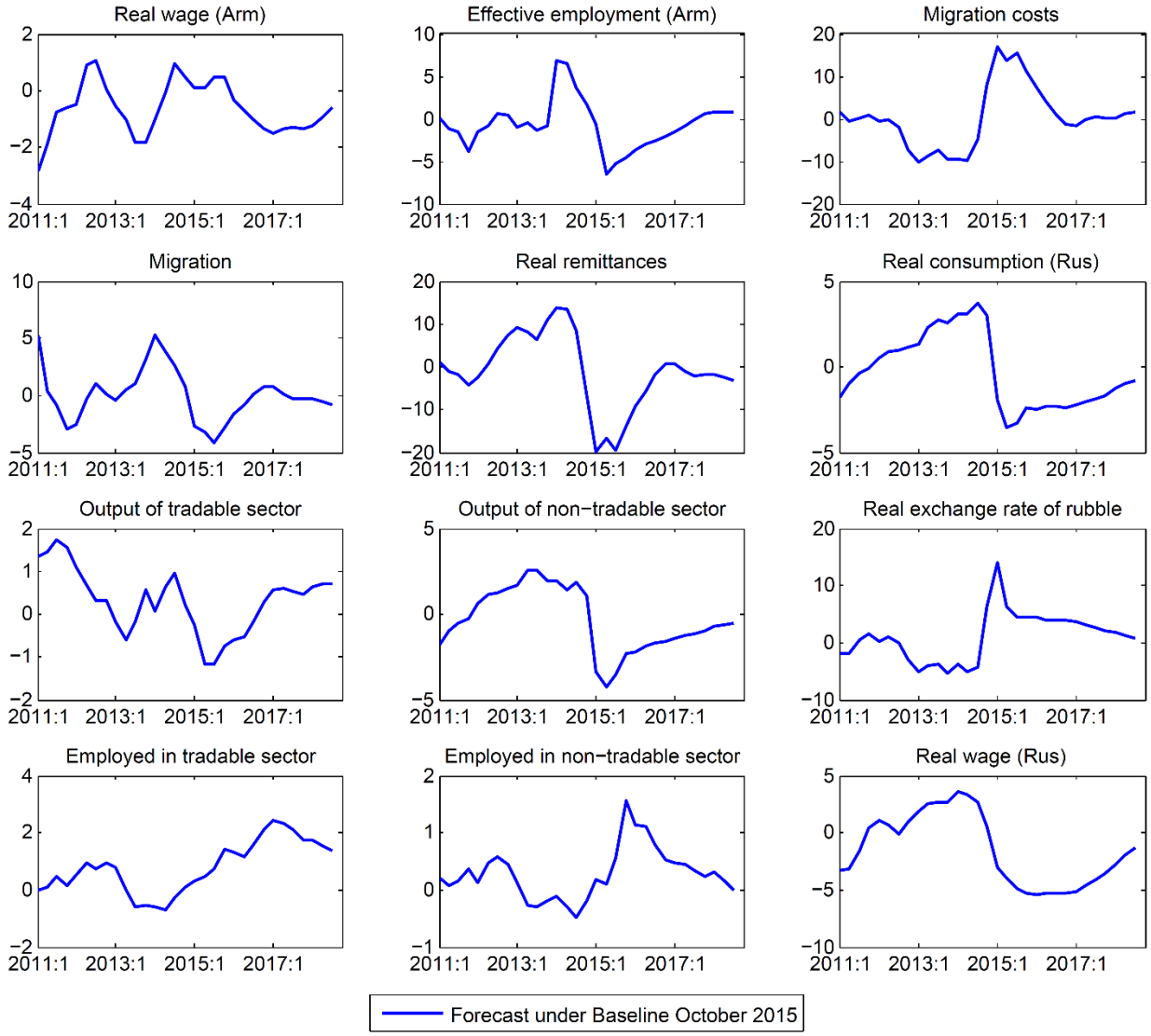


Figure 6.2. Forecasts of DSGE model



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