## Knowledge licensing in a Model of R&D-driven Endogenous Growth

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### Early growth theory

Solow (1956), one of the seminal papers, discusses how physical capital accumulation can affect short- and long-run growth and welfare

Solow uses neoclassical framework/production function

Y = F(K, AL)

where F is homogeneous of degree 1 in capital K and labor L and A is labor augmenting technology.

Solow also uses the standard capital accumulation rule

$$\dot{K} = I - \delta K$$

where  $\delta \in (0,1)$  and I = sY with  $s \in (0,1)$ 

## Microeconomics: Homogeneity of degree 1

Homogeneity of degree 1 in K and L is motivated by standard replication argument given that K and L are rival inputs

- Rival goods: whose use by one prevents the use by other
- Replication argument: in order to double output firm needs to hire twice more K and L

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Homogeneity of degree 1 supports competitive equilibrium; in equilibrium firms hire K and L and make zero profits

• Euler thoerem: 
$$Y = \frac{\partial F}{\partial K}K + \frac{\partial F}{\partial L}L$$

Implication of homogeneity of degree 1

That F is homogenous of degree 1 in K and L implies that the accumulation of K bears decreasing returns

- As capital K increases, the returns to its accumulation decline to zero
- ▶ In long-run, output per capita Y/L is constant if A is fixed
  - This is not in line with the observation that many developed countries grow at relatively constant rates (Kaldor stylized facts)

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• A needs to grow in order to have long run growth

Can A grow endogenously a neoclassical model?

In neoclassical framework F is homogenous of degree 1 in K and LThis implies that

- ► All revenues are spent compensating K and L, and A cannot be compensated
- Therefore, A cannot be accumulated endogenously (by firms/market mechanisms)

Solow (1956) assumes that A grows exogenously

- He acknowledges that this is a significant shortcoming since changes in A involve trade-offs
  - e.g., time and physical resources allocated to research

Early endogenous growth theory - Romer (1986)

Romer (1986) "endogenizes" the accumulation of A assuming that the latter is proportional to the stock of physical capital (per capita)

Romer (1986) assumes that in equilibrium

$$A = \frac{K}{L}$$

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 Microeconomics: there are learning-by-doing externalities; workers learn and become more productive as they interact with capital Early endogenous growth theory - Lucas (1988)

Lucas (1988) interprets AL as human capital H

- "endogenizes" the accumulation of A assuming that the household does it through schooling
- household allocates part of human capital to production (u<sub>Y</sub>H) and part to schooling (u<sub>H</sub>H)

$$Y = F(K, u_Y H)$$
$$\dot{H} = \lambda u_H H$$

$$u_Y + u_H \leqslant 1$$

## Issues in Romer (1986) and Lucas (1988)

- In Romer (1986) changes in A are semi-endogenous/not driven by (rational) decisions of agents
- In Lucas (1988) it is hard to motivate the linear structure of schooling function
  - human capital is a rival input; constant returns are hard to justify

# Romer (1990)

Romer (1990) assumes that private firms' intentional investments in R&D are the driver of long-run growth and welfare

- R&D generates knowledge/ideas that can be used for subsequent innovations
- Knowledge is not rival and is partly non-excludable
  - Non-excludable: there are knowledge spillovers and R&D builds on a pool of knowledge
  - Excludable: firms can use it in order to secure (at least temporary) monopoly position in product market

Final goods sector

$$Y = (u_Y L)^{1-\sigma} \int_0^A x^{\sigma}(i) di$$

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where x are intermediate goods

Intermediate/capital goods sector

- intermediate goods producers are price setters
- production of 1 unit of an intermediate good x requires 1 unit of final goods
- intermediate goods producers are infinitely lived
- they maximize their discounted sum of profits (value)

$$\max V = \int_{t}^{+\infty} \pi(\tau) \exp \left[-\int_{0}^{\tau} r(s) ds\right] d\tau$$

R&D sector

- "researchers produce blueprints of intermediate goods
- research builds on previous knowledge (which is non-rival good! here it is also non-excludable in R&D)

$$\dot{A} = \lambda A(u_A L)$$

There is free entry into intermediate goods industry

- in order to enter the industry an entrepreneur needs to buy a blueprint
- it borrows the resources for that investment from household at the market interest rate r

## Motivation - Non-rivalry and partly excludability

Rivalry of a good is purely technological attribute

Knowledge is non-rival good because its use by a firm or a person does not preclude its use by another

Excludability depends also on the legal framework and detection mechanisms

## Motivation - Interpretation of the assumption

Partly non-excludability: patenting and patent enforcement frameworks and mechanisms for detection of patent infringements are imperfect

- They are weak since firms or researchers can avoid citing or paying license fees for current patents while generating new patents
- They are strong to the extent that firms can maintain exclusive rights on their type of good that is part of the patents

## Motivation - Patenting in high-tech industries

In high-tech industries (e.g., ISIC 32) patenting and patent enforcement frameworks and mechanisms for detection of patent infringements seem to be not so imperfect

- In these industries citing, licensing, and establishing consortiums for exchanging patents is common and has played and currently plays significant role for innovation
  - Grindley & Teece (1997), Hagedoorn (1993, 2002), Shapiro (2001), Clark, Piccolo, Stanton & Tyson (2001)
  - e.g., establishment of RCA Corporation patent consortium in the Radio, Television and Communication Equipment industry

## Motivation - High-tech industries' contribution to growth

High-tech industries are the top private R&D performers and have significant contribution to economic growth

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Helpman (1998), Jorgenson, Ho & Stiroh (2005)

Jerbashian (2016) models knowledge (patent) licensing between high-tech firms in an endogenous growth framework

- Shows how market concentration, intensity of competition in high-tech industry can matter for innovation in that industry and aggregate performance
- Compares the inference to a setup with knowledge spillovers

### The model

- There are N high-tech firms (N > 1)
- ▶ Firms produce differentiated goods {*x*} and set prices {*p<sub>x</sub>*}
- Each firm can invest in R&D which improves its knowledge on the production process (or the quality of its x)
- $\blacktriangleright$  The knowledge of the production process of a high-tech firm is measured by its productivity  $\lambda$ 
  - Each firm has its knowledge of the production process
- The production function of a high-tech good x is

$$x = \lambda L_x$$

### R&D Processes

- In order to improve its knowledge (increase λ) a high-tech firm needs to hire "researchers" L<sub>r</sub>
- Researchers use the current knowledge of the firm in order to create a better one
  - Process innovation: The firm is able to produce more of x
  - Quality upgrade: The firm is able to produce the same amount of higher quality x

# Knowledge Licensing (S.1)

- S.1: Knowledge licensing
  - In this setup, knowledge can be licensed
  - Intellectual property regulation facilitates excludability of knowledge and grants bargaining power to the licensors
  - If a high-tech firm licenses knowledge from other firms, researchers combine it with the knowledge available in the firm in order to produce new knowledge
  - The knowledge of a firm is the only essential knowledge input in the R&D process of the firm

## Knowledge Licensing (S.1)

The R&D process of a firm  $j, j \in (1, N]$ , is given by

$$\dot{\lambda}_{j} = \xi \left[ \sum_{i=1}^{N} (u_{i,j}\lambda_{i})^{\alpha} \right] \lambda_{j}^{1-\alpha} L_{r_{j}}$$
  
$$\xi > 0, 1 > \alpha > 0,$$

where  $u_{i,j}$  is the share of knowledge of firm  $i(\lambda_i)$  that firm j licenses, and  $u_{j,j} \equiv 1$ .

Details

Knowledge Spillovers (S.2)

S.2: Knowledge spillovers

- Intellectual property regulation does not enforce excludability and firms cannot maintain secrecy
  - Firms obtain others' knowledge for free/There are knowledge spillovers among high-tech firms
  - In a firm the researchers combine the knowledge that spills over from other firms with the knowledge of their own firm for generating new knowledge

Knowledge Spillovers (S.2)

The R&D process is given by

$$\dot{\lambda}_j = \xi \tilde{\Lambda} \lambda_j^{1-\alpha} L_{r_j}$$

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Knowledge Spillovers (S.2)

The R&D process is given by

$$\dot{\lambda}_j = \xi \tilde{\Lambda} \lambda_j^{1-\alpha} L_{r_j}$$

where I assume that in equilibrium

$$\tilde{\Lambda} = \sum_{i=1}^{N} \lambda_i^{\alpha}$$

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### The problem of High-tech Firm *j*

The problem of high-tech firm j is

$$V_{j}(t) = \max_{p_{x_{j}}, L_{r_{j}}, \left\{u_{i,i}, u_{i,j}\right\}_{i=1; (i\neq j)}^{N}} \left\{ \int_{t}^{+\infty} \pi_{j}\left(\hat{t}\right) \exp\left[-\int_{t}^{\hat{t}} r(s) ds\right] d\hat{t} \right\}$$

s.t.

$$\pi_{j} = \boldsymbol{p}_{x_{j}} x_{j} - \boldsymbol{w} \left( \boldsymbol{L}_{x_{j}} + \boldsymbol{L}_{r_{j}} \right) + \left[ \sum_{i=1, i \neq j}^{N} \boldsymbol{p}_{u_{j,i}\lambda_{j}} \left( \boldsymbol{u}_{j,i}\lambda_{j} \right) - \sum_{i=1, i \neq j}^{N} \boldsymbol{p}_{u_{i,j}\lambda_{i}} \left( \boldsymbol{u}_{i,j}\lambda_{i} \right) \right],$$

 $x_j, \dot{\lambda}_j, p_{x_j}$ 

#### The Final Goods Sector

Final goods are homogenous Y

Final goods producers form the demand for high-tech goods

The problem of the representative producer is

$$\pi_{Y} = \max_{\{x_{i}\}_{i=1}^{N}, L_{Y}} \left\{ Y - \sum_{i=1}^{N} p_{x_{i}} x_{i} - w L_{Y} \right\}$$
  
s.t.  
$$Y = X^{\sigma} L_{Y}^{1-\sigma}$$
  
$$X = \left( \sum_{i=1}^{N} x_{i}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$
  
$$1 > \sigma > 0, \varepsilon > 1$$

### Households

There is a continuum of identical and infinitely lived households of mass  $\ensuremath{\mathbf{1}}$ 

Each is endowed with a constant amount of labor L

The representative household's optimal problem is

$$U = \max_{C,L} \left\{ \int_{0}^{+\infty} \frac{C_t^{1-\theta} - 1}{1 - \theta} \exp(-\rho t) dt \right\}$$
  
s.t.  
$$\dot{A} = rA + wL - C$$

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 $\theta, \rho > 0$ 

Firm j's demand for labor for production and R&D are

$$w = \lambda_j \rho_{x_j} \left( 1 - rac{1}{e_j} 
ight)$$
  
 $w = q_{\lambda_j} rac{\dot{\lambda}_j}{L_{r_j}}$ 

where  $e_i$  is the perceived elasticity of substitution Elaborate

Demand for and supply of knowledge, S.1

Firm j's demand for and supply of knowledge are

$$p_{u_{i,j}\lambda_i} = q_{\lambda_j} \xi \alpha \left(\frac{\lambda_j}{u_{i,j}\lambda_i}\right)^{1-\alpha} L_{r_j}, \ \forall i \neq j$$
$$u_{j,i} = 1, \ \forall i \neq j$$

#### Returns on knowledge accumulation, S.1

When there is licensing, high-tech firm's returns on knowledge accumulation are given by

$$\frac{\dot{q}_{\lambda_j}}{q_{\lambda_j}} = r - \left(\frac{e_j^k - 1}{e_j} \frac{p_{x_j}}{q_{\lambda_j}} L_{x_j} + \frac{\partial \dot{\lambda}_j}{\partial \lambda_j} + \sum_{i=1, i \neq j}^{N} \frac{p_{u_j, i\lambda_j} u_{j, i}}{q_{\lambda_j}}\right)$$
$$\frac{\partial \dot{\lambda}_j}{\partial \lambda_j} = \xi L_{r_j} \left[1 + (1 - \alpha) \sum_{i=1, i \neq j}^{N} \left(\frac{u_{i, j} \lambda_i}{\lambda_j}\right)^{\alpha}\right]$$

When there are spillovers, high-tech firm's returns on knowledge accumulation are given by

$$\begin{split} \frac{\dot{q}_{\lambda_j}}{q_{\lambda_j}} &= r - \left( \frac{e_j^k - 1}{e_j} \frac{p_{x_j}}{q_{\lambda_j}} L_{x_j} + \frac{\partial \dot{\lambda}_j}{\partial \lambda_j} \right) \\ \frac{\partial \dot{\lambda}_j}{\partial \lambda_j} &= (1 - \alpha) \frac{\dot{\lambda}_j}{\lambda_j} \end{split}$$

### Growth rates

The growth rates of final output (Y) and productivity ( $\lambda$ ) are

$$g_{Y} = \sigma g_{\lambda}$$
$$g_{\lambda} = \frac{\xi DL - \rho}{(\theta - 1)\sigma + \alpha I_{5.2}^{1} + D}$$

where

$$I_{5.2}^{1} = \begin{cases} 0 \text{ for knowledge licensing (S.1);} \\ 1 \text{ otherwise} \end{cases}$$

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### Growth rates

 ${\cal D}$  summarizes the effect of competitive pressures on innovation and growth:

$$D = \sigma \frac{e - 1}{e - \sigma}$$
$$e = \varepsilon - \frac{\varepsilon - 1}{N}$$

where

$$\frac{\partial D}{\partial N}, \frac{\partial D}{\partial \varepsilon} > 0; \frac{\partial^2 D}{\partial N^2}, \frac{\partial^2 D}{\partial \varepsilon^2} < 0$$
$$\frac{\partial g_{\lambda}}{\partial D} > 0; \frac{\partial^2 g_{\lambda}}{\partial D^2} < 0$$

### Welfare

Total (consumer) welfare can be expressed as

$$\tilde{U} = -\left[N^{\frac{\sigma}{\varepsilon-1}}\left(NL_{x}\right)^{\sigma}\left(L_{Y}\right)^{1-\sigma}\right]^{-(\theta-1)}\frac{1}{(\theta-1)\sigma g_{\lambda}+\rho}.$$

• the term in square brackets increases and  $g_{\lambda}$  declines with  $I_{S,2}^1$ 

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•  $\tilde{U}$  declines with  $I_{S.2}^1$ 

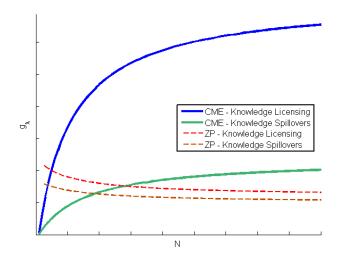
### Firm entry: Cost-free entry

- I assume zero entry (exit) costs. In such a case, firms enter (exit) as long as profits net of R&D expenditures are non-negative (negative)
- Zero value/profit condition determines the number of firms given g<sub>λ</sub>:

$$\pi = 0 \Leftrightarrow$$

$$g_{\lambda} = \frac{\rho}{e - 1 - \alpha I_{5,2-3}^{1} - (\theta - 1)\sigma}$$

## $g_{\lambda}$ and the number of firms



The number of firms is given by the following two expressions

$$e = \varepsilon - \frac{\varepsilon - 1}{N},$$

$$e = \frac{\xi \sigma L \left[ 1 + \alpha I_{S,2}^1 + (\theta - 1) \sigma \right]}{\xi \sigma L - \rho}.$$

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Innovation and growth do not depend on  $\varepsilon$  since so does the right-hand side of the second expression

### Welfare with cost-free entry

Total (consumer) welfare can be expressed as

$$\tilde{U} = -\left[N^{\frac{\sigma}{\varepsilon-1}} \left(NL_{x}\right)^{\sigma} \left(L_{Y}\right)^{1-\sigma}\right]^{-(\theta-1)} \frac{1}{(\theta-1)\sigma g_{\lambda}+\rho}.$$

•  $(NL_x)^{\sigma} (L_Y)^{1-\sigma}$  and  $N^{\frac{\sigma}{\varepsilon-1}}$  (love-for-variety effect) increase with  $I_{5.2}^1$ 

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•  $g_{\lambda}$  declines with  $I_{S.2}^1$ 

• 
$$\tilde{U}^{S.1} < \tilde{U}^{S.2}$$
 for  $\alpha \approx 0$ 

-  $ilde{U}^{S.1} > ilde{U}^{S.2}$  if there is no love-for-variety effect

Thank you!

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Perceived elasticity of substitution

It can be shown that

$$e_{j}^{B}\equiv e_{j}=arepsilon-\left[rac{\left(arepsilon-1
ight)oldsymbol{p}_{\mathbf{X}_{j}}^{1-arepsilon}}{\sum_{i=1}^{N}oldsymbol{p}_{\mathbf{X}_{i}}^{1-arepsilon}}
ight]$$

Back to S.1-3

#### This R&D process can be rewritten as

$$\dot{\lambda}_{j} = \xi \left[ \sum_{i=1, i \neq j}^{N} (u_{i,j}\lambda_{i})^{\alpha} + \lambda_{j}^{\alpha} \right] \lambda_{j}^{1-\alpha} L_{r_{j}}$$

Back to S.1 (2)

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